LECTURE # 7

In this lecture we will learn about:

- Harmonic Force
- Response of Undamped Systems Subjected to Harmonic Forces

Course Name:

"Introduction To Earthquake Engineering"

Course Code: CT-634

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HARMONIC FORCE

A harmonic force is one whose variation which with time is defined by any one of the following equations

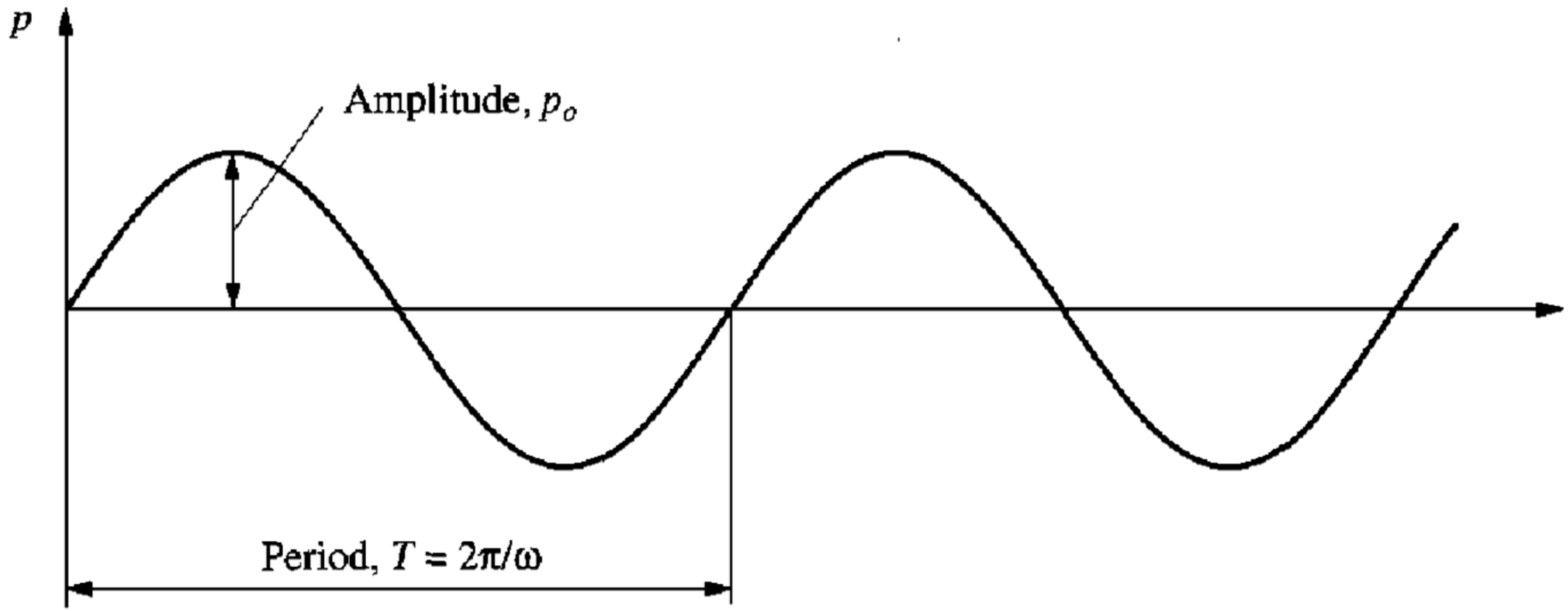
$$\mathbf{p(t) = p_o Sin(\omega t)\ p_o Cos(\omega t)}$$

Where

- $\mathbf{p_o}$ is the amplitude or maximum value of force
- $\boldsymbol{\omega}$ is its frequency also called as *exciting frequency* or *forcing frequency*.
- $\mathbf{T=2\pi/\omega}$ is the *exciting period* or *forcing period.*

The equations used in this module are strictly applicable to $p_o Sin(\omega t)$

HARMONIC FORCE



HARMONIC FORCE

A common source of such a sinusoidal force is unbalance in a rotating machines (such as turbines, electric motors and electric generators, as well as fans, or rotating shafts).

