$$= (-1) \int_{\infty}^{-\infty} x(t-r)\, h(r)\, (-dr)$$

$$= \int_{-\infty}^{\infty} x(t-r)\, h(r)\, dr.$$

$$= \int_{-\infty}^{\infty} h(r)\, x(t-r)\, dr$$

$$\boxed{x(t) * h(t) = h(t) * x(t)}$$

Hence Proved.

$\longleftrightarrow$

2) Distributive Property:

Distributive property holds true in case of convolution. Hence, Convolutio distributes over addition.

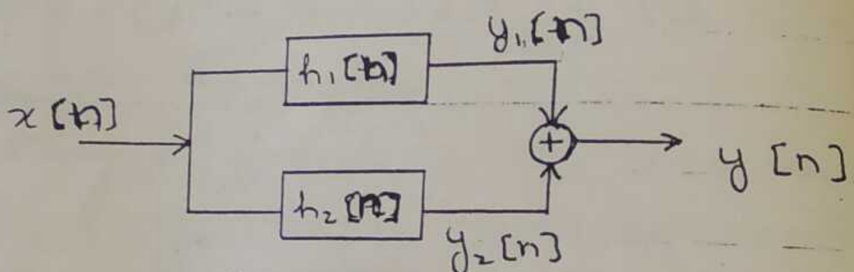
Case Of Discrete-Time Signals:

In case of discrete-time signals:

$$\boxed{x[n] * [h_1[n] + h_2[n]] = x[n] * h_1[n] + x[n] + h_2[n]}$$

Proof:

Consider an LTI system:-



(A)

Here, the output is given by:

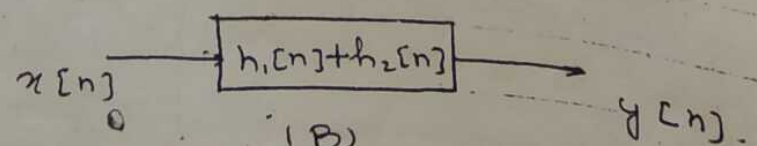
$y_1[n] = x[n] * h_1[n]$ & $y_2[n] = x[n] * h_2[n]$

Now if we combine the two signals:

$$y[n] = y_1[n] + y_2[n].$$

$$y[n] = x[n] * h_1[n] + x[n] \; h_2[n] \longrightarrow (1)$$

Now again we draw the block diagram:



(B)

Both the block diagrams are identical & give the same response.

Hence from block diagram (B)

$$y[n] = x[n] * [h_1[n] + h_2[n]] \longrightarrow (2)$$

3) Associative Property:-

Associative property holds true in case of convolution.

Case of Discrete-Time Systems:-

In case of discrete-time systems:-

$$\boxed{x[n] * [h_1[n] * h_2[n]] = [x[n] * h_1[n]] * h_2[n]}$$

Consider.

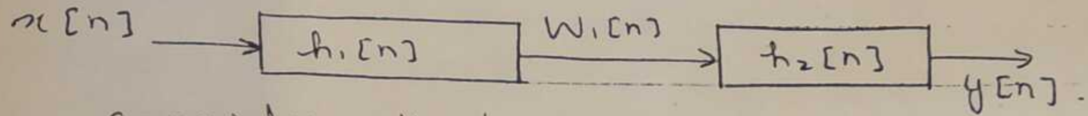
$$y[n] = x[n] * h_1[n] * h_2[n].$$

Let, $x[n] * h_1[n] = W_1[n]$.

$\therefore$

$$y[n] = [x[n] * h_1[n]] * h_2[n] \longrightarrow (1)$$

$$y[n] = W_1[n] * h_2 n$$

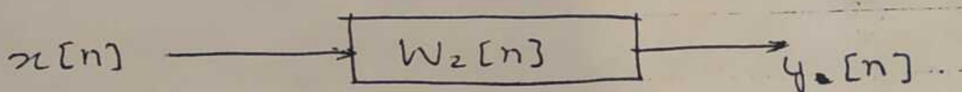


Now consider that

$$W_2[n] = h_1[n] * h_2[n].$$

$\therefore$

$$y[n] = x[n] * [h_1[n] * h_2[n]].$$
$$= x[n] * W_2[n].$$



Both block diagrams give the same response.

