Lecture 5

Electric Field Intensity

1. For further Assessment of lecture taken through Zoom, Watch the YouTube video available on following link:

   https://www.youtube.com/watch?v=jBzoDplsT4Q

2. Read out chapter 2.2 and 2.3 for full assessment;

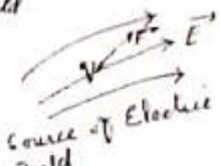
   Home Work: Try to solve D 2.2, D 2.3, and 2.4.

3. As well the Notes of Lecture is available below

Electric field intensity :- ($\vec{E}$)
↳ vector quantity

→ The vector force on a unit positive test charge. or. $\vec{E}$ is the force per unit charge when placed in an electric field

$$F_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2} a_{1t}$$



Writing this force as a force per unit charge gives

$$\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} a_{1t}$$

Electric field intensity must be measured by the unit newtons per coulomb - the force per unit charge. (volts/meter)

Using a Capital letter E for electric field intensity, we have finally.

$$\vec{E} = \frac{F_t}{Q_t} = \frac{\vec{F}}{q} \quad \text{—①}$$

$$E = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} a_{1t} \quad \text{—②}$$

Eq① is the defining expression for electric field intensity, and eq② is the expression for the electric field intensity due to a single point charge Q_1 in a vacuum

$$E = \frac{Q}{4\pi\epsilon_0 R^2} a_R \qquad a_R = \frac{\vec{R}}{R}$$

R is the magnitude of the vector R, the directed

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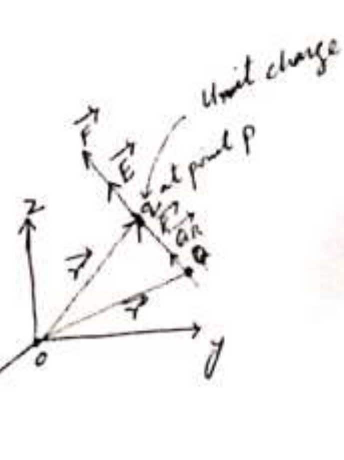
Electrical field intensity at some point :

Electric field intensity $\vec{E}$ at point $\vec{r}$ due to a point



$$\vec{r'} + R = \vec{r}$$

or $\vec{R} = \vec{r} - \vec{r'}$

$$\vec{F} = \frac{Qq(\vec{r} - \vec{r'})}{4\pi\epsilon_0 |(\vec{r} - \vec{r'})|^3}$$

$$\vec{F} = \frac{Qq}{4\pi\epsilon_0 R^2} \vec{a_R}$$

$$\vec{F} = \frac{Qq}{4\pi\epsilon_0} \frac{\vec{R}}{R^3}$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\epsilon_0 R^2} \vec{a_R}$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\epsilon_0 R^2}\left(\frac{\vec{R}}{R}\right) = \frac{Q(\vec{r} - \vec{r'})}{4\pi\epsilon_0 R^3} = \frac{Q(\vec{r} - \vec{r'})}{4\pi\epsilon_0 |(\vec{r} - \vec{r'})|^3} \checkmark$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \vec{a_R} = \frac{Q(\vec{r} - \vec{r'})}{4\pi\epsilon_0 |\vec{r} - \vec{r'}|^3}$$

Special Case: – $\vec{E}$ at point $\vec{r}$ due to 'N' point charges $Q_1, Q_2, \ldots, Q_N$ located at $\vec{r_1}, \vec{r_2}, \ldots, \vec{r_N}$ is given as 2 (b)

$$\vec{E} = \frac{Q_1(\vec{r}-\vec{r_1})}{4\pi\varepsilon_0|\vec{r}-\vec{r_1}|^3} + \frac{Q_2(\vec{r}-\vec{r_2})}{4\pi\varepsilon_0|\vec{r}-\vec{r_2}|^3} + \cdots + \frac{Q_N(\vec{r}-\vec{r_N})}{4\pi\varepsilon_0|\vec{r}-\vec{r_N}|^3}$$

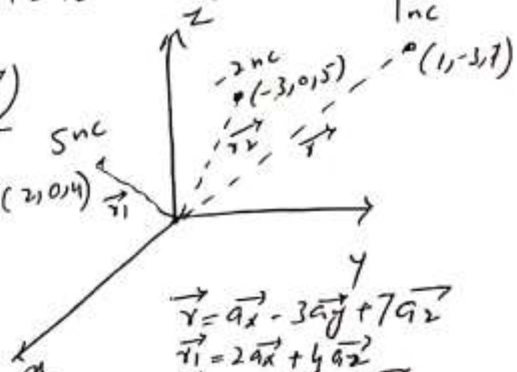
or
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\vec{r}-\vec{r_k})}{|\vec{r}-\vec{r_k}|^3}$$

Q:- Point charges 5nc and −2nc are located at (2,0,4) and (−3,0,5) respectively.

a) Determine the force on a 1nc point charge located at (1,−3,7)

b) Find the electric field $\vec{E}$ at (1,−3,7).