Unbalance cloth in a rotating drum of a washing machine is also an harmonic force.

When the wheels of a car are not balanced, harmonic forces are developed in the rotating wheels. If the rotational speed of the wheels is close to the natural frequency of the car's suspension system in vertical direction, amplitude of vertical displacement in the car's suspension system increases and violent shaking occur in car due to match of frequency of the force (due to vertical component of harmonic forces acting at unbalanced mass centre) with natural frequency of car's suspension system in vertical direction, $\boldsymbol{\omega_n}$

RESPONSE OF UNDAMPED SYSTEMS SUBJECTED TO HARMONIC FORCES

The equation of motion for harmonic vibration of Undamped system is:

$$\mathbf{m\ddot{u} + ku = p_o Sin(\omega t)}$$

The solution to the equation is made up of two parts.

- The first part is the solution which correspond to forced vibration and is known as the Particular Solution. The corresponding vibration is known as Steady State Vibration, for its present because of the applied force no matter what the initial conditions.
- The second part is the solution to the free vibration, which does not require any forcing function, this part is known as the Complimentary Solution. The corresponding vibration is known as Transient Vibration, which depends on the initial conditions

PARTICULAR SOLUTION OF UNDAMPED HARMONIC VIBRATIONS

It can be derived that the particular solution of Undamped vibration is as follows:

$$\Rightarrow u_p(t) = \frac{p_o}{k}\left[\frac{1}{1 - r_\omega^{\ 2}}\right]\mathrm{Sin}(\omega t) \qquad \text{where } \omega \neq \omega_n$$

ω/ω_n is termed as frequency ratio

For the sack of simplicity, we will use $\mathbf{r_\omega}$ in our lectures to represent $\boldsymbol{\omega/\omega_n}$

$$u_p(t) = \frac{p_o}{k}.\frac{1}{1 - \left(\omega/\omega_n\right)^2}\mathrm{Sin}(\omega t) \qquad \text{where } \omega \neq \omega_n$$

$u_p(t)$ is the displacements corresponding to the *Particular solution* (i.e. due to forced vibration).

COMPLIMENTARY SOLUTION OF UNDAMPED HARMONIC VIBRATIONS

Complementary solution of Undamped vibration is given as follows:

$$\mathbf{u_c(t) = BSin(\omega_n t) + ACos(\omega_n t)}$$

$u_c(t)$ is the displacements corresponding to the *Complimentary solution* (i.e. due to free vibration) and depends on initial conditions.

COMPLETE SOLUTION OF UNDAMPED HARMONIC VIBRATIONS

Complete solution is the sum of complementary solution, $u_c(t)$, and particular solution, $u_p(t)$

$$u(t) = B\mathrm{Sin}(\omega_n t) + A\mathrm{Cos}(\omega_n t) + \frac{p_o}{k}\frac{1}{1-r_\omega^2}\mathrm{Sin}(\omega t)$$

The constants 'A' and 'B' are determined by imposing initial conditions i.e. $u = u(0)$ and $\dot{u} = \dot{u}(0)$

$$u(t) = \underbrace{u(0)\mathrm{Cos}(\omega_n t) + \left[\frac{\dot{u}(0)}{\omega_n} - \frac{p_o}{k}\frac{r_\omega}{1-r_\omega^2}\right]\mathrm{Sin}(\omega_n t)}_{\text{Transient}}$$

$$+ \underbrace{\frac{p_o}{k}\frac{1}{1-r_\omega^2}\mathrm{Sin}(\omega t)}_{\text{Steady state}}$$

