Lecture 6:

Electric Flux Density; Guass's Law (CLO2)

1. **For further Assessment of lecture taken through Zoom, Watch the YouTube video available on following link:**

   a. https://www.youtube.com/watch?v=6STe-VIZUcs
   b. https://www.youtube.com/watch?v=Ctn3chDWdSI
   c. https://www.youtube.com/watch?v=4x2kvSFCrIl&t=44s

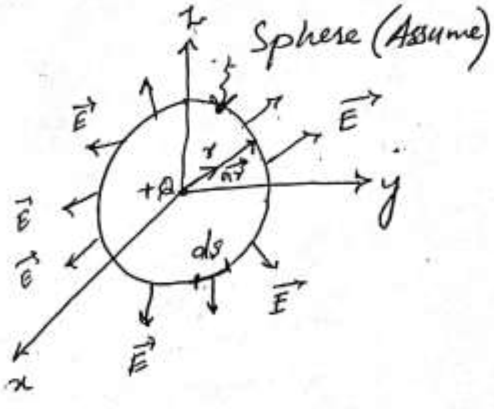
2. **Read book in chapter 3 from 3.1 to 3.4**

3. **The class notes of lecture is available below:**

(3a)

Electric flux Density:-

$$\vec{E} = \frac{Q}{4\pi \mathcal{E}_0 r^2} \vec{a_r} \ N/C$$



$$\mathcal{E}_0 \vec{E} = \frac{Q}{4\pi r^2} \vec{a_r}$$

$4\pi r^2$ → Surface area of Sphere

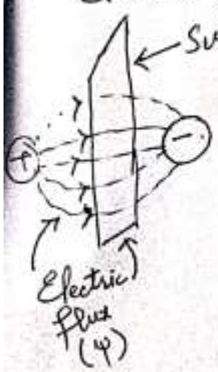
So $\frac{Q}{4\pi r^2} = \frac{\text{Charge}}{\text{Area}} = \vec{D}$ → Electric flux density

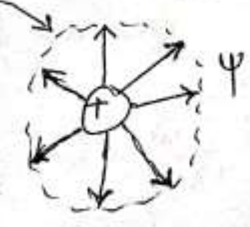
$\boxed{\vec{D} = \mathcal{E}_0 \vec{E}}$ — vector

$\vec{D} \rightarrow (C/m^2)$

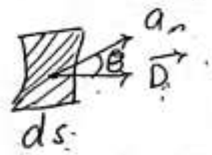
Also called Surface charge density (ρ_s)

Electric flux is denoted by $\boxed{\text{Electric Flux } (\Psi)}$





(Terminate to ∞) In absence of −ve charge.



$\vec{D}$ over a small area ds

Small flux $d\Psi$:

$$d\Psi = \vec{D} \cdot \vec{ds}$$

$$\Psi = \int_s \vec{D} \cdot \vec{ds}$$

Ψ → Measured in Coloumbs (Scaler)

$\vec{D}$ → Measured in (C/m^2).

"Flux ≈ Number of lines that passes through the Surface".

Flux → Scaler

N:- $\vec{D} = \mathcal{E}_0 \vec{E}$ → free space or Vacuum
→ Also Called displacement flux density

$$\vec{D} = \mathcal{E}\vec{E} = \mathcal{E}_0 \mathcal{E}_r \vec{E} \longrightarrow \text{dielectric medium}$$

So, $\boxed{\mathcal{E} = \mathcal{E}_0 \mathcal{E}_r}$

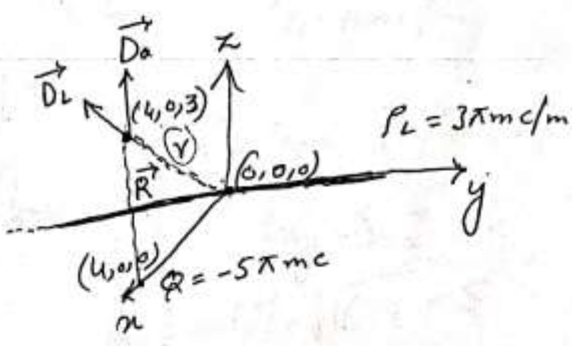
(3-b)

Infinite line:- $\vec{E} = \dfrac{\rho_L}{2\pi\epsilon_0 r}\vec{a_r}$ $\qquad \boxed{\vec{D} = \epsilon_0 \vec{E} = \dfrac{\rho_L}{2\pi r}\vec{a_r}}$