Sol:- (a)
$$\vec{F} = \frac{(1\times10^{-9})(5\times10^{-9})}{4\pi\left(\frac{10^{-9}}{36\pi}\right)}\frac{(\vec{a_x}-3\vec{a_y}+7\vec{a_z})-(2\vec{a_x}+4\vec{a_z})}{\left(\sqrt{-\vec{a_x}-3\vec{a_y}+3\vec{a_z}}\right)^3}$$
$$+\frac{(1\times10^{-9})(-2\times10^{-9})}{4\pi\left(\frac{10^{-9}}{36\pi}\right)}\frac{(\vec{a_x}-3\vec{a_y}+7\vec{a_z})-(-3\vec{a_x}+5\vec{a_z})}{\left(\sqrt{4\vec{a_x}-3\vec{a_y}+2\vec{a_z}}\right)^3}$$



$\vec{r} = \vec{a_x} - 3\vec{a_y} + 7\vec{a_z}$
$\vec{r_1} = 2\vec{a_x} + 4\vec{a_z}$
$\vec{r_2} = -3\vec{a_x} + 5\vec{a_z}$

$$= (45\times10^{-9})\frac{(-\vec{a_x}+3\vec{a_y}+3\vec{a_z})}{(19)^{3/2}} - (18\times10^{-9})\frac{(4\vec{a_x}-3\vec{a_y}+2\vec{a_z})}{(29)^{3/2}} = \frac{(-45,-135,135)}{(19)^{3/2}} - \frac{(72,-54,36)}{(29)^{3/2}}\text{ nN}$$

$$= \frac{(-45,-135,135)}{82.819} - \frac{(72,-54,36)}{156.17}\text{ nN}$$

$= (-0.543, -1.630, 1.630) - (0.461, -0.346, 0.231)$.

$\vec{F} = -1.004\vec{a_x} - 1.284\vec{a_y} + 1.4\vec{a_z}$ nN Ans.

b) $$\vec{E} = \frac{\vec{F}}{q} = \frac{-1.004\vec{a_x} - 1.284\vec{a_y} + 1.4\vec{a_z}}{(1\times10^{-9})}\times10^{-9}$$

$\vec{E} = -1.004\vec{a_x} - 1.284\vec{a_y} + 1.4\vec{a_z}$ V/m Ans

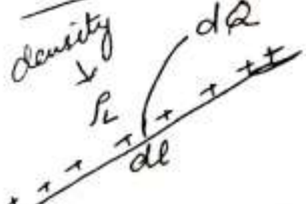
(C)

⇒ Electric field due to Continuous Charge distribution

$\cdot +Q$ Point charge.

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

Line charge:-

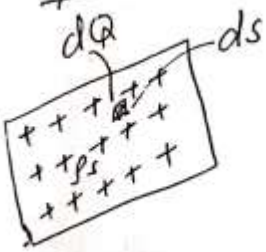


$\rho_L \approx$ Line charge density

$dQ = \rho_L\, dl$ or $\rho_L = \frac{dQ}{dl}$

$Q = \int_L \rho_L\, dl$ so, $$E = \int_L \frac{\rho_L\, dl}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

Surface charge:-

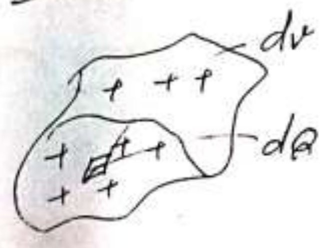


ρ_s = surface charge density $= \frac{dQ}{ds}$

$dQ = \rho_s\, ds$

$Q = \int_s \rho_s\, ds$ so, $$\vec{E} = \int_s \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

Volume charge:-



$\rho_v \approx$ volume charge density $= \frac{dQ}{dv}$

$Q = \int_v \rho_v\, dv$

So

$$\vec{E} = \int_v \frac{\rho_v\, dv}{4\pi\varepsilon_0 R^2}\vec{a_R}$$