Lecture No.12

LECTURE OUTLINE

6–6 Encoders

An **encoder** is a combinational logic circuit that essentially performs a "reverse" decoder function. An encoder accepts an active level on one of its inputs representing a digit, such as a decimal or octal digit, and converts it to a coded output, such as BCD or binary. Encoders can also be devised to encode various symbols and alphabetic characters. The process of converting from familiar symbols or numbers to a coded format is called *encoding*.

The Decimal-to-BCD Encoder

This type of encoder has ten inputs—one for each decimal digit—and four outputs corresponding to the BCD code, as shown in Figure 6–36. This is a basic 10-line-to-4-line encoder.

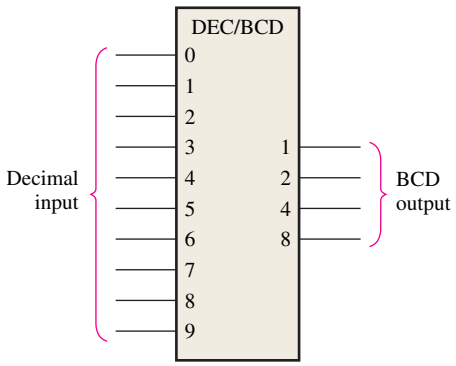


FIGURE 6–36 Logic symbol for a decimal-to-BCD encoder.

The BCD (8421) code is listed in Table 6–6. From this table you can determine the relationship between each BCD bit and the decimal digits in order to analyze the logic. For instance, the most significant bit of the BCD code, A_3, is always a 1 for decimal digit 8 or 9. An OR expression for bit A_3 in terms of the decimal digits can therefore be written as

$$A_3 = 8 + 9$$

TABLE 6–6

Decimal Digit	BCD Code			
	A_3	A_2	A_1	A_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

Bit A_2 is always a 1 for decimal digit 4, 5, 6 or 7 and can be expressed as an OR function as follows:

$$A_2 = 4 + 5 + 6 + 7$$

Bit A_1 is always a 1 for decimal digit 2, 3, 6, or 7 and can be expressed as

$$A_1 = 2 + 3 + 6 + 7$$

Finally, A_0 is always a 1 for decimal digit 1, 3, 5, 7, or 9. The expression for A_0 is

$$A_0 = 1 + 3 + 5 + 7 + 9$$

Now let's implement the logic circuitry required for encoding each decimal digit to a BCD code by using the logic expressions just developed. It is simply a matter of ORing the appropriate decimal digit input lines to form each BCD output. The basic encoder logic resulting from these expressions is shown in Figure 6–37.

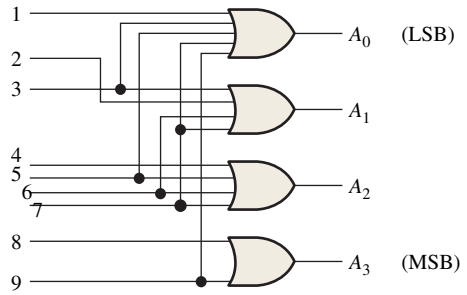


FIGURE 6–37 Basic logic diagram of a decimal-to-BCD encoder. A 0-digit input is not needed because the BCD outputs are all LOW when there are no HIGH inputs.

The basic operation of the circuit in Figure 6–37 is as follows: When a HIGH appears on *one* of the decimal digit input lines, the appropriate levels occur on the four BCD output lines. For instance, if input line 9 is HIGH (assuming all other input lines are LOW), this condition will produce a HIGH on outputs A_0 and A_3 and LOWs on outputs A_1 and A_2, which is the BCD code (1001) for decimal 9.

The Decimal-to-BCD Priority Encoder

This type of encoder performs the same basic encoding function as previously discussed. A **priority encoder** also offers additional flexibility in that it can be used in applications that require priority detection. The priority function means that the encoder will produce a BCD output corresponding to the *highest-order decimal digit* input that is active and will ignore any other lower-order active inputs. For instance, if the 6 and the 3 inputs are both active, the BCD output is 0110 (which represents decimal 6).

