## Lecture No.4

## **Lecture Outlines**

## 2.1 Designing for Performance

Microprocessor Speed Performance Balance Improvements in Chip Organization and Architecture

## 2.2 Multicore, MICs, and GPGPUs

## 2.3 Two Laws that Provide Insight: Amdahl's Law and Little's Law

Amdahl's Law Little's Law

## 2.4 Basic Measures of Computer Performance

Clock Speed Instruction Execution Rate

## LEARNING OBJECTIVES

After studying this lecture, you should be able to:

- ◆ Understand the key performance issues that relate to computer design.
- ◆ Explain the reasons for the move to multicore organization, and understand the trade-off between cache and processor resources on a single chip.
- ◆ Distinguish among multicore, MIC, and GPGPU organizations.
- ◆ Summarize some of the issues in computer performance assessment.
- ◆ Discuss the SPEC benchmarks.
- ◆ Explain the differences among arithmetic, harmonic, and geometric means.

### 2.1 DESIGNING FOR PERFORMANC

Year by year, the cost of computer systems continues to drop dramatically, while the performance and capacity of those systems continue to rise equally dramatically. Today's laptops have the computing power of an IBM mainframe from 10 or 15 years ago. Thus, we have virtually "free" computer power. Processors are so inexpensive that we now have microprocessors we throw away. The digital pregnancy test is an example (used once and then thrown away). And this continuing technological revolution has enabled the development of applications of astounding complexity and power. For example, desktop applications that require the great power of today's microprocessor-based systems include

- Image processing
- Speech recognition
- Multimedia authoring
- Simulation modeling
- Three-dimensional rendering
- Videoconferencing
- Voice and video annotation of files

Workstation systems now support highly sophisticated engineering and scientific applications and have the capacity to support image and video applications. In addition, businesses are relying on increasingly powerful servers to handle transaction and database processing and to support massive client/server networks that have replaced the huge mainframe computer centers of yesteryear. As well, cloud service providers use massive high-performance banks of servers to satisfy high-volume, high-transaction-rate applications for a broad spectrum of clients.

What is fascinating about all this from the perspective of computer organization and architecture is that, on the one hand, the basic building blocks for today's computer miracles are virtually the same as those of the IAS computer from over 50 years ago, while on the other hand, the techniques for squeezing the maximum performance out of the materials at hand have become increasingly sophisticated.

This observation serves as a guiding principle for the presentation in this book. As we progress through the various elements and components of a computer, two objectives are pursued. First, the book explains the fundamental functionality in each area under consideration, and second, the book explores those techniques required to achieve maximum performance. In the remainder of this section, we highlight some of the driving factors behind the need to design for performance.

## Microprocessor Speed

What gives Intel x86 processors or IBM mainframe computers such mind-boggling power is the relentless pursuit of speed by processor chip manufacturers. The evolution of these machines continues to bear out Moore's law, described in Chapter 1. So long as this law holds, chipmakers can unleash a new generation of chips every three years—with four times as many transistors. In memory chips, this has quadrupled the capacity of **dynamic random-access memory (DRAM)**, still the basic technology for computer main memory, every three years. In microprocessors, the addition of new circuits, and the speed boost that comes from reducing the distances between them, has improved performance four- or fivefold every three years or so since Intel launched its x86 family in 1978.

But the raw speed of the microprocessor will not achieve its potential unless it is fed a constant stream of work to do in the form of computer instructions. Anything that gets in the way of that smooth flow undermines the power of the processor. Accordingly, while the chipmakers have been busy learning how to fabricate chips of greater and greater density, the processor designers must come up with ever more elaborate techniques for feeding the monster. Among the techniques built into contemporary processors are the following:

■ Pipelining: The execution of an instruction involves multiple stages of operation, including fetching the instruction, decoding the opcode, fetching operands, performing a calculation, and so on. Pipelining enables a processor to work simultaneously on multiple instructions by performing a different phase for each of the multiple instructions at the same time. The processor overlaps operations by moving data or instructions into a conceptual pipe with all stages of the pipe processing simultaneously. For example, while one instruction is being executed, the computer is decoding the next instruction. This is the same principle as seen in an assembly line.

