Lecture 7:

Flux Density due to Line Charges, Flux Density due to Volume Charge (CLO2)

1. For further Assessment of lecture taken through Zoom, Watch the YouTube video

Available on following link:

a. https://www.youtube.com/watch?v=8yYdxf0y5Bs

b. https://www.youtube.com/watch?v=6Y2v8BEXLEM

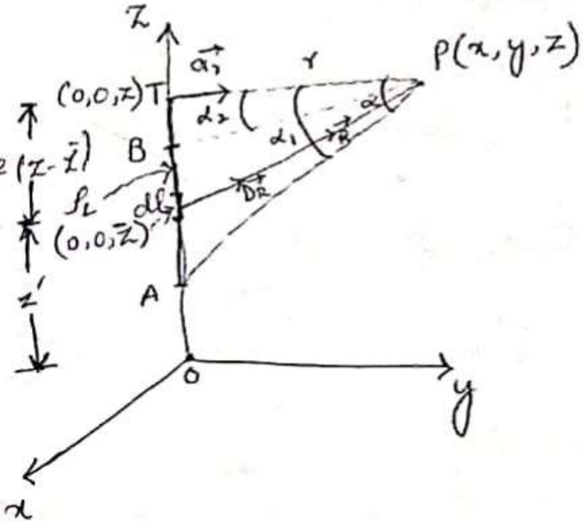
2. Read book in chapter 3 from 2.3 to 2.5

3. The class notes of lecture are available below:

⇒ Lecture # 07

Flux Density due to line charge, }
Flux Density due to volume charge } CLO2

⇒ A Line Charge:-



Line charge with uniform charge density ρ_L (A to B) along z-axis.

$$dQ = \rho_L \, dl$$

$$\vec{E} = \int_L \frac{\rho_L \, dl}{4\pi\epsilon_0 R^2} \vec{a_R} \quad \text{--- (1)}$$

$$\vec{R} = (x,y,z) - (0,0\,z')$$

$$= \underbrace{x\vec{a_x} + y\vec{a_y}}_{r\vec{a_r}} + (z-z')\vec{a_z}$$

$$\vec{R} = r\vec{a_r} + (z-z')\vec{a_z}$$

$$R^2 = |\vec{R}|^2 = r^2 + (z-z')^2$$

Since, $\vec{a_R} = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{\vec{R}}{R}$

$$\frac{\vec{a_R}}{R^2} = \frac{\vec{R}}{|\vec{R}|^3} = \frac{r\vec{a_r} + (z-z')\vec{a_z}}{\left[r^2 + (z-z')^2\right]^{3/2}}$$

$$\left[\begin{array}{l} \vec{dl} = dz' \, \vec{a_z} \\ dl = dz' \end{array}\right]$$

From eq (1)

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0} \int_L \frac{r\vec{a_r} + (z-z')\vec{a_z}}{\left[r^2 + (z-z')\right]^{3/2}} \, dz'$$

$$R = \left[r^2 + (z-z')^2\right]^{1/2}$$

$$= \left[r^2 + r^2 \tan^2\alpha\right]^{1/2}$$

From Fig.

$$\tan\alpha = \frac{z-z'}{r}$$

or $(z-z') = r\tan\alpha$

Also, $r = R\cos\alpha$

$(z-z') = R\sin\alpha$

$$R = r\sqrt{1+\tan^2\alpha} = r\sqrt{\sec^2\alpha}$$

$$R = r\sec\alpha$$

$$z' = OT - (z-z')$$
$$z' = OT - r\tan\alpha$$
$$dz' = -r\sec^2\alpha\, d\alpha$$

From eq (2)

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{r\sec^2\alpha\left[R\cos\alpha\,\vec{a_r} + R\sin\alpha\,\vec{a_z}\right]}{[r\sec\alpha]^3}\,d\alpha$$

Thus, for a line charge

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon_0 r}\int_{\alpha_1}^{\alpha_2} \frac{r\sec^2\alpha\,\overset{R}{(r\sec\alpha)}\left[\cos\alpha\,\vec{a_r} + \sin\alpha\,\vec{a_z}\right]}{[r\sec\alpha]^3}\,d\alpha$$

$$= \frac{-\rho_L}{4\pi\epsilon_0 r}\int_{\alpha_1}^{\alpha_2}\left[\cos\alpha\,\vec{a_r} + \sin\alpha\,\vec{a_z}\right]d\alpha$$

$$\boxed{\vec{E} = \frac{\rho_L}{4\pi\epsilon_0 r}\left[-(\sin\alpha_2 - \sin\alpha_1)\vec{a_r} + (\cos\alpha_2 - \cos\alpha_1)\vec{a_z}\right]}$$

→ Electric field for a line charge.