An Application

The ten decimal digits on a numeric keypad must be encoded for processing by the logic circuitry. In this example, when one of the keys is pressed, the decimal digit is encoded to the corresponding BCD code. Figure 6–39 shows a simple keyboard encoder arrangement using a priority encoder. The keys are represented by ten push-button switches, each with a **pull-up resistor** to $+V$. The pull-up resistor ensures that the line is HIGH when a key is not depressed. When a key is depressed, the line is connected to ground, and a LOW is applied to the corresponding encoder input. The zero key is not connected because the BCD output represents zero when none of the other keys is depressed.

The BCD complement output of the encoder goes into a storage device, and each successive BCD code is stored until the entire number has been entered. Methods of storing BCD numbers and binary data are covered in Chapter 11.

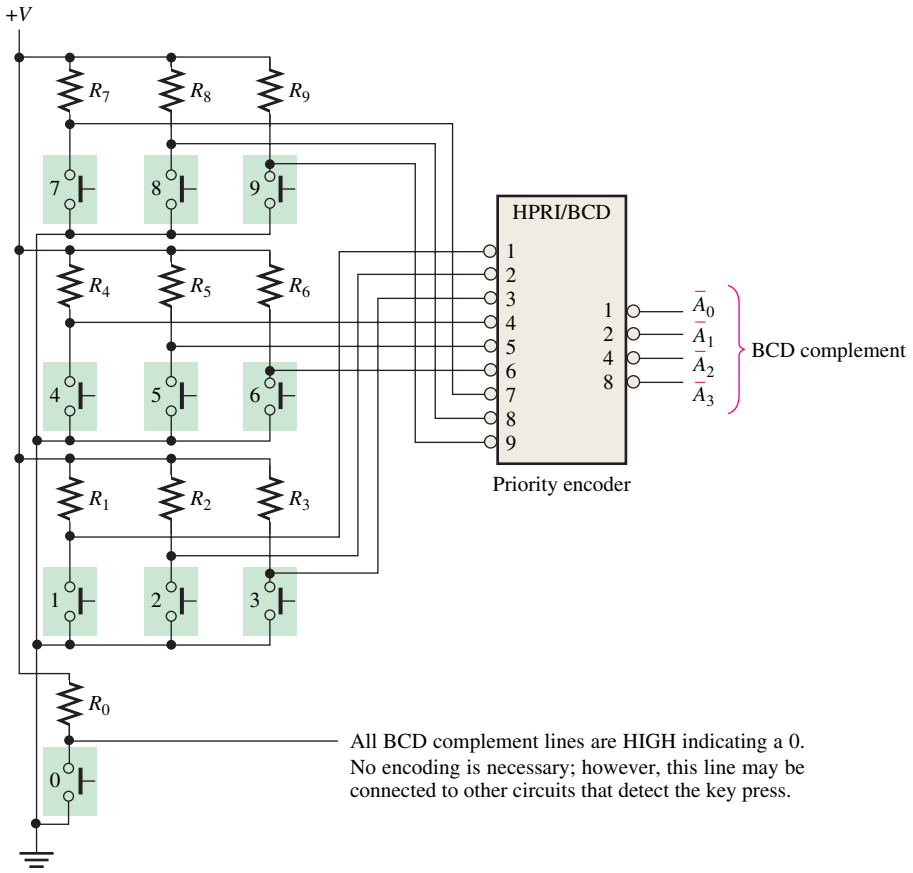


FIGURE 6–39 A simplified keyboard encoder.

SECTION 6–6 CHECKUP

1. Suppose the HIGH levels are applied to the 2 input and the 9 input of the circuit in Figure 6–37.
   - **(a)** What are the states of the output lines?
   - **(b)** Does this represent a valid BCD code?
   - **(c)** What is the restriction on the encoder logic in Figure 6–37?
2. **(a)** What is the $\overline{A}_3\,\overline{A}_2\,\overline{A}_1\,\overline{A}_0$ output when LOWs are applied to pins 1 and 5 of the 74HC147 in Figure 6–38?
   - **(b)** What does this output represent?