- Branch prediction: The processor looks ahead in the instruction code fetched from memory and predicts which branches, or groups of instructions, are likely to be processed next. If the processor guesses right most of the time, it can prefetch the correct instructions and buffer them so that the processor is kept busy. The more sophisticated examples of this strategy predict not just the next branch but multiple branches ahead. Thus, branch prediction potentially increases the amount of work available for the processor to execute.
- **Superscalar execution:** This is the ability to issue more than one instruction in every processor clock cycle. In effect, multiple parallel pipelines are used.
- **Data flow analysis:** The processor analyzes which instructions are dependent on each other's results, or data, to create an optimized schedule of instructions. In fact, instructions are scheduled to be executed when ready, independent of the original program order. This prevents unnecessary delay.
- **Speculative execution:** Using branch prediction and data flow analysis, some processors speculatively execute instructions ahead of their actual appearance in the program execution, holding the results in temporary locations. This enables the processor to keep its execution engines as busy as possible by executing instructions that are likely to be needed.

These and other sophisticated techniques are made necessary by the sheer power of the processor. Collectively they make it possible to execute many instructions per processor cycle, rather than to take many cycles per instruction.

#### **Performance Balance**

While processor power has raced ahead at breakneck speed, other critical components of the computer have not kept up. The result is a need to look for performance balance: an adjustment/tuning of the organization and architecture to compensate for the mismatch among the capabilities of the various components.

The problem created by such mismatches is particularly critical at the interface between processor and main memory. While processor speed has grown rapidly, the speed with which data can be transferred between main memory and the processor has lagged badly. The interface between processor and main memory is the most crucial pathway in the entire computer because it is responsible for carrying a constant flow of program instructions and data between memory chips and the processor. If memory or the pathway fails to keep pace with the processor's insistent demands, the processor stalls in a wait state, and valuable processing time is lost.

A system architect can attack this problem in a number of ways, all of which are reflected in contemporary computer designs. Consider the following examples:

- Increase the number of bits that are retrieved at one time by making DRAMs "wider" rather than "deeper" and by using wide bus data paths.
- Change the DRAM interface to make it more efficient by including a cache or other buffering scheme on the DRAM chip.
- Reduce the frequency of memory access by incorporating increasingly complex and efficient cache structures between the processor and main memory. This includes the incorporation of one or more caches on the processor chip as well as on an off-chip cache close to the processor chip.

■ Increase the interconnect bandwidth between processors and memory by using higher-speed buses and a hierarchy of buses to buffer and structure data flow.

Another area of design focus is the handling of I/O devices. As computers become faster and more capable, more sophisticated applications are developed that support the use of peripherals with intensive I/O demands. Figure 2.1 gives some examples of typical peripheral devices in use on personal computers and workstations. These devices create tremendous data throughput demands. While the current generation of processors can handle the data pumped out by these devices, there remains the problem of getting that data moved between processor and peripheral. Strategies here include caching and buffering schemes plus the use of higher-speed interconnection buses and more elaborate interconnection structures. In addition, the use of multiple-processor configurations can aid in satisfying I/O demands.

The key in all this is balance. Designers constantly strive to balance the throughput and processing demands of the processor components, main memory, I/O devices, and the interconnection structures. This design must constantly be rethought to cope with two constantly evolving factors:

- The rate at which performance is changing in the various technology areas (processor, buses, memory, peripherals) differs greatly from one type of element to another.
- New applications and new peripheral devices constantly change the nature of the demand on the system in terms of typical instruction profile and the data access patterns.