Infinite Sheet:- $\vec{E} = \dfrac{\rho_s}{2\epsilon_0}\vec{a_n}$ $\qquad \boxed{\vec{D} = \epsilon_0 \vec{E} = \dfrac{\rho_s}{2}\vec{a_n}}$

Point charge $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}\vec{a_r}$ $\qquad \boxed{\vec{D} = \epsilon_0 \vec{E} = \dfrac{Q}{4\pi r^2}\vec{a_r}}$

Q:- Determine $\vec{D}$ at $(4,0,3)$ if there is a point charge $-5\pi\, mC$ at $(4,0,0)$ and a line charge $3\pi\, mC/m$ along the y-axis



S:- $\vec{D} = \vec{D_Q} + \vec{D_L}$

due to point charge (for $\vec{D_Q}$); Due to line charge (for $\vec{D_L}$).

$$\vec{D_Q} = \epsilon_0 \vec{E} = \frac{Q}{4\pi R^2}\vec{a_R} = \frac{Q}{4\pi R^3}\vec{R}$$

$$= \frac{-5\pi\times10^{-3}(3\vec{a_z})}{4\pi(3)^2} = \frac{-5\times10^{-3}}{4\times27}(3\vec{a_z})$$

$$\boxed{\vec{D_Q} = -0.138\,\vec{a_z}\ mC/m^2}$$

$$\vec{D_L} = \frac{\rho_L}{2\pi r}\vec{a_r} = \frac{\rho_L \vec{r}}{2\pi r^2}$$

$$= \frac{3\pi(4\vec{a_x}+3\vec{a_z})}{2\pi(5)(5)}$$

$$= 0.24\vec{a_x} + 0.18\vec{a_z}\ mC/m^2$$

Thus $\vec{D} = \vec{D_Q} + \vec{D_L}$

$$= 240\vec{a_x} + 42\vec{a_z}\ \mu C/m^2$$

Ans.

$\vec{R} = (4,0,3) - (4,0,0)$

$= (0,0,3)$

$\vec{R} = 3\vec{a_z}$

$\boxed{R = 3}$ ✓

$\vec{a_R} = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{3\vec{a_z}}{3} = \vec{a_z}$

$\vec{a_r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{4\vec{a_x}+3\vec{a_z}}{\sqrt{16+9}}$

$\vec{a_r} = \dfrac{4\vec{a_x}+3\vec{a_z}}{5}$

$\boxed{r = |\vec{r}| = 5}$

3c

Q:- A point charge of 30nC is located at the origin, while plane $y=3$ carries charge 10nC/m². Find $\vec{D}$ at $(0, 4, 3)$.

Sol:- $\vec{D} = \vec{D}_Q + \vec{D}_{\rho_s}$

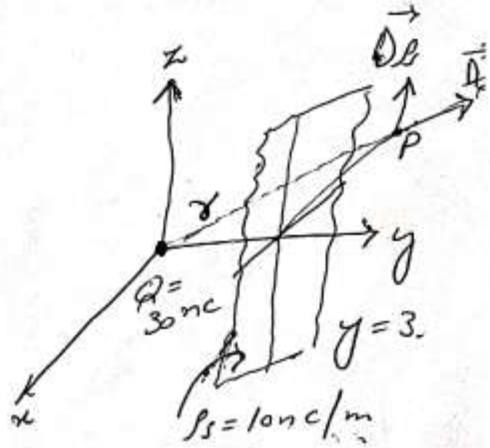
$$= \frac{Q}{4\pi r^2}\vec{a}_r + \frac{\rho_s}{2}\vec{a}_n$$

$$= \frac{Q\vec{r}}{4\pi r^3} + \frac{10\times 10^{-9}}{2}\vec{a}_y \quad [\because \vec{a}_n = \vec{a}_y]$$

$$= \frac{3\times 10^{-9}(4\vec{a}_y + 3\vec{a}_z)}{4\pi(5)^3} + 5\times 10^{-9}\vec{a}_y$$

$$= \frac{30}{500\pi}(4\vec{a}_y + 3\vec{a}_z) + 5\vec{a}_y \text{ nC/m}^2$$

$$\boxed{\vec{D} = 5.076\,\vec{a}_y + 0.0573\,\vec{a}_z \text{ nC/m}^2}$$ Ans.