AMPLITUDE OF 'STATIC' DEFLECTION DUE TO HARMONIC FORCE

If the force is applied slowly then $\dot{u} = 0$ and the equation of motion under harmonic force $m\ddot{u} + ku = p_o\text{Sin}(\omega t)$ becomes: $ku = p_o\text{Sin}(\omega t)$ or

$$u_{st} = {p_o}/{k}\ \text{Sin}(\omega t)$$

The subscript "st" (standing for static) indicate the elimination of acceleration's effect

The maximum value of static deformation, $(u_{st})_o$ can be interpreted as the deformation corresponding to the amplitude of **p** of the force $\mathbf{p_o}$ **:**

$$\boxed{(u_{st})_o = {p_o}/{k}}$$

For brevity we will refer to $(u_{st})_o$ as the *static deformation*

EFFECT OF FREQUENCY RATIO, r_ω, ON THE DIRECTION OF STRUCTURAL DISPLACEMENTS

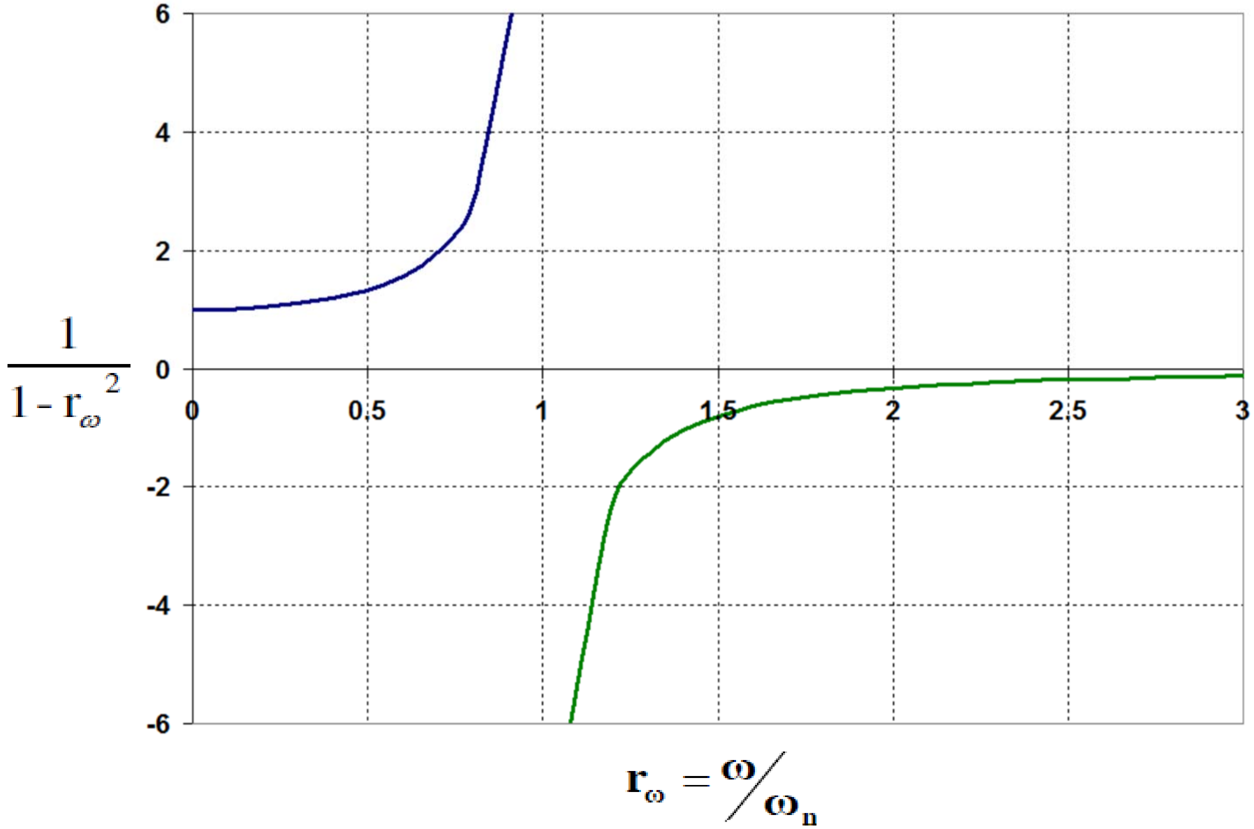
$$u_p(t) = \frac{p_o}{k}\left[\frac{1}{1 - r_\omega^{\ 2}}\right]\mathrm{Sin}(\omega t) \text{ can be written as :}$$