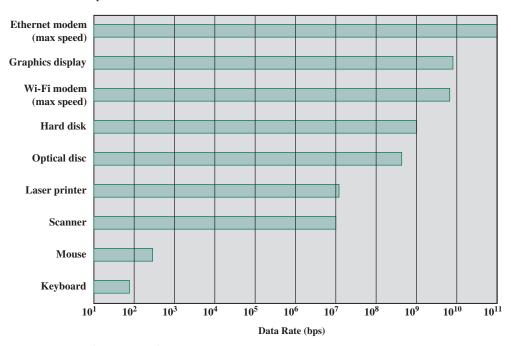


Figure 2.1 Typical I/O Device Data Rates

Thus, computer design is a constantly evolving art form. This book attempts to present the fundamentals on which this art form is based and to present a survey of the current state of that art.

## Improvements in Chip Organization and Architecture

As designers wrestle with the challenge of balancing processor performance with that of main memory and other computer components, the need to increase processor speed remains. There are three approaches to achieving increased processor speed:

- Increase the hardware speed of the processor. This increase is fundamentally due to shrinking the size of the logic gates on the processor chip, so that more gates can be packed together more tightly and to increasing the clock rate. With gates closer together, the propagation time for signals is significantly reduced, enabling a speeding up of the processor. An increase in clock rate means that individual operations are executed more rapidly.
- Increase the size and speed of caches that are interposed between the processor and main memory. In particular, by dedicating a portion of the processor chip itself to the cache, cache access times drop significantly.
- Make changes to the processor organization and architecture that increase the effective speed of instruction execution. Typically, this involves using parallelism in one form or another.

Traditionally, the dominant factor in performance gains has been in increases in clock speed due and logic density. However, as clock speed and logic density increase, a number of obstacles become more significant:

- **Power:** As the density of logic and the clock speed on a chip increase, so does the power density (Watts/cm²). The difficulty of dissipating the heat generated on high-density, high-speed chips is becoming a serious design issue.
- RC delay: The speed at which electrons can flow on a chip between transistors is limited by the resistance and capacitance of the metal wires connecting them; specifically, delay increases as the RC product increases. As components on the chip decrease in size, the wire interconnects become thinner, increasing resistance. Also, the wires are closer together, increasing capacitance.
- **Memory latency and throughput:** Memory access speed (latency) and transfer speed (throughput) lag processor speeds, as previously discussed.

Thus, there will be more emphasis on organization and architectural approaches to improving performance. These techniques are discussed in later chapters of the text.

Beginning in the late 1980s, and continuing for about 15 years, two main strategies have been used to increase performance beyond what can be achieved simply by increasing clock speed. First, there has been an increase in cache capacity. There are now typically two or three levels of cache between the processor and main memory. As chip density has increased, more of the cache memory has been incorporated on the chip, enabling faster cache access. For example, the original Pentium

chip devoted about 10% of on-chip area to a cache. Contemporary chips devote over half of the chip area to caches. And, typically, about three-quarters of the other half is for pipeline-related control and buffering.

Second, the instruction execution logic within a processor has become increasingly complex to enable parallel execution of instructions within the processor. Two noteworthy design approaches have been pipelining and superscalar. A pipeline works much as an assembly line in a manufacturing plant enabling different stages of execution of different instructions to occur at the same time along the pipeline. A superscalar approach in essence allows multiple pipelines within a single processor, so that instructions that do not depend on one another can be executed in parallel.

By the mid to late 90s, both of these approaches were reaching a point of diminishing returns. The internal organization of contemporary processors is exceedingly complex and is able to squeeze a great deal of parallelism out of the instruction stream. It seems likely that further significant increases in this direction will be relatively modest. With three levels of cache on the processor chip, each level providing substantial capacity, it also seems that the benefits from the cache are reaching a limit.

However, simply relying on increasing clock rate for increased performance runs into the power dissipation problem already referred to. The faster the clock rate, the greater the amount of power to be dissipated, and some fundamental physical limits are being reached.