$\vec{r} = (0, 4, 3) - (0, 0, 0)$

$\vec{r} = 4\vec{a}_y + 3\vec{a}_z$

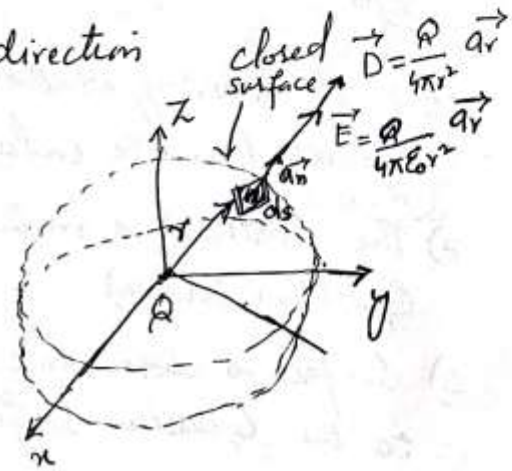
$r = |\vec{r}| = \sqrt{16+9} = 5$

GAUSS'S LAW – MAXWELL'S EQUATION:- 4a

"Gauss's law states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface".

$$\boxed{\psi = Q_{enc} = \oint_S \vec{D} \cdot \vec{ds}} \quad \text{— (1)}$$

, $\vec{D}$ and $\vec{E}$ has the same direction



$$D \cdot \vec{ds} = \frac{Q}{4\pi r^2}\vec{a_r} \cdot ds\,\vec{a_n}$$

$$= \frac{Q}{4\pi r^2}\vec{a_r} \cdot ds\,\vec{a_r} \quad [\because \vec{a_n} = \vec{a_r}]$$

$$D \cdot \vec{ds} = \frac{Q\,ds}{4\pi r^2}$$

Integration both sides. (i.e. closed surface integration).

$$\oint_S \vec{D} \cdot \vec{ds} = \oint_S \frac{Q\,ds}{4\pi r^2} = \frac{Q}{4\pi r^2}\oint_S ds = \frac{Q}{4\pi r^2} \times 4\pi r^2 = Q$$

Since, $Q = \int_v \rho_v\,dv$ ---- total charge enclosed.

$$\boxed{\therefore \oint_S \vec{D} \cdot \vec{ds} = \int_v \rho_v\,dv = Q} \quad \text{— (2)}$$

Area of sphere is $4\pi r^2$

By Divergence theorem applying, we get

$$\oint_S \vec{D} \cdot \vec{ds} = \int_v \nabla \cdot \vec{D}\,dv \quad \text{— (3)}$$

Comparing eq. no. (2) & (3)

$$\boxed{\rho_v = \nabla \cdot D}$$ — first form of Maxwell.

" The volume charge density is the same as the divergence of the electric flux density".

Applications of GAUSS'S LAW:-

steps:-

1) First knowing whether Symmetry exists, then apply Gauss's law to calculate Electric Field.

2) The construct a mathematical closed surface (known as Gaussian Surface).

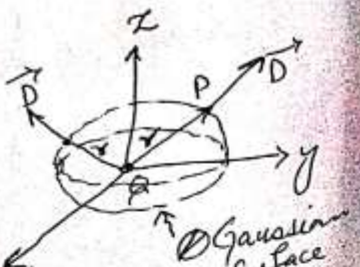
3) Surface is chosen such that $\vec{D}$ is normal or tangential to the Gaussian Surface.

$$\vec{D}\cdot\vec{ds} = D\,ds \;,\quad \theta = 0^\circ \quad [\vec{D} \text{ normal to surface}]$$
$$\vec{D}\cdot\vec{ds} = 0 \;,\quad \theta = 90^\circ \quad [\vec{D} \text{ tangential to Surface}]$$

POINT CHARGE:-

Q:- Find electric flux density "$\vec{D}$" in case of point charge by applying Gauss's law?

Sol:-



$\vec{D} = D_r\,\vec{a_r} \Rightarrow D_r = \vec{D}/\vec{a_r}$

$\vec{ds} = ds\,\vec{a_r}$

$Q = \oint_s \vec{D}\cdot\vec{ds} = D_r \oint_s ds = D_r\, 4\pi r^2$

Thus, $\boxed{\vec{D} = \frac{Q}{4\pi r^2}\,\vec{a_r}}$

$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$

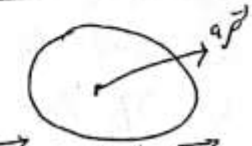
$\therefore \oint \vec{ds} = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta\, d\theta\, d\phi$

$= 4\pi r^2$ ⇒ Surface area of Gaussian surface.

The cylindrical surface $\rho = 8\,cm$ contain the surface charge density $\rho_s = 5 \times 10^{-20|z|}\ nc/m^2$. (4c)

a) what is the total amount of charge present?

b) How much electric flux leaves the surface $\rho = 8\,cm$, $1\,cm < z < 5\,cm$, $30^\circ < \phi < 90^\circ$?