6–7 Code Converters

In this section, we will examine some methods of using combinational logic circuits to convert from one code to another.

BCD-to-Binary Conversion

One method of BCD-to-binary code conversion uses adder circuits. The basic conversion process is as follows:

1. The value, or weight, of each bit in the BCD number is represented by a binary number.
2. All of the binary representations of the weights of bits that are 1s in the BCD number are added.
3. The result of this addition is the binary equivalent of the BCD number.

A more concise statement of this operation is

The binary numbers representing the weights of the BCD bits are summed to produce the total binary number.

Let's examine an 8-bit BCD code (one that represents a 2-digit decimal number) to understand the relationship between BCD and binary. For instance, you already know that the decimal number 87 can be expressed in BCD as

$$\underbrace{1000}_{8} \quad \underbrace{0111}_{7}$$

The left-most 4-bit group represents 80, and the right-most 4-bit group represents 7. That is, the left-most group has a weight of 10, and the right-most group has a weight of 1. Within each group, the binary weight of each bit is as follows:

	Tens Digit				**Units Digit**			
Weight:	80	40	20	10	8	4	2	1
Bit designation:	B_3	B_2	B_1	B_0	A_3	A_2	A_1	A_0

The binary equivalent of each BCD bit is a binary number representing the weight of that bit within the total BCD number. This representation is given in Table 6–7.

TABLE 6–7

Binary representations of BCD bit weights.

		(MSB)		**Binary Representation**				**(LSB)**
BCD Bit	**BCD Weight**	**64**	**32**	**16**	**8**	**4**	**2**	**1**
A_0	1	0	0	0	0	0	0	1
A_1	2	0	0	0	0	0	1	0
A_2	4	0	0	0	0	1	0	0
A_3	8	0	0	0	1	0	0	0
B_0	10	0	0	0	1	0	1	0
B_1	20	0	0	1	0	1	0	0
B_2	40	0	1	0	1	0	0	0
B_3	80	1	0	1	0	0	0	0

If the binary representations for the weights of all the 1s in the BCD number are added, the result is the binary number that corresponds to the BCD number. Example 6–12 illustrates this.

EXAMPLE 6–12

Convert the BCD numbers 00100111 (decimal 27) and 10011000 (decimal 98) to binary.

Solution

Write the binary representations of the weights of all 1s appearing in the numbers, and then add them together.

80	40	20	10	8	4	2	1		
0	0	1	0	0	1	1	1		
								0000001	1
								0000010	2
								0000100	4
								+ 0010100	20
								0011011	Binary number for decimal 27

80	40	20	10	8	4	2	1		
1	0	0	1	1	0	0	0		
								0001000	8
								0001010	10
								+ 1010000	80
								1100010	Binary number for decimal 98

Related Problem

Show the process of converting 01000001 in BCD to binary.

MultiSim Open file EX06-12 and run the simulation to observe the operation of a BCD-to-binary logic circuit.

Binary-to-Gray and Gray-to-Binary Conversion

The basic process for Gray-binary conversions was covered in Chapter 2. Exclusive-OR gates can be used for these conversions. Programmable logic devices (PLDs) can also be programmed for these code conversions. Figure 6–40 shows a 4-bit binary-to-Gray code converter, and Figure 6–41 illustrates a 4-bit Gray-to-binary converter.

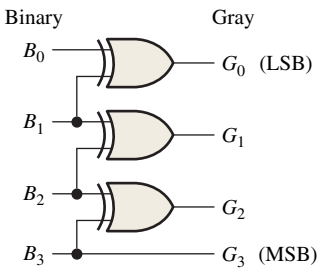


FIGURE 6–40 Four-bit binary-to-Gray conversion logic. Open file F06-40 to verify operation.