Figure 2.2 illustrates the concepts we have been discussing. The top line shows that, as per Moore's Law, the number of transistors on a single chip continues to

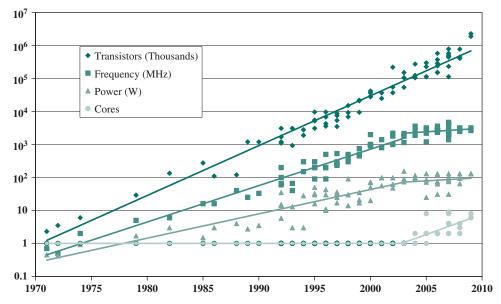


Figure 2.2 Processor Trends

grow exponentially. Meanwhile, the clock speed has leveled off, in order to prevent a further rise in power. To continue increasing performance, designers have had to find ways of exploiting the growing number of transistors other than simply building a more complex processor. The response in recent years has been the development of the multicore computer chip.

## 2.2 MULTICORE, MICS, AND GPGPUS

With all of the difficulties cited in the preceding section in mind, designers have turned to a fundamentally new approach to improving performance: placing multiple processors on the same chip, with a large shared cache. The use of multiple processors on the same chip, also referred to as multiple cores, or **multicore**, provides the potential to increase performance without increasing the clock rate. Studies indicate that, within a processor, the increase in performance is roughly proportional to the square root of the increase in complexity. But if the software can support the effective use of multiple processors, then doubling the number of processors almost doubles performance. Thus, the strategy is to use two simpler processors on the chip rather than one more complex processor.

In addition, with two processors, larger caches are justified. This is important because the power consumption of memory logic on a chip is much less than that of processing logic.

As the logic density on chips continues to rise, the trend for both more cores and more cache on a single chip continues. Two-core chips were quickly followed by four-core chips, then 8, then 16, and so on. As the caches became larger, it made performance sense to create two and then three levels of cache on a chip, with initially, the first-level cache dedicated to an individual processor and levels two and three being shared by all the processors. It is now common for the second-level cache to also be private to each core.

Chip manufacturers are now in the process of making a huge leap forward in the number of cores per chip, with more than 50 cores per chip. The leap in performance as well as the challenges in developing software to exploit such a large number of cores has led to the introduction of a new term: **many integrated core (MIC)**.

The multicore and MIC strategy involves a homogeneous collection of general-purpose processors on a single chip. At the same time, chip manufacturers are pursuing another design option: a chip with multiple general-purpose processors plus **graphics processing units (GPUs)** and specialized cores for video processing and other tasks. In broad terms, a GPU is a core designed to perform parallel operations on graphics data. Traditionally found on a plug-in graphics card (display adapter), it is used to encode and render 2D and 3D graphics as well as process video.

Since GPUs perform parallel operations on multiple sets of data, they are increasingly being used as vector processors for a variety of applications that require repetitive computations. This blurs the line between the GPU and the CPU.

When a broad range of applications are supported by such a processor, the term **general-purpose computing on GPUs (GPGPU)** is used.

# 2.3 TWO LAWS THAT PROVIDE INSIGHT: AHMDAHL'S LAW AND LITTLE'S LAW

In this section, we look at two equations, called "laws." The two laws are unrelated but both provide insight into the performance of parallel systems and multicore systems.

#### Amdahl's Law

Computer system designers look for ways to improve system performance by advances in technology or change in design. Examples include the use of parallel processors, the use of a memory cache hierarchy, and speedup in memory access time and I/O transfer rate due to technology improvements. In all of these cases, it is important to note that a speedup in one aspect of the technology or design does not result in a corresponding improvement in performance. This limitation is succinctly expressed by Amdahl's law.

Amdahl's law was first proposed by Gene Amdahl in 1967 and deals with the potential speedup of a program using multiple pro-cessors compared to a single processor. Consider a program running on a single processor such that a fraction (1-f) of the execution time involves code that is inherently sequential, and a fraction f that involves code that is infinitely paralleliz-able with no scheduling overhead. Let T be the total execution time of the program using a single processor. Then the speedup using a parallel processor with N pro-cessors that fully exploits the parallel portion of the program is as follows:

Speedup = 
$$\frac{\text{Time to execute program on a single processor}}{\text{Time to execute program on } N \text{ parallel processors}}$$
$$= \frac{T(1-f) + Tf}{T(1-f) + \frac{Tf}{N}} = \frac{1}{(1-f) + \frac{f}{N}}$$

This equation is illustrated in Figures 2.3 and 2.4. Two important conclusions can be drawn:

- 1. When f is small, the use of parallel processors has little effect.
- **2.** As N approaches infinity, speedup is bound by 1/(1 f), so that there are diminishing returns for using more processors.

