LECTURE # 9

In this lecture you will learn about:

Numerical of:

- Lecture 6
- Lecture 7
- Lecture 8

Course Name:

"Introduction To Earthquake Engineering"

Course Code: CT-634

Credit Hours: 3

Semester: 6^{TH}

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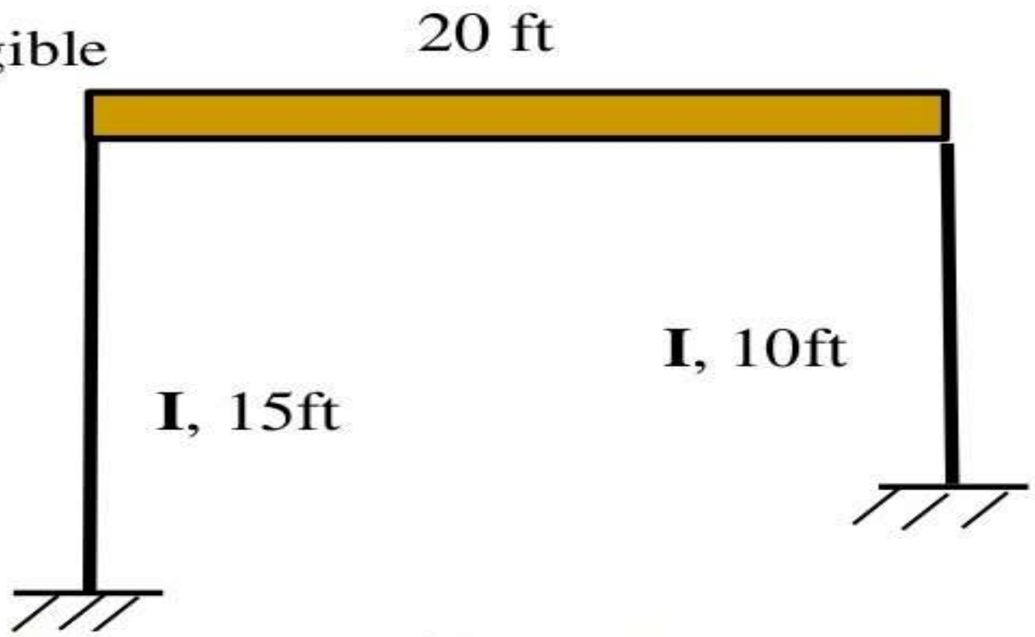
PROBLEM # 1

Determine lateral stiffness of the frame if a lateral load is applied at beam level. Assume:

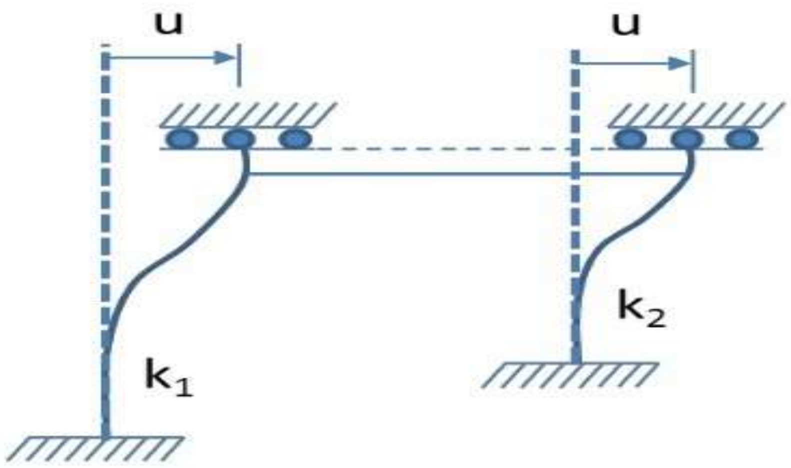
1. The flexural stiffness of beam is too high as compared to that of connected columns.
2. Axial deformations in beam is negligible