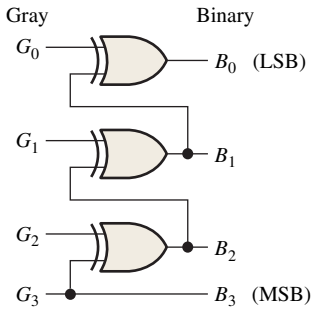


FIGURE 6–41 Four-bit Gray-to-binary conversion logic. Open file F06-41 to verify operation.

EXAMPLE 6–13

(a) Convert the binary number 0101 to Gray code with exclusive-OR gates.

(b) Convert the Gray code 1011 to binary with exclusive-OR gates.

Solution

(a) 0101_2 is 0111 Gray. See Figure 6–42(a).

(b) 1011 Gray is 1101_2. See Figure 6–42(b).

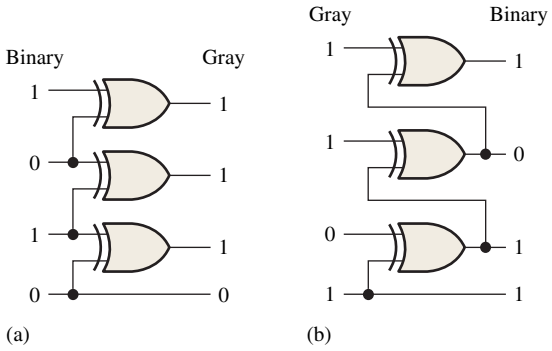


FIGURE 6–42

Related Problem

How many exclusive-OR gates are required to convert 8-bit binary to Gray?

SECTION 6–7 CHECKUP

1. Convert the BCD number 10000101 to binary.

2. Draw the logic diagram for converting an 8-bit binary number to Gray code.

6–8 Multiplexers (Data Selectors)

A **multiplexer (MUX)** is a device that allows digital information from several sources to be routed onto a single line for transmission over that line to a common destination. The basic multiplexer has several data-input lines and a single output line. It also has data-select inputs, which permit digital data on any one of the inputs to be switched to the output line. Multiplexers are also known as data selectors.

A logic symbol for a 4-input multiplexer (MUX) is shown in Figure 6–43. Notice that there are two data-select lines because with two select bits, any one of the four data-input lines can be selected.

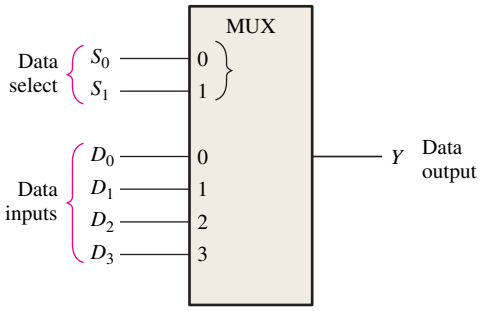


FIGURE 6–43 Logic symbol for a 1-of-4 data selector/multiplexer.

In Figure 6–43, a 2-bit code on the data-select (S) inputs will allow the data on the selected data input to pass through to the data output. If a binary 0 ($S_1 = 0$ and $S_0 = 0$) is applied to the data-select lines, the data on input D_0 appear on the data-output line. If a binary 1 ($S_1 = 0$ and $S_0 = 1$) is applied to the data-select lines, the data on input D_1 appear on the data output. If a binary 2 ($S_1 = 1$ and $S_0 = 0$) is applied, the data on D_2 appear on the output. If a binary 3 ($S_1 = 1$ and $S_0 = 1$) is applied, the data on D_3 are switched to the output line. A summary of this operation is given in Table 6–8.

TABLE 6–8

Data selection for a 1-of-4-multiplexer.

Data-Select Inputs		
S_1	S_0	Input Selected
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3

Now let's look at the logic circuitry required to perform this multiplexing operation. The data output is equal to the state of the *selected* data input. You can therefore, derive a logic expression for the output in terms of the data input and the select inputs.