These conclusions are too pessimistic, an assertion first put forward in [GUST88]. For example, a server can maintain multiple threads or multiple tasks to handle multiple clients and execute the threads or tasks in parallel up to the limit of the number of processors. Many database applications involve computations on massive amounts of data that can be split up into multiple parallel tasks.

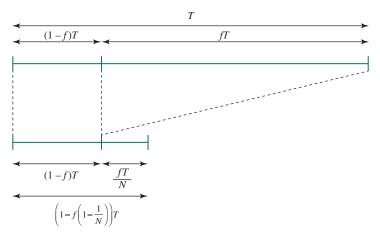


Figure 2.3 Illustration of Amdahl's Law

Nevertheless, Amdahl's law illustrates the problems facing industry in the development of multicore machines with an ever-growing number of cores: The software that runs on such machines must be adapted to a highly parallel execution environment to exploit the power of parallel processing.

Amdahl's law can be generalized to evaluate any design or technical improvement in a computer system. Consider any enhancement to a feature of a system that results in a speedup. The speedup can be expressed as

$$Speedup = \frac{Performance after enhancement}{Performance before enhancement} = \frac{Execution time before enhancement}{Execution time after enhancement}$$
(2.1)

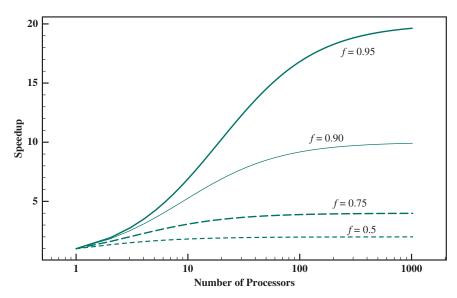


Figure 2.4 Amdahl's Law for Multiprocessors

Suppose that a feature of the system is used during execution a fraction of the time f, before enhancement, and that the speedup of that feature after enhancement is  $SU_f$ . Then the overall speedup of the system is

Speedup = 
$$\frac{1}{(1-f) + \frac{f}{SU_f}}$$

**EXAMPLE 2.1** Suppose that a task makes extensive use of floating-point operations, with 40% of the time consumed by floating-point operations. With a new hardware design, the floating-point module is sped up by a factor of K. Then the overall speedup is as follows:

Speedup = 
$$\frac{1}{0.6 + \frac{0.4}{K}}$$

Thus, independent of K, the maximum speedup is 1.67.

#### Little's Law

A fundamental and simple relation with broad applications is Little's Law. We can apply it to almost any system that is statistically in steady state, and in which there is no leakage. Specifically, we have a steady state system to which items arrive at an average rate of  $\lambda$  items per unit time. The items stay in the system an average of W units of time. Finally, there is an average of L items in the system at any one time. Little's Law relates these three variables as  $L = \lambda W$ .

Using queuing theory terminology, Little's Law applies to a queuing system. The central element of the system is a server, which provides some service to items. Items from some population of items arrive at the system to be served. If the server is idle, an item is served immediately. Otherwise, an arriving item joins a waiting line, or queue. There can be a single queue for a single server, a single queue for multiple servers, or multiples queues, one for each of multiple servers. When a server has completed serving an item, the item departs. If there are items waiting in the queue, one is immediately dispatched to the server. The server in this model can represent anything that performs some function or service for a collection of items. Examples: A processor provides service to processes; a transmission line provides a transmission service to packets or frames of data; and an I/O device provides a read or write service for I/O requests.