Special case:- For a infinite line charge: $\begin{cases} B(0,0,\infty) \\ A(0,0,-\infty) \end{cases}$

So $\alpha_1 = \frac{\pi}{2}$; $\alpha_2 = -\frac{\pi}{2}$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 r}\vec{a_r}}$$ → (z - component vanishes)

Surface charge:-

For Surface ①

$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{a_R}$ —①

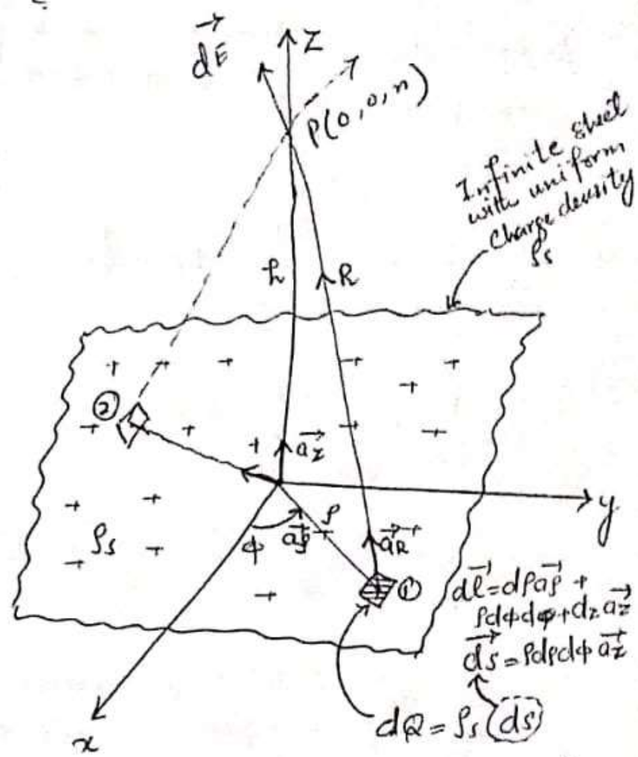
$\rho\vec{a_\rho} + \underbrace{R\vec{a_R}}_{\vec{R}} = h\vec{a_z}$

$\vec{R} = \rho(-\vec{a_\rho}) + h\vec{a_z}$

$R = |\vec{R}| = \sqrt{\rho^2 + h^2}$

$\vec{a_R} = \frac{\vec{R}}{R}$

$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0}\frac{\vec{R}}{R^3}$ → ②



$dQ = \rho(\rho d\rho d\phi)$

From Eq 2

$$d\vec{E} = \frac{\rho_s \rho d\rho d\phi}{4\pi\varepsilon_0}\frac{[\rho(-\vec{a_\rho}) + h\vec{a_z}]}{[\rho^2 + h^2]^{3/2}}$$

$$d\vec{E_z} = \frac{\rho_s \rho\, d\rho\, d\phi}{4\pi\varepsilon_0}\frac{h\vec{a_z}}{[\rho^2 + h^2]^{3/2}}$$

$$\vec{E} = \int_s d\vec{E_z} = \frac{\rho_s}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty}\frac{\rho d\rho\, d\phi\, h\vec{a_z}}{(\rho^2 + h^2)^{3/2}}.$$

Due to Symmetry of charge distribution. For every element ①, there is corresponding element ②, ~~we~~ so contribution along $\vec{a_\rho}$ cancels, (i.e. add upto zero).
So $\vec{E}$ has only z-component.

(4)

Let $\rho^2 + h^2 = u^2$

$2\rho\, d\rho = 2u\, du$

$\rho\, d\rho = u\, du$

when $\rho = 0 \ldots u = h$

$\rho = \infty \ldots u = \infty$

Also, $\vec{a_\rho} = \cos\phi\, \vec{a_x} + \sin\phi\, \vec{a_y}$

Integration of $\cos\phi$ and $\sin\phi$ over $0 < \phi < 2\pi$ gives zero.

$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi} \int_{u=h}^{\infty} \frac{u\,du}{(u^2)^{3/2}}\, d\phi\; h\,\vec{a_z}$$

$$= \frac{\rho_s}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi} d\phi \int_{u=h}^{\infty} \frac{du}{u^2}\; h\,\vec{a_z}$$

$$= \frac{\rho_s}{4\pi\varepsilon_0}(2\pi)\left[-\frac{1}{u}\right]_h^{\infty} h\,\vec{a_z} = \frac{\rho_s}{2\varepsilon_0} h(\vec{a_z})\left(-\frac{1}{\infty} + \frac{1}{h}\right)$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a_z}}$$

$\vec{E}$ has only z-component if the charge is in x-y plane. (valid for $h > 0$)

Note: for $h < 0$, $\vec{a_z}$ is replaced by $(-\vec{a_z})$

Also, $\vec{a_z} \approx \vec{a_n}$ (unit normal vector to the sheet)

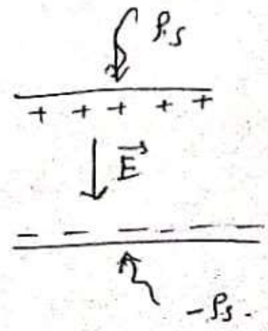
$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a_n}}$$

— normal to the sheet and independent of the distance between the sheet and point P.

<u>Special Case</u> :- Parallel Plate Capacitor.

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a_n} + \frac{-\rho_s}{2\varepsilon_0}(-\vec{a_n}).$$

$$\boxed{\vec{E} = \frac{\rho_s}{\varepsilon_0}\vec{a_n}}$$



Lecture 8:

Divergence Theorem (CLO2)

1. For further Assessment of lecture taken through Zoom, Watch the YouTube video

Available on following link:

- **a.** https://www.youtube.com/watch?v=vrj9wPUI13A&t=105s
- **b.** https://www.youtube.com/watch?v=x5nDX369Ra0

2. Read book in chapter 3 from 3.5 and 3.6

3. The class notes of lecture are available in next Slides.