The data output is equal to D_0 only if $S_1 = 0$ and $S_0 = 0$: $Y = D_0\overline{S}_1\overline{S}_0$.
The data output is equal to D_1 only if $S_1 = 0$ and $S_0 = 1$: $Y = D_1\overline{S}_1S_0$.
The data output is equal to D_2 only if $S_1 = 1$ and $S_0 = 0$: $Y = D_2S_1\overline{S}_0$.
The data output is equal to D_3 only if $S_1 = 1$ and $S_0 = 1$: $Y = D_3S_1S_0$.

When these terms are ORed, the total expression for the data output is

$$Y = D_0\overline{S}_1\overline{S}_0 + D_1\overline{S}_1S_0 + D_2S_1\overline{S}_0 + D_3S_1S_0$$

The implementation of this equation requires four 3-input AND gates, a 4-input OR gate, and two inverters to generate the complements of S_1 and S_0, as shown in Figure 6–44. Because data can be selected from any one of the input lines, this circuit is also referred to as a **data selector**.

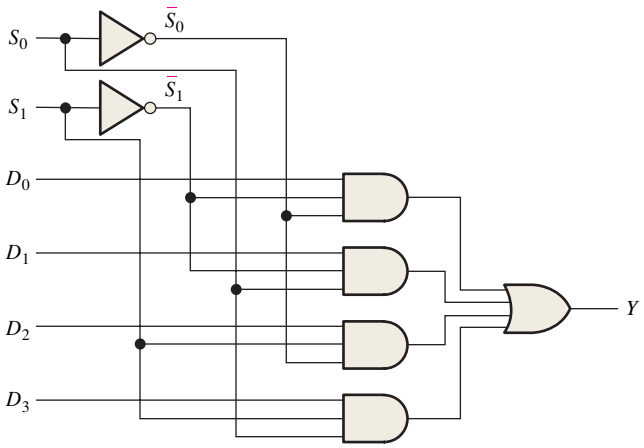


FIGURE 6–44 Logic diagram for a 4-input multiplexer. Open file F06-44 to verify operation.

EXAMPLE 6–14

The data-input and data-select waveforms in Figure 6–45(a) are applied to the multiplexer in Figure 6–44. Determine the output waveform in relation to the inputs.

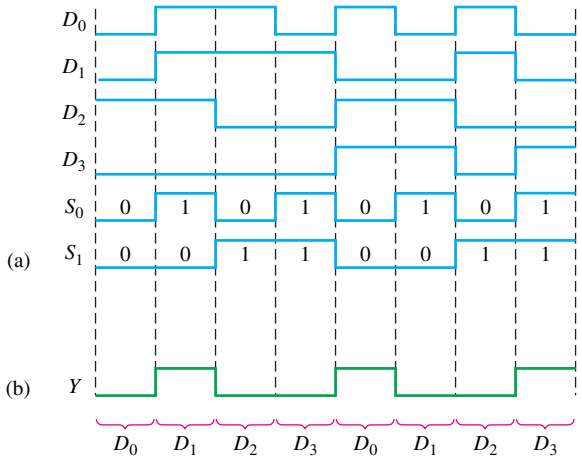


FIGURE 6–45

Solution

The binary state of the data-select inputs during each interval determines which data input is selected. Notice that the data-select inputs go through a repetitive binary sequence 00, 01, 10, 11, 00, 01, 10, 11, and so on. The resulting output waveform is shown in Figure 6–45(b).

Related Problem

Construct a timing diagram showing all inputs and the output if the S_0 and S_1 waveforms in Figure 6–45 are interchanged.

6–9 Demultiplexers

A **demultiplexer (DEMUX)** basically reverses the multiplexing function. It takes digital information from one line and distributes it to a given number of output lines. For this reason, the demultiplexer is also known as a data distributor. As you will learn, decoders can also be used as demultiplexers.

Figure 6–52 shows a 1-line-to-4-line demultiplexer (DEMUX) circuit. The data-input line goes to all of the AND gates. The two data-select lines enable only one gate at a time, and the data appearing on the data-input line will pass through the selected gate to the associated data-output line.