To understand Little's formula, consider the following argument, which focuses on the experience of a single item. When the item arrives, it will find on

average L items ahead of it, one being serviced and the rest in the queue. When the item leaves the system after being serviced, it will leave behind on average the same number of items in the system, namely L, because L is defined as the average number of items waiting. Further, the average time that the item was in the system was W. Since items arrive at a rate of  $\lambda$ , we can reason that in the time W, a total of  $\lambda W$  items must have arrived. Thus  $L = \lambda W$ .

To summarize, under steady state conditions, the average number of items in a queuing system equals the average rate at which items arrive multiplied by the average time that an item spends in the system. This relationship requires very few assumptions. We do not need to know what the service time distribution is, what the distribution of arrival times is, or the order or priority in which items are served. Because of its simplicity and generality, Little's Law is extremely useful and has experienced somewhat of a revival due to the interest in performance problems related to multicore computers.

A very simple example, illustrates how Little's Law might be applied. Consider a multicore system, with each core supporting multiple threads of execution. At some level, the cores share a common memory. The cores share a common main memory and typically share a common cache memory as well. In any case, when a thread is executing, it may arrive at a point at which it must retrieve a piece of data from the common memory. The thread stops and sends out a request for that data. All such stopped threads are in a queue. If the system is being used as a server, an analyst can determine the demand on the system in terms of the rate of user requests, and then translate that into the rate of requests for data from the threads generated to respond to an individual user request. For this purpose, each user request is broken down into subtasks that are implemented as threads. We then have  $\lambda$  = the average rate of total thread processing required after all members' requests have been broken down into whatever detailed subtasks are required. Define L as the average number of stopped threads waiting during some relevant time. Then W = average response time. This simple model can serve as a guide to designers as to whether user requirements are being met and, if not, provide a quantitative measure of the amount of improvement needed.

## 2.4 BASIC MEASURES OF COMPUTER PERFORMANCE

In evaluating processor hardware and setting requirements for new systems, performance is one of the key parameters to consider, along with cost, size, security, reliability, and, in some cases, power consumption.

It is difficult to make meaningful performance comparisons among different processors, even among processors in the same family. Raw speed is far less important than how a processor performs when executing a given application. Unfortunately, application performance depends not just on the raw speed of the processor but also on the instruction set, choice of implementation language, efficiency of the compiler, and skill of the programming done to implement the application.

In this section, we look at some traditional measures of processor speed. The following section discusses how to average results from multiple tests.

## Clock Speed

Operations performed by a processor, such as fetching an instruction, decoding the instruction, performing an arithmetic operation, and so on, are governed by a system clock. Typically, all operations begin with the pulse of the clock. Thus, at the most fundamental level, the speed of a processor is dictated by the pulse frequency produced by the clock, measured in cycles per second, or Hertz (Hz).

Typically, clock signals are generated by a quartz crystal, which generates a constant sine wave while power is applied. This wave is converted into a digital voltage pulse stream that is provided in a constant flow to the processor circuitry (Figure 2.5). For example, a 1-GHz processor receives 1 billion pulses per second. The rate of pulses is known as the **clock rate**, or **clock speed**. One increment, or pulse, of the clock is referred to as a **clock cycle**, or a **clock tick**. The time between pulses is the **cycle time**.

The clock rate is not arbitrary, but must be appropriate for the physical layout of the processor. Actions in the processor require signals to be sent from one processor element to another. When a signal is placed on a line inside the processor, it takes some finite amount of time for the voltage levels to settle down so that an accurate value (logical 1 or 0) is available. Furthermore, depending on the physical layout of the processor circuits, some signals may change more rapidly than others. Thus, operations must be synchronized and paced so that the proper electrical signal (voltage) values are available for each operation.

The execution of an instruction involves a number of discrete steps, such as fetching the instruction from memory, decoding the various portions of the instruction, loading and storing data, and performing arithmetic and logical operations. Thus, most instructions on most processors require multiple clock cycles to complete. Some instructions may take only a few cycles, while others require dozens. In addition, when pipelining is used, multiple instructions are being executed simultaneously. Thus, a straight comparison of clock speeds on different processors does not tell the whole story about performance.

