# Chapter # 03 (a)

## Dimensional Analysis and Similitude

### Background :-

* Although many practical engineering problem involving Hydraulics engineering can be solved by "Equation and Analytical procedures". but yet a Large number of problems rely on experimental data for their Solution.
* In fact very few problems can be solved by only analytical procedures.
* In general, solution is obtained through the use of a Combination of analysis and experimental data.
* An obvious goal of any experiment is to make the results widely applicable.
* To achieve this goal, the concept of Similitude is often used so that measurements made on one system (Laboratory) can be used to describe the behaviour of other system (out side of Laboratory).

* The Laboratory systems are usally thought of as "models" and are used to study the phenomenon of any thing under carefully controlled system or condition.

* From these model studies emperical formulations can be developed or specific prediction of one or more characteristics of some other similar system can be made.

* However to do this it is necessary to establish the relationship between the laboratory model and the outside system or other system.

## Dimensional Analysis :-

Dimensional Analysis is a mathematical technique making use of study of dimensions.

* It deals with the dimensions of physical quantities involved in the phenomenon.

* In dimensional analysis, one first predicts the physical parameters that will influence the flow, and then by grouping these parameters in dimension less cambinations a better understanding of the flow phenomenon is made possible".

It is particularly helpful in experimental work because it provides a guide to those things that significantly influence the phenomenon.

* This mathematical technique is used in research work for design and for Conducting model tests.

## Types of Dimensions:-

### 1. Fundamental Dimensions or Fundamental Quantities

These are basic quantities. For example.

Time, T
Distance, L
Mass, M

$\Rightarrow$

Time, T
Distance, L
Force, F.

Force = Mass × Acceleration.

$$F = \text{Mass} \times \frac{\text{meter}}{\text{sec}^2}$$

$$F = \text{Mass} \times \text{m sec}^{-2}$$

meter = distance
Sec = Time

$$F = M \times LT^{-2}$$

$$\boxed{F = MLT^{-2}}$$

### 2. Secondary Dimensions or Derived Quantities

These are those quantities which posses more than one fundamental dimension.

For Example;

- Velocity is denoted by distance per unit time $L/T$
- Acceleration is denoted by distance per unit time Square $L/T^2$
- Density is denoted by mass per unit Volume $M/L^3$

Since Velocity, density and acceleration involve more than one fundamental quantities. So these are called derived quantities.

Flow characteristics, units and Dimensions.

| Characteristics | | Units (SI) | Dimension (M L T) | Dimension (F L T) |
|---|---|---|---|---|
| Geometry | Length | m | $L$ | |
| | Area | $m^2$ | $L^2$ | |
| | Volume | $m^3$ | $L^3$ | |
| Kinematic | Time | T | $T$ | |
| | Velocity | m/sec | $L/T$ | |
| | Acceleration | $m/sec^2$ | $L/T^2$ | |
| | Discharge | $m^3/sec$ | $L^3/T$ | |
| Dynamic | Mass | Kg | $M$ | $FL^{-1}T^2$ |
| | Force | N ($Kg-m/s^2$) | $MLT^{-2}$ | $F$ |
| | Pressure | Pa ($N/m^2$) | $ML^{-1}T^{-2}$ | $FL^{-2}$ |
| | Energy | J (N-m) | $ML^2T^{-2}$ | $FL$ |
| | Power. | Watt (N-m/sec) | $ML^2T^{-3}$ | $FLT^{-1}$ |

| Quantity | Symbol | Dimensions | |
|---|---|---|---|
| | | $MLT\Theta$ | $FLT\Theta$ |
| Length | $L$ | $L$ | $L$ |
| Area | $A$ | $L^2$ | $L^2$ |
| Volume | $\mathcal{V}$ | $L^3$ | $L^3$ |
| Velocity | $V$ | $LT^{-1}$ | $LT^{-1}$ |
| Acceleration | $dV/dt$ | $LT^{-2}$ | $LT^{-2}$ |
| Speed of sound | $a$ | $LT^{-1}$ | $LT^{-1}$ |
| Volume flow | $Q$ | $L^3T^{-1}$ | $L^3T^{-1}$ |
| Mass flow | $\dot{m}$ | $MT^{-1}$ | $FTL^{-1}$ |
| Pressure, stress | $p$, $\sigma$ | $ML^{-1}T^{-2}$ | $FL^{-2}$ |
| Strain rate | $\dot{\epsilon}$ | $T^{-1}$ | $T^{-1}$ |
| Angle | $\theta$ | None | None |
| Angular velocity | $\omega$ | $T^{-1}$ | $T^{-1}$ |
| Viscosity | $\mu$ | $ML^{-1}T^{-1}$ | $FTL^{-2}$ |
| Kinematic viscosity | $\nu$ | $L^2T^{-1}$ | $L^2T^{-1}$ |
| Surface tension | $\Upsilon$ | $MT^{-2}$ | $FL^{-1}$ |
| Force | $F$ | $MLT^{-2}$ | $F$ |
| Moment, torque | $M$ | $ML^2T^{-2}$ | $FL$ |
| Power | $P$ | $ML^2T^{-3}$ | $FLT^{-1}$ |
| Work, energy | $W$, $E$ | $ML^2T^{-2}$ | $FL$ |
| Density | $\rho$ | $ML^{-3}$ | $FT^2L^{-4}$ |
| Temperature | $T$ | $\Theta$ | $\Theta$ |
| Specific heat | $c_p$, $c_v$ | $L^2T^{-2}\Theta^{-1}$ | $L^2T^{-2}\Theta^{-1}$ |
| Specific weight | $\gamma$ | $ML^{-2}T^{-2}$ | $FL^{-3}$ |
| Thermal conductivity | $k$ | $MLT^{-3}\Theta^{-1}$ | $FT^{-1}\Theta^{-1}$ |
| Expansion coefficient | $\beta$ | $\Theta^{-1}$ | $\Theta^{-1}$ |