$$u_p(t) = (u_{st})_o\left[\frac{1}{1 - r_\omega^{\ 2}}\right]\mathrm{Sin}(\omega t) \quad \text{where } p_o/k = (u_{st})_o$$

It can be observed from this equation that $u_p(t)$ has negative sign when frequency ratio, $r_\omega > 1$ (i.e. $\omega > \omega_n$), and vice versa. A Graph on next slide is plotted between frequency ratio, r_ω and $\frac{1}{1 - r_\omega^{\ 2}}$

u_p is positive if this term is positive and vice versa

EFFECT OF FREQUENCY RATIO, r_ω, ON THE DIRECTION OF STRUCTURAL DISPLACEMENTS



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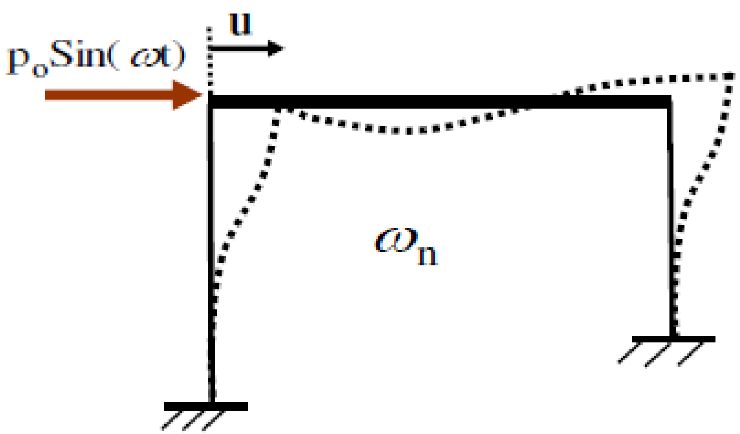
EFFECT OF FREQUENCY RATIO, r_ω, ON THE DIRECTION OF STRUCTURAL DISPLACEMENTS

Following observation can be made from the plot given on previous slide

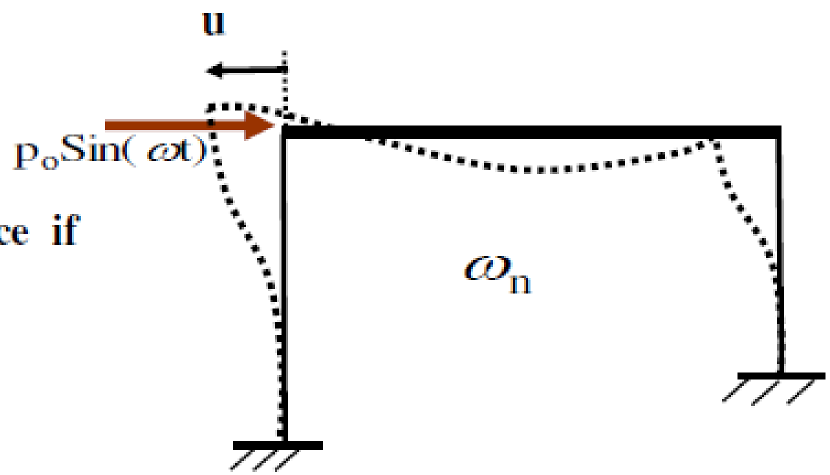
- When $r_\omega < 1$ (i.e. $\omega < \omega_n$), the displacement is positive, indicating that $u_p(t)$ and $p(t)$ has same directions. The displacement is said to be *in phase* with the applied force.