Sol:- (a) $Q = \int_s \rho_s ds = \int_s (5e^{-20|z|} \times 10^{-9})\, \rho\, d\phi\, dz$

$\vec{ds} = \rho\, d\phi\, dz\, \vec{a_\rho}$ ↳ Cylinder Surface (curve)

$$= 5\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=-\infty}^{\infty} e^{-20|z|} dz \ \ nc.$$

$$= 5\rho \int_0^{2\pi} d\phi \left\{ 2\int_0^{\infty} e^{-20z} dz \right\} nc.$$

$$= 5(0.08)(2\pi)\left\{ 2\int_0^{\infty} e^{-20z} dz \right\} nc.$$

$$= 5.026 \int_0^{\infty} e^{-20z} dz \ nc.$$

$$= 5.026 \left[\frac{e^{-20z}}{-20} \right]_0^{\infty} = 5.026\left(0 + \frac{1}{20}\right)$$

$$\boxed{Q = 0.25\,nC \ \ \text{Ans.}}$$

(b) $\rho = 8\,cm$, $1\,cm < z < 5\,cm$, $30^\circ < \phi < 90^\circ$.

Electric flux leaves the surface = Total charge enclosed.

So, $Q = \int_s \rho_s ds = \int_s 5\, e^{-20|z|} ds \ nc = 5\rho \int_{30^\circ}^{90^\circ} d\phi \left\{ \int_{0.01}^{0.05} e^{-20z} dz \right\} nc$

$$= 5(0.08)\, |\phi|_{\pi/6}^{\pi/2} \left| \frac{e^{-20z}}{-20} \right|_{0.01}^{0.05} = 0.4\left(\frac{\pi}{2} - \frac{\pi}{6}\right)\left[\frac{-e^{-20(0.05)} + e^{-20(0.01)}}{20}\right]$$

$$= 0.4\left(\frac{3\pi - 2\pi}{6}\right)\left(\frac{-0.37 + 0.819}{20}\right) = 0.42 \times 0.02$$

$$= 9.429 \times 10^{-3}\ nc$$

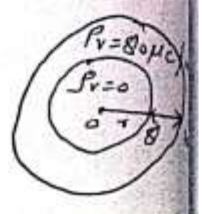
$$= 9.429\ pC \ \ \text{Ans}$$

GAUSS'S LAW NUMERICAL.

Q:- A uniform volume charge density of $80\mu C/m^3$ is present throughout the region $8mm < r < 10mm$. Let $\rho_v = 0$ for $0 < r < 8mm$.

(a) Find the total charge inside the spherical surface $r = 10$
(b) Find D_r at $r = 10mm$.
(c) If there is no charge for $r > 10mm$, find D_r at $r = 20mm$

(a):- $Q = \int \rho_v \, dv$ at $r = 10mm$



$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=8mm}^{10mm} (80\times10^{-6}) \underbrace{r^2 \sin\theta \, dr \, d\theta \, d\phi}_{dv}$$

$$= 80\times10^{-6} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_{0.008}^{0.010} r^2 dr = 80\times10^{-6}(2\pi)\left[-\cos\theta\right]_0^{\pi} \left[\frac{r^3}{3}\right]_{0.008}^{0.010}$$

$$= 80\times10^{-6}(2\pi)(1+1)\left(\frac{0.010^3}{3} - \frac{0.008^3}{3}\right)$$

$$= 80\times10^{-6} \times 4\pi \times (1.63\times10^{-7})$$

$$= 1.64\times10^{-10} C$$

$$Q = 164\times10^{-12} C = 164 \, pC \text{ Ans.}$$

(b) $D_r = \dfrac{Q}{4\pi r^2}$ $\Rightarrow$ $D_r(r = 0.10m) = \dfrac{164\times10^{-12}}{4\pi(0.010)^2} = \dfrac{164\times10^{-12}}{1.26\times10^{-3}}$
$\hookrightarrow$ (G.S)

$$= 1.30\times10^{-7} = 130 \, nC/m^2 \text{ Ans}$$

(c) $D_r(r = 0.020m) = \dfrac{Q}{4\pi r^2} = \dfrac{164\times10^{-12}}{4\pi(0.02)^2} = \dfrac{164\times10^{-12}}{5\times10^{-3}}$
$\hookrightarrow$ G.S

$$= 32.8\times10^{-9} \, C/m^2$$

$$= 32.8 \, nC/m^2 \text{ Ans.}$$