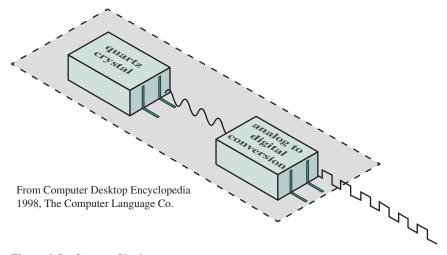


Figure 2.5 System Clock

#### Instruction Execution Rate

A processor is driven by a clock with a constant frequency f or, equivalently, a constant cycle time  $\tau$ , where  $\tau=1/f$ . Define the instruction count,  $I_c$ , for a program as the number of machine instructions executed for that program until it runs to completion or for some defined time interval. Note that this is the number of instruction executions, not the number of instructions in the object code of the program. An important parameter is the average cycles per instruction (CPI) for a program. If all instructions required the same number of clock cycles, then CPI would be a constant value for a processor. However, on any given processor, the number of clock cycles required varies for different types of instructions, such as load, store, branch, and so on. Let  $CPI_i$  be the number of cycles required for instruction type i, and  $I_i$  be the number of executed instructions of type i for a given program. Then we can calculate an overall CPI as follows:

$$CPI = \frac{\sum_{i=1}^{n} (CPI_i \times I_i)}{I_c}$$
 (2.2)

The processor time T needed to execute a given program can be expressed as

$$T = I_c \times CPI \times \tau$$

We can refine this formulation by recognizing that during the execution of an instruction, part of the work is done by the processor, and part of the time a word is being transferred to or from memory. In this latter case, the time to transfer depends on the memory cycle time, which may be greater than the processor cycle time. We can rewrite the preceding equation as

$$T = I_c \times [p + (m \times k)] \times \tau$$

where p is the number of processor cycles needed to decode and execute the instruction, m is the number of memory references needed, and k is the ratio between memory cycle time and processor cycle time. The five performance factors in the preceding equation  $(I_c, p, m, k, \tau)$  are influenced by four system attributes: the design of the instruction set (known as *instruction set architecture*); compiler technology (how effective the compiler is in producing an efficient machine language program from a high-level language program); processor implementation; and cache and memory hierarchy. Table 2.1 is a matrix in which one dimension shows the five performance factors and the other dimension shows the four system attributes. An X in a cell indicates a system attribute that affects a performance factor.

Table 2.1 P	erformance I	Factors a	and Sy	stem A	Attributes
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	$I_c$	р	m	k	au
Instruction set architecture	X	X			
Compiler technology	X	X	X		
Processor implementation		X			X
Cache and memory hierarchy				X	X

A common measure of performance for a processor is the rate at which instructions are executed, expressed as millions of instructions per second (MIPS), referred to as the **MIPS rate**. We can express the MIPS rate in terms of the clock rate and *CPI* as follows:

MIPS rate 
$$=\frac{I_c}{T \times 10^6} = \frac{f}{CPI \times 10^6}$$
 (2.3)

**EXAMPLE 2.2** Consider the execution of a program that results in the execution of 2 million instructions on a 400-MHz processor. The program consists of four major types of instructions. The instruction mix and the *CPI* for each instruction type are given below, based on the result of a program trace experiment:

Instruction Type	CPI	<b>Instruction Mix (%)</b>
Arithmetic and logic	1	60
Load/store with cache hit	2	18
Branch	4	12
Memory reference with cache miss	8	10

The average *CPI* when the program is executed on a uniprocessor with the above trace results is  $CPI = 0.6 + (2 \times 0.18) + (4 \times 0.12) + (8 \times 0.1) = 2.24$ . The corresponding MIPS rate is  $(400 \times 10^6)/(2.24 \times 10^6) \approx 178$ .

Another common performance measure deals only with floating-point instructions. These are common in many scientific and game applications. Floating-point performance is expressed as millions of floating-point operations per second (MFLOPS), defined as follows:

MFLOPS rate = 
$$\frac{\text{Number of executed floating - point operations in a program}}{\text{Execution time } \times 10^{6}}$$