- When $r_\omega > 1$ (i.e. $\omega > \omega_n$), the displacement is negative, indicating that the $u(t)$ and $p(t)$ has opposite direction directions. The displacement is said to be *out of phase* with the applied force.

EFFECT OF FREQUENCY RATIO, r_ω, ON THE DIRECTION OF STRUCTURAL DISPLACEMENTS



Structure displaces in the direction of force if $\omega/\omega_n < 1$



<u>Structure displaces opposite to direction of force if $\omega/\omega_n > 1$</u>

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DEFORMATION RESPONSE FACTOR, R_d

$$u_p(t) = \frac{p_o}{k}\frac{1}{1 - r_\omega^{\;2}}\mathrm{Sin}(\omega t) = (u_{st})_o\frac{1}{1 - r_\omega^{\;2}}\mathrm{Sin}(\omega t)$$

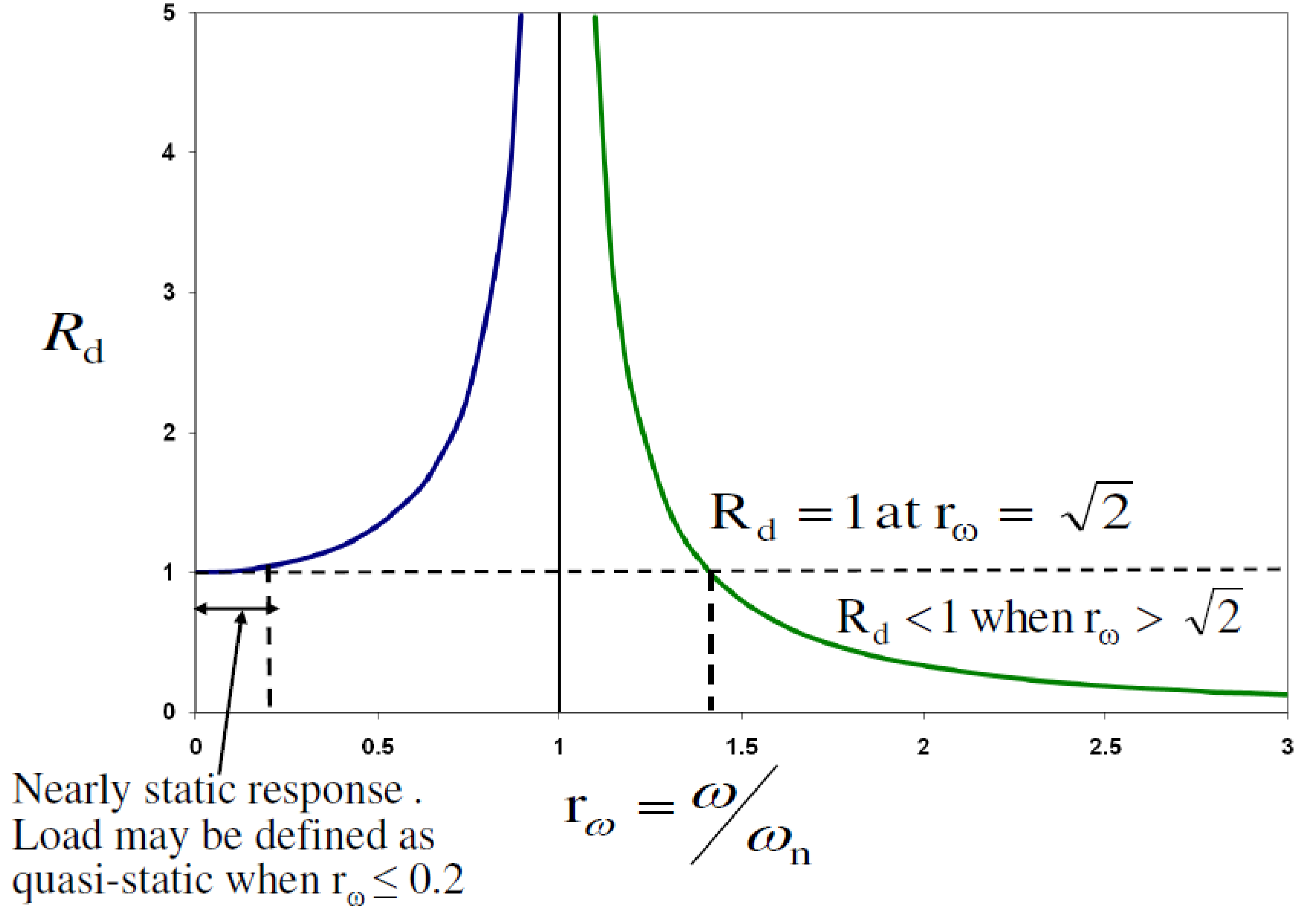
Another mathematical form of the above mentioned equation is:

$$u_p(t) = u_o\mathrm{Sin}(\omega t - \phi) = (u_{st})_o R_d\mathrm{Sin}(\omega t - \phi)$$

$$\boxed{\text{Where } R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{\left|1 - r_\omega^{\;2}\right|}} \text{ and } \phi = \begin{cases} 0^o & \omega < \omega_n \text{ i.e., } r_\omega < 1 \\ 180^o & \omega > \omega_n \text{ i.e., } r_\omega > 1 \end{cases}$$

Where R_d = Dynamic Magnification factor or Deformation (or displacement) response factor, u_o=Amplitude of dynamic displacement, and, φ= Phase angle

INFLUENCE OF FREQUENCY RATIO, r_ω, ON DEFORMATIONRESPONSE FACTOR, R_d



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INFLUENCE OF FREQUENCY RATIO, r_ω, ON DEFORMATION RESPONSE FACTOR, R_d

Following observation can be made from the plot

When r_ω is small (i.e. force is 'slowly varying'), Rd is only slightly greater than 1 or in the other words amplitude of dynamic deformation, u_o, is almost same as amplitude of static deformation, $(u_{st})_o$

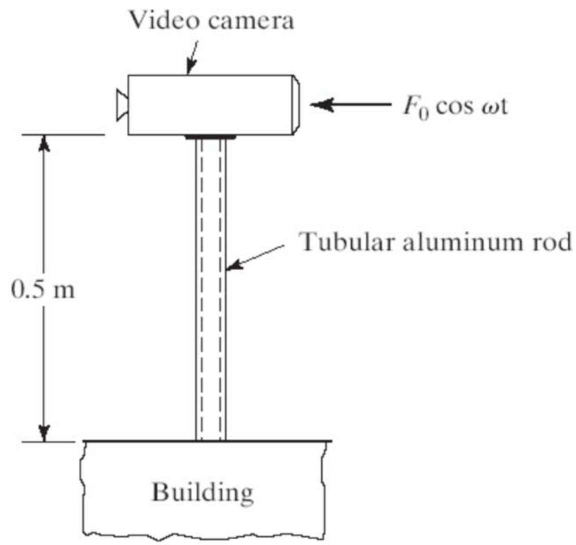
When $r_\omega > \sqrt{2}$ (i.e $\omega > \sqrt{2}\omega_n$), $R_d < 1$ and the dynamic deformation amplitude is less than static deformation.

When r_ω increases beyond $\sqrt{2}$, R_d become smaller and becomes zero as $r_\omega \rightarrow \infty$

When r_ω is close to 1.0, R_d is many times larger than 1.

PROBLEM

A video camera, of mass 2.0 kg, is mounted on the top of a bank building for surveillance. The video camera is fixed at one end of a tubular aluminium rod whose other end is fixed to the building as shown in Fig. The wind-induced force acting on the video camera, is found to be harmonic with p(t) = 25 sin 75t N. Determine the cross-sectional dimensions of the aluminium tube if the maximum amplitude of vibration of the video camera is to be limited to 0.005 m. E Aluminium = 71 GPa.



Thank You