## Melkodology of Dimensional Analysis

The basic principle is Dimensional Homogeneity which means the dimensions of each terms in an equation on both sides are equal. So such an equation in which dimensions of each term on both sides of equations are same is known as Dimensionaly Homogeneous Equation -

Let us consider the equation $V = (2gH)^{\frac{1}{2}}$

$$L.H.S \quad V = \frac{m}{sec} = \frac{L}{T}$$

$$\boxed{V = LT^{-1}}$$

$$R.H.S = (2gH)^{\frac{1}{2}} = \left(\frac{m}{sec^2} \cdot H\right)^{\frac{1}{2}}$$

$$= \left(\frac{L}{T^2} \cdot L\right)^{\frac{1}{2}}$$

$$= \left(\frac{L^2}{T^2}\right)^{\frac{1}{2}} = \frac{L}{T} = LT^{-1}$$

$$\boxed{= LT^{-1}}$$

So the equation is dimensionally homogenous equation.

If the number of Variables involved in a physical phenomenon are Known, then the relation among the Variables can be determined by the following two Methods

1- Rayleigh's Method
2- Buckingham's $\pi$ - Theorm.

## (1) Rayleigh's Method

It is used for determing expression for a Variable (dependent) which depends upon maximum three to four variables (independent) only.

* If the number of independent Variables are more than 4 then it is very difficult to obtain expression for dependent Variables.

* Let $X$ is a dependent Variable which depends upon $X_1$, $X_2$, and $X_3$ as independent Variables Then according to Rayleigh's Method

$$X = f(X_1, X_2, X_3)$$
$$X = K X_1^a, X_2^b, X_3^c$$

where $k$ is a non dimensional constant and $a, b, c$ are arbitany powers which are obtained by Comparing the powers of fundamental dimensions (Dimensional Homogenerity

Problem:- The resisting Force R of a supersonic plane during flight can be considered as dependent upon the length of the air craft $l$, velocity $v$, air viscosity $\mu$, air density $\rho$ and bulk modulus of air K. Express the functional relationship b/w the variables and the resisting Force -

Solution:-
$$R_F = f(l, v, \mu, \rho, K)$$
$$R_F = A(l^a, v^b, \mu^c, \rho^d, K^e) \rightarrow (Y)$$
$$MLT^{-2} = A(L)^a, (LT^{-1})^b (ML^{-1}T^{-1})^c (ML^{-3})^d (ML^{-1}T^{-2})^e \rightarrow (X)$$

Equating the powers of MLT on both sides.

power of M $\Rightarrow 1 = c + d + e \rightarrow (1)$

power of L $\Rightarrow 1 = a + b + (-c) + (-3d) + (-e)$
$$1 = a + b - c - 3d - e \rightarrow (2)$$

power of T = $-2 = -b - c - 2e$ — (3)

Using (1) $1 = c + d + e$
$$\boxed{d = 1 - c - e}$$

Using (2)
$$\boxed{a = 1 - b + c + 3d + e}$$

Using (3)
$$\boxed{b = 2 - c - 2e}$$

put b and d in (a)

$a = 1 - (2-c-2e) + c + 3(1-c-e) + e$

$a = 1 - 2 + c + 2e + c + 3 - 3c - 3e + e$

$a = 1 - 2 + 2c + \cancel{2e} + 3 - 3c - \cancel{2e}$

$a = -1 - c + 3$

$\boxed{a = 2 - c}$

Substituting a, b, d in eq (4).

$R_F = A\,(l^a, v^b, \mu^c, \rho^d, K^e)$

$R_F = A\,(l^{2-c}\ V^{2-c-2e}\ \mu^c\ \rho^{1-c-e}\ K^e)$

$R_F = A\,\rho l^2 V^2\,(l^{-c}\ V^{-c}\ \mu^c \rho^{-c})\,(V^{-2e}\rho^{-e} K^e)$

$R_F = A\rho l^2 V^2\left[\left(\frac{\mu}{\rho V l}\right)^c\left(\frac{K}{V^2\rho}\right)^e\right]$

OR

$$\boxed{R_F = A\rho l^2 V^2\left[\left(\frac{\mu}{\rho V L}\right)\left(\frac{K}{V^2\rho}\right)\right]}$$

Required relation ship