Hence.

$$\boxed{[x[n] * h_1[n]] * h_2[n] = x[n] * [h_1[n] * h_2[}$$

Hence proved.

Case Of Continuous Time-Systems:-

In case of continuous-time systems:

$$\boxed{[x(t) * h_1(t)] * h_2[t] = x(t) * [h_1(t) * h_2(t)]}$$

Similarly, it can also be proved.

Block Diagram Representations Of First Order Systems Described By Differential & Difference Equations:

An important property of the systems described by linear constant co-efficients ~~and~~ differential equations is that, they can be represented in a very simple and natural way in terms of block diagram interconnections of elementary operations.

Case Of Discrete-Time Signals:

Consider a discrete-time signal represented by a difference eq.

$$y[n] = -a\,y[n-1] + b\,x[n] \longrightarrow (1)$$

First we define different components of block diagram.

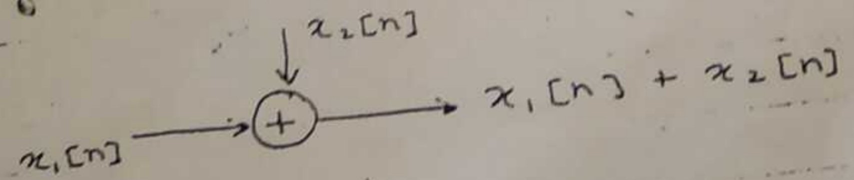
Multiplier:-

In a multiplier, a signal is multipli[ed] by a scalar i.e.

$$x[n] \xrightarrow{\quad b \quad} b\,x[n]$$

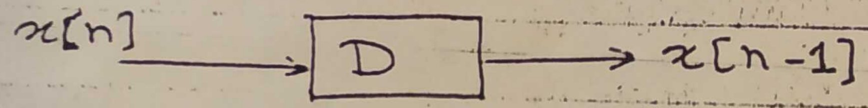
Adder:-

In an adder, two or more input signals are applied to get the algebraic sum of them.



Delayed Circuit:

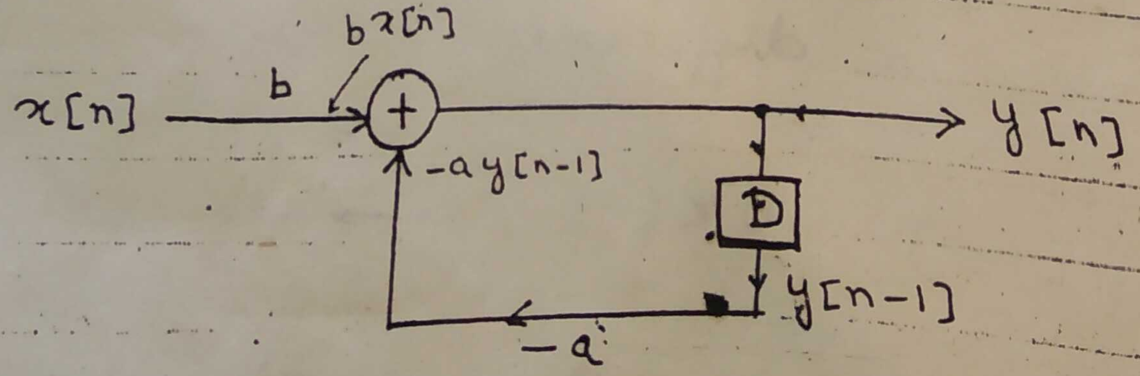
Delayed circuit produces a unit delay.



Now again consider eq. (1)

$$y[n] = -a\,y[n-1] + b\,x[n]$$

$$y[n] = b\,x[n] - a\,y[n-1]$$



(Block Diagram)