Take **E** = 29,000 ksi, **I** = 1200 in^4



SOLUTION

$$k_{eq} = k_1 + k_2$$



$$k = \frac{12EI}{h_1^3} + \frac{12EI}{h_2^3}$$

$$= 12EI\left[\frac{1}{h_1^3} + \frac{1}{h_2^3}\right]$$

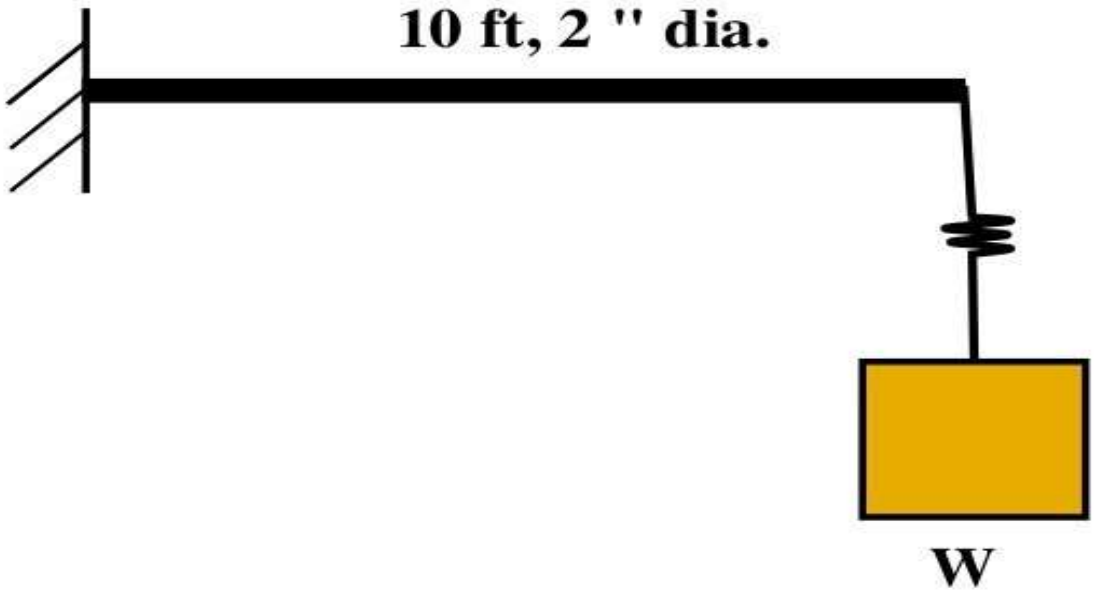
$$= 12 \times (29000\ k/in^2) \times (1200in^4)\left[\frac{1}{(15 \times 12\ in)^3} + \frac{1}{(10 \times 12\ in)^3}\right]$$

$$= 313.29\, k/in$$

$$= 3759\, k/ft$$

PROBLEM # 2

Determine the stiffness of cantilever beam by assuming that the self weight of beam is negligible Take **E** = 29,000 ksi, $\mathbf{k}_{spring}$= 200 lb/ft.



SOLUTION

$$k_1 = 200\, lb/ft$$

$$k_2 = \frac{3EI}{l^3} = \frac{3 \times \left(29000\, {}^{k}/_{in^2}\right) \times \left(\frac{\pi}{64} \times (2\ in)^2\right)}{(10 \times 12\ in)^3}$$

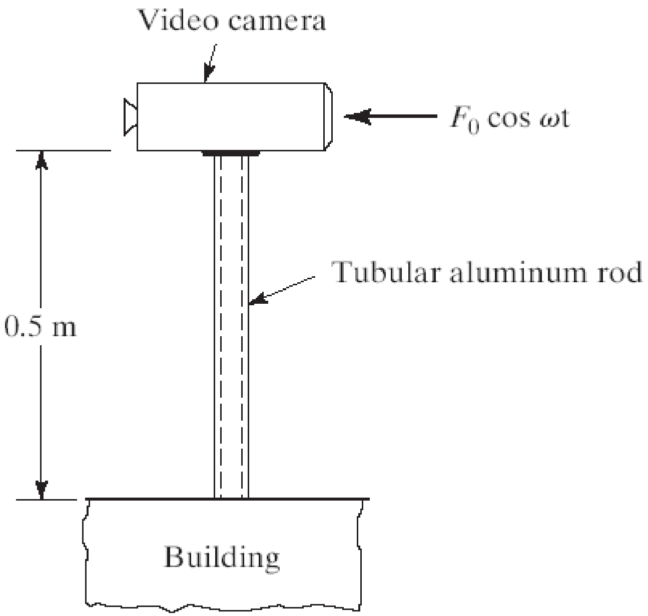
$$= 0.0396\, k/in^2 = 474.7\, lb/ft$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{200 \times 474.7}{200 + 474.7}$$

$$k_{eq} = 140.7\, lb/ft$$

PROBLEM # 3

A video camera, of mass 2.0 kg, is mounted on the top of a bank building for surveillance. The video camera is fixed at one end of a tubular aluminium rod whose other end is fixed to the building as shown in Fig. The wind-induced force acting on the video camera, is found to be harmonic with p(t) = 25 sin 75t N. Determine the cross-sectional dimensions of the aluminium tube if the maximum amplitude of vibration of the video camera is to be limited to 0.005 m. E Aluminium = 71 GPa.



PB: GIVEN DATA:

Mass, $m = 2$ kg

Harmonic Force, $p(t) = 25 \sin 75t$ N

Amplitude, $P_o = 25$ N N

Force Frequency, $\omega = 75$ rad/sec

$U_o = 0.005$ m

Modulus of Elasticity, $E_{AL} = 73$ GPa

$= 73 \times 10^9$ Pa

Length, $L = 0.5$ m

REQUIRED:

Diameter, $d = ?$

SOLUTION:

For Undamped Structure

$$R_d = \frac{U_o}{(U_{st})_o} = \frac{1}{(1 - \delta_\omega^2)} \quad \text{----} (1)$$

$$(U_{st})_o = \frac{P_o}{K} \Rightarrow (U_{st})_o = \frac{25}{K}$$

$$\omega_n = \sqrt{\frac{K}{m}} \Rightarrow \omega_n = \sqrt{\frac{K}{2}} \rightarrow \text{Natural Frequency}$$

$$\text{Frequency Ratio, } \delta_\omega = \frac{\omega}{\omega_n} = \frac{75}{\sqrt{\frac{K}{m}}} = \frac{75 \times \sqrt{2}}{\sqrt{K}}$$

put the value of "$(U_{st})_o$" and "δ_ω" in eq (1)

$$\frac{0.005}{\frac{25}{k}} = \frac{1}{\left(1-\left(\frac{75\times\sqrt{2}}{\sqrt{k}}\right)^2\right)}$$

$$0.005\times\left(1-\left(\frac{11250}{k}\right)\right) = \frac{25}{k}$$

$$0.005\times\frac{56.25}{k} = \frac{25}{k}$$

$$0.005 = \frac{56.25}{k} + \frac{25}{k}$$

$$0.005 = \frac{81.25}{k}$$

$$k = \frac{81.25}{0.005}$$

$$k = 16250 \text{ N/m}$$

Now $K = \frac{3EI}{L^3}$ → 

$$I = \frac{K\times L^3}{3E} = \frac{16250 \times (0.5)^3}{(3\times(71\times 10^9))}$$

$$I = 9.54\times 10^{-9} \text{ m}^4$$

So, $I = \frac{\pi}{64}\times d^4$

$$d = \left(\frac{T \times 64}{\pi} \right)^{1/4}$$

$$d = \left(\frac{(9.54 \times 10^{-9}) \times (64)}{3.14} \right)$$

$$d = 0.021 \text{ m}$$

$$d = 0.021 \times 1000$$

$$\boxed{d = 21 \text{ mm}}$$

PROBLEM # 4

A rotating machine with a 600 kg mass operating at a constant speed produces harmonic force in vertical direction. The harmonic force is expressed as p(t)= 5000 Sin 150t, where p(t) is in N. If the damping ratio of isolators at the foundation of machine is 7.5%, determine the stiffness of isolators so that the Transmissibility at the operating speed does not exceed 0.15. Also determine the amplitude of force transmitted to the foundation

PB : GIVEN DATA

Mass, $m = 600$ Kg
Harmonic Force, $p(t) = 5000 \times \sin \times 150 \times t$ N
Amplitude, $P_0 = 5000$ N
Force Frequency, $\omega = 150$ rad/sec

Damping Ratio, $\xi = 7.5\ \%$
$= 0.075$
Transmissibility, $TR = 0.15$

REQUIRED:
Force Transmitted: Amplitude $= (f_T)_0 = ?$
Stiffness, $K = ?$

SOLUTION:

$$TR = \frac{(f_T)_0}{P_0} = \sqrt{\frac{1+(2\xi\gamma_\omega)^2}{(1-\gamma_\omega^2)^2+(2\times\xi\times\gamma_\omega)^2}} \quad ---- (1)$$

$$TR = \sqrt{\frac{1+(2\xi\gamma_\omega)^2}{(1-\gamma_\omega^2)^2+(2\times\xi\times\gamma_\omega)^2}}$$

$$(0.15)^2 = \left(\sqrt{\frac{1+(2\times0.075\times\gamma_\omega)^2}{(1-\gamma_\omega^2)^2+(2\times0.075\times\gamma_\omega)^2}}\right)^2$$

$$0.0225 = \frac{1+(0.15\times\gamma_\omega)^2}{(1-\gamma_\omega^2)^2+(0.15\times\gamma_\omega)^2}$$

$$0.0225 = \frac{1+(0.0225\times\gamma_\omega^2)}{(1-\gamma_\omega^2)^2+(0.0225\times\gamma_\omega^2)}$$ put $\gamma_\omega^2 = x$

$$0.0225 = \frac{1+0.0225x}{(1-x)^2+(0.0225x)}$$

$$0.0225 = \frac{1+0.0225x}{1+x^2-2x+0.0225x}$$

$$0.0225 = \frac{1+0.0225x}{x^2-1.9775+1}$$

$$x^2 - 1.9775x + 1 = \frac{1 + 0.0225x}{0.0225}$$

$$x^2 - 1.9775x + 1 = \frac{1}{0.0225} + \frac{0.0225 \times x}{0.0225}$$

$$x^2 - 1.9775x + 1 = 44.44 + x$$

$$x^2 - 1.9775x + 1 - 44.44 - x$$

$$x^2 - 2.9775x - 43.44$$

By quadratic formula:

$$x = 8.25$$

$$r_w^2 = 8.25$$

$$\sqrt{r_w^2} = \sqrt{8.25}$$

$$r_w = 2.87$$

$$r_w = \frac{\omega}{\omega_n}$$

$$2.87 = \frac{150}{\sqrt{\frac{K}{m}}}$$

$$\sqrt{\frac{K}{m}} = \frac{150}{2.87}$$

$$\left(\sqrt{\frac{K}{600}}\right)^2 = (52.66)^2$$

$$2731.61 = \frac{K}{600}$$

$$K = 2731.61 \times 600$$

$$K = 1638966 \quad N/m$$

put all the values in eq (1)

$$0.15 = \frac{(F_T)_0}{5000}$$

$$(F_T)_0 = 0.15 \times 5000$$

$$(F_T)_0 = 750$$

Thank You