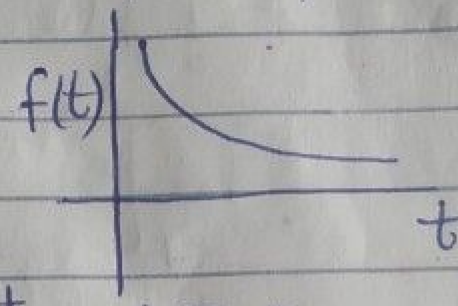
Laplace transform :-

Let there be a function $f(t)$, then

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

s is a variable in complex plane such that $s = \sigma + j\omega$



$$f(t) = e^{-at}, \quad t \geq 0$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-at} e^{-st}\, dt$$

$$= \int_0^\infty e^{(-a-s)t}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= -\left| \frac{e^{-(s+a)t}}{(s+a)} \right|_0^\infty$$

$$= \frac{-1}{(s+a)} \left| e^{-(s+a)t} \right|_0^\infty$$

$$\mathcal{L}[f(t)] = \frac{-1}{s+a}(-1) = \frac{1}{s+a}$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}$$

Trigonometric function :-

$$f(t) = \sin\omega t$$

$$L[f(t)] = \int_0^\infty \sin\omega t \, e^{-st} \, dt$$

$$= \int_0^\infty \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) e^{-st} dt \quad \because \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$= \int_0^\infty \frac{1}{2j}\left[e^{-(s-j\omega)t} dt - e^{-(s+j\omega)t} dt\right]$$

$$= \frac{1}{2j}\left[\frac{-e^{-(s-j\omega)t}}{s-j\omega} + \frac{e^{-(s+j\omega)}}{s+j\omega}\right]_0^\infty$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{s}+j\omega - \cancel{s}+j\omega}{(s-j\omega)(s+j\omega)}\right]$$

$$= \frac{1}{\cancel{2j}} \frac{\cancel{2j}\omega}{s^2+\omega^2}$$

$$L[f(t)] = \frac{\omega}{s^2+\omega^2}$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$L[f(t)] = F(s) = \frac{1}{2}\int_0^\infty \left(e^{j\omega t} e^{-st} + e^{-j\omega t} e^{-st}\right) dt$$

$$= \frac{1}{2}\int \left(e^{-(s-j\omega)t} + e^{-(s+j\omega)t}\right) dt$$

$$= +\frac{1}{2}\left[-\frac{e^{-(s-j\omega)}}{(s-j\omega)} - \frac{e^{-(s+j\omega)}}{(s+j\omega)}\right]_0^\infty$$

$$= +\frac{1}{2}\left[\frac{+1}{(s-j\omega)} + \frac{1}{(s+j\omega)}\right]$$

$$= \cancel{-s - j\omega + s - j\omega}$$

$$= \frac{1}{2}\left[\frac{1}{(s-j\omega)} + \frac{1}{s+j\omega}\right]$$

$$= \frac{s + \cancel{j\omega} + s - \cancel{j\omega}}{2(s-j\omega)(s+j\omega)}$$

$$= \frac{1}{\cancel{2}} \frac{\cancel{2}s}{s^2+\omega^2}$$

$$F(s) = \frac{s}{s^2+\omega^2}$$

	Time Function	Laplace transform
Step	1	$1/s$
Impulse	t	$1/s^2$
	t^n	$\frac{n!}{s^{n+1}}$
	e^{-at}	$1/s+a$
	$\sin \omega t$	$\omega/s^2+\omega^2$
	$\cos \omega t$	$s/s^2+\omega^2$
	$\sinh(\omega t)$	$\omega/s^2-\omega^2$
	$\cosh(\omega t)$	$s/s^2-\omega^2$

$$f^{n}(s) = s^{n} L f(s) - s^{n-1} f(0) - s^{n-2} f'(0) \ldots - f^{(n-1)}(0)$$

At $n=2$,

$$f^{2}(s) = s^{2} L f(s) - s f(0) - f'(0) \ldots - f^{(n-1)}(0)$$

$$f^{3}(s) = s^{3} L f(s) - s^{2} f(0) - s f'(0) - f''(0)$$

$$f^{4}(s) = s^{4} L f(s) - s^{3} f(0) - s^{2} f'(0) - s f''(0) - f'''(0)$$