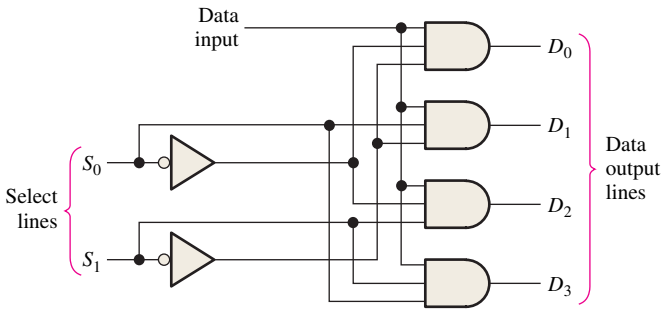


FIGURE 6–52 A 1-line-to-4-line demultiplexer.

EXAMPLE 6–18

The serial data-input waveform (Data in) and data-select inputs (S_0 and S_1) are shown in Figure 6–53. Determine the data-output waveforms on D_0 through D_3 for the demultiplexer in Figure 6–52.

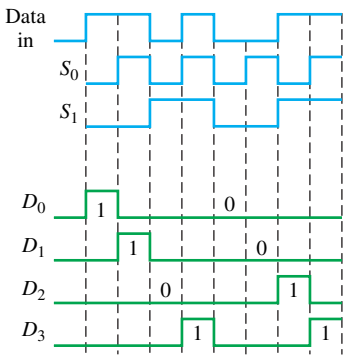


FIGURE 6–53

Solution

Notice that the select lines go through a binary sequence so that each successive input bit is routed to D_0, D_1, D_2, and D_3 in sequence, as shown by the output waveforms in Figure 6–53.

Related Problem

Develop the timing diagram for the demultiplexer if the S_0 and S_1 waveforms are both inverted.

SECTION 6–9 CHECKUP

1. Generally, how can a decoder be used as a demultiplexer?
2. The demultiplexer in Figure 6–54 has a binary code of 1010 on the data-select lines, and the data-input line is LOW. What are the states of the output lines?

6–10 Parity Generators/Checkers

Errors can occur as digital codes are being transferred from one point to another within a digital system or while codes are being transmitted from one system to another. The errors take the form of undesired changes in the bits that make up the coded information; that is, a 1 can change to a 0, or a 0 to a 1, because of component malfunctions or electrical noise. In most digital systems, the probability that even a single bit error will occur is very small, and the likelihood that more than one will occur is even smaller. Nevertheless, when an error occurs undetected, it can cause serious problems in a digital system.

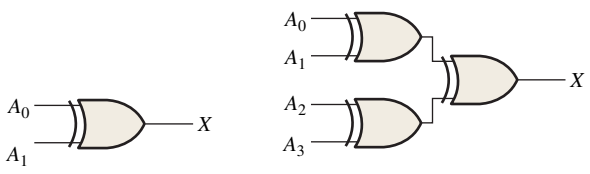
The parity method of error detection in which a **parity bit** is attached to a group of information bits in order to make the total number of 1s either even or odd (depending on the system) was covered in Chapter 2. In addition to parity bits, several specific codes also provide inherent error detection.

Basic Parity Logic

In order to check for or to generate the proper parity in a given code, a basic principle can be used:

The sum (disregarding carries) of an even number of 1s is always 0, and the sum of an odd number of 1s is always 1.

Therefore, to determine if a given code has **even parity** or **odd parity,** all the bits in that code are summed. As you know, the modulo-2 sum of two bits can be generated by an exclusive-OR gate, as shown in Figure 6–55(a); the modulo-2 sum of four bits can be formed by three exclusive-OR gates connected as shown in Figure 6–55(b); and so on. When the number of 1s on the inputs is even, the output X is 0 (LOW). When the number of 1s is odd, the output X is 1 (HIGH).



(a) Summing of two bits (b) Summing of four bits

FIGURE 6–55