

Maxima and Minima Questions

Q1 what positive number added to its reciprocal gives the minimum sum?

Solution 2

Let

x = the required positive number and

$1/x$ = the reciprocal of the number

y = sum of x and $1/x$

$$y = x + 1/x$$

$$y = x + x^{-1}$$

$$y' = 1 - x^{-2} = 0$$

$$x = 1 \quad \text{answer}$$

What number exceeds its square by the maximum amount?

Solution 1

Let

x = the number and

x^2 = the square of the number

y = the difference between x and x^2

$$y = x - x^2$$

$$y' = 1 - 2x = 0$$

$$x = 1/2 \text{ answer}$$

Problem 3

Q3 The sum of two numbers is k . Find the minimum value of the sum of their squares.

Solution 3

Let

x and y = the numbers

z = sum of their squares

$$k=x+y$$

$$y=k-x$$

$$z=x^2+y^2$$

$$z=x^2+(k-x)^2$$

$$dz/dx = 2x+2(k-x)(-1)=0$$

$$2x-k=0$$

$$x=1/2 k$$

$$y=k-1/2 k$$

$$y=1/2k$$

$$z = (1/2k)^2+(1/2k)^2$$

$$z=1/2 k^2$$

Q4: The sum of two numbers is k. Find the minimum value of the sum of their cubes.

Solution 4

Let

x and y = the numbers

z = sum of their cubes

$$k=x+y$$

$$y=k-x$$

$$z=x^3+y^3$$

$$z=x^3+(k-x)^3$$

$$dz/dx = 3x^2+3(k-x)^2(-1)=0$$

$$x^2-(k^2-2kx+x^2)=0$$

$$x=1/2k$$

$$y=k-1/2k$$

$$y=1/2k$$

$$z= (1/2k)^3+(1/2k)^3$$

$$z=1/4k^3$$

Q5: The sum of two positive numbers is 2. Find the smallest value possible for the sum of the cube of one number and the square of the other.

Solution

Let x and y = the numbers

$$x+y=2 \rightarrow \text{Equation (1)}$$

$$1+y'=0 \quad y'=-1$$

$$z=x^3+y^2 \rightarrow \text{Equation (2)}$$

$$dz/dx=3x^2+2yy'=0$$

$$3x^2+2y(-1)=0$$

$$y=3/2 x^2$$

From Equation (1)

$$x+3/2x^2=2$$

$$2x+3x^2=4$$

$$3x^2+2x-4=0$$

$$x=0.8685 \text{ \& } -1.5352$$

Use

$$x=0.8685$$

$$y=3/2(0.8685^2)$$

$$y=1.1315$$

$$z=0.8685^3+1.1315^2$$

$$z=1.9354 \quad \text{answer}$$

Q7: Find two numbers whose sum is a , if the product of one to the square of the other is to be a minimum.

Let x and y = the numbers

$$x+y=a$$

$$x=a-y$$

$$z=xy^2$$

$$z=(a-y)y^2$$

$$z=ay^2-y^3$$

$$dz/dy=2ay-3y^2=0$$

$$y=2/3a$$

$$x=a-2/3a$$

$$x=1/3a$$

The numbers are $1/3 a$, and $2/3 a$. *Answer*