Lecture 8:

Divergence Theorem

- Related YouTube Video Link:

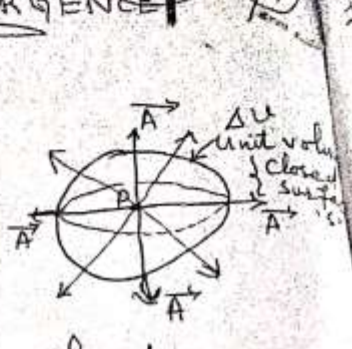
1) https://www.youtube.com/watch?v=vrj9wPUI13A&t=110s
2) https://www.youtube.com/watch?v=t5qB3Ha4q9A

- Read 3.5 and 3.7 of the Given Book

Lecture Notes on next Page

DIVERGENCE OF A VECTOR AND DIVERGENCE THEOREM:-

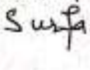
" The divergence of $\vec{A}$ at a given Point "P" is the outward flux per unit volume as the volume shrink about point 'P'."



Closed Surface ≃ volume

→ Result is Scalar.

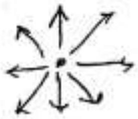
→ net outward flux $= \oint_S \vec{A} \cdot \vec{ds}$



So,

$$\text{div}\,\vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{A} \cdot \vec{ds}}{\Delta v}$$

→ Measures how much the field diverges or emerges from the points:



+ve divergence [Source point)

(-ve divergence) {Sink Point)

(Zero divergence) (neither Sink or Source).

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \rightarrow \text{Cartesian}$$

$$\nabla \cdot \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \rightarrow \text{Cylindrical}$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta} (A_\theta \sin\theta) + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

→ Spherical.

While
A vector whose divergence is Zero is called Solenoidal.

$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B}$$

$$\nabla \cdot (V\vec{A}) = V(\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla V$$

Divergence Theorem:-

4e

$$\nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A} \cdot d\vec{s}}{\Delta v}$$

$$\boxed{\int_v \nabla \cdot \vec{A}\, dv = \oint_S \vec{A} \cdot \vec{ds}}$$

↳ Relationship of (closed surface = volume)

$$\oint_s = \int_v$$

" The divergence theorem states that the total outward flux of a vector $\vec{A}$ through the closed surface S is the same as the volume integral of the divergence of $\vec{A}$ ".

Consider Cartesian Coordinate system.

$$\int_v \nabla \cdot \vec{A}\, dv = \int_v \left(\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z} \right) \cdot \left(A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z} \right) dv$$

$$= \int_v \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dv$$

$$= \iiint \frac{\partial A_x}{\partial x} dx\, dy\, dz + \iiint \frac{\partial A_y}{\partial y} dy\, dz\, dx + \iiint \frac{\partial A_z}{\partial z} dz\, dx\, dy$$

$$= \iint A_x\, dy\, dz + \iint A_y\, dz\, dx + \iint A_z\, dx\, dy$$

$$= \int_s A_x\, ds_x + \int_s A_y\, ds_y + \int_s A_z\, ds_z$$

$$= \int_s \left(A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z} \right) \cdot \left(ds_x \vec{a_x} + ds_y \vec{a_y} + ds_z \vec{a_z} \right)$$

$$= \oint_s \vec{A} \cdot \vec{ds}$$

Verified.

Lecture 9: Energy and Potential in moving point Charge in an electric field

- Related YouTube Video Link:

  1. https://www.youtube.com/watch?v=C7jjaqt7E-A&pbjreload=10

- Read 4.1 and 4.3 of the given book

Lecture Notes

Energy Expended in Moving a Point Charge in an Electric Field

- The electric field intensity was defined as the force on a unit test charge at that point where we wish to find the value of the electric field intensity.

- To move the test charge against the electric field, we have to exert a force equal and opposite in magnitude to that exerted by the field. ► We must expend energy or do work.

- To move the charge in the direction of the electric field, our energy expenditure turns out to be negative. ► We do not do the work, the field does.

- To move a charge Q a distance d**L** in an electric field **E**, the force on Q arising from the electric field is:

$$\mathbf{F}_E = Q\mathbf{E}$$

- The component of this force in the direction dL is:

$$F_{EL} = \mathbf{F}_E \cdot \mathbf{a}_L = Q\mathbf{E} \cdot \mathbf{a}_L$$

- The force that we apply must be equal and opposite to the force exerted by the field:

$$F_{\text{appl}} = -Q\mathbf{E} \cdot \mathbf{a}_L$$

- Differential work done by external source to Q is equal to:

$$dW = -Q\mathbf{E} \cdot \mathbf{a}_L dL = -Q\mathbf{E} \cdot d\mathbf{L}$$

If E and L are perpendicular, the differential work will be zero

- Differential work done by external source to Q is equal to:

$$W = \int_{\text{init}}^{\text{final}} dW$$

$$W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

  - The path must be specified beforehand
  - The charge is assumed to be at rest at both initial and final positions
  - ***W* > 0 means we expend energy or do work**
  - ***W* < 0 means the field expends energy or do work**

Definition of Potential Difference and Potential

- We already find the expression for the work *W* done by an external source in moving a charge *Q* from one point to another in an electric field E:

$$W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

- Potential difference V is defined as the work done by an external source in moving a unit positive charge from one point to another in an electric field:

$$\text{Potential difference} = V = -\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

- We shall now set an agreement on the direction of movement. V_{AB} signifies the potential difference between points *A* and *B* and is the work done in moving the unit charge from *B* (last named) to *A* (first named).

- We shall now set an agreement on the direction of movement. VAB signifies the potential difference between points A and B and is the work done in moving the unit charge from B (last named) to A (first named).

$$V_{AB} = -\int_{B}^{A} \mathbf{E} \cdot d\mathbf{L} \text{ V}$$

  - VAB is positive if work is done in carrying the positive charge from B to A

- From the line-charge example, we found that the work done in taking a charge *Q* from $\rho = a$ to $\rho = b$ was:

$$W = -\frac{Q\rho_L}{2\pi\varepsilon_0} \ln \frac{b}{a}$$

- Or, from $\rho = b$ to $\rho = a$,

$$W = -\frac{Q\rho_L}{2\pi\varepsilon_0}\ln\frac{a}{b} \quad = \frac{Q\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

- Thus, the potential difference between points at $\rho = a$ to $\rho = b$ is:

$$V_{ab} = \frac{W}{Q} = \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

- For a point charge, we can find the potential difference between points A and B at radial distance r_A and r_B, choosing an origin at Q:

$$\mathbf{E} = E_r\mathbf{a}_r = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r$$

$$V_{AB} = -\int_B^A \mathbf{E}\cdot d\mathbf{L}$$

$$= -\int_{r_B}^{r_A}\frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$r_B > r_A \rightarrow V_{AB} > 0$, $W_{AB} > 0$,
Work expended by the external source (us)
$r_B < r_A \rightarrow V_{AB} < 0$, $W_{AB} < 0$,
Work done by the electric field

- It is often convenient to speak of *potential*, or *absolute potential*, of a point rather than the potential difference between two points.

- For this purpose, we must first specify the reference point which we consider to have zero potential.

- The most universal zero reference point is “ground”, which means the potential of the surface region of the earth.

- Another widely used reference point is “infinity.”

- For cylindrical coordinate, in discussing a coaxial cable, the outer conductor is selected as the zero reference for potential.

- If the potential at point *A* is V_A and that at *B* is V_B, then:

$$V_{AB} = V_A - V_B$$