Z-transform :-

$$X(z) = \sum_{k=0}^{N} x[k] z^{-k}$$

Q $x[n] = 2\delta[n] + 3\delta[n-1] + 5\delta[n-2] + 2\delta[n-3]$

$$X(z) = 2 + 3z^{-1} + 5z^{-2} + 2z^{-3}$$

Properties :-

i) Linearity

$$ax_1[n] + bx_2[n] \overset{Z}{\longleftrightarrow} aX_1(z) + bX_2(z)$$

Let $x[n] = ax_1[n] + bx_2[n]$

$$X(z) = \sum_{k=0}^{N} x[k] z^{-k}$$

$$= \sum_{n=0}^{N} (ax_1[n] + bx_2[n]) z^{-1}$$

$$= a\sum_{n=0}^{N} x_1[n] z^{-1} + b\sum_{n=0}^{N} x_2[n] z^{-1}$$

$$X(z) = aX_1(z) + bX_2(z)$$

ii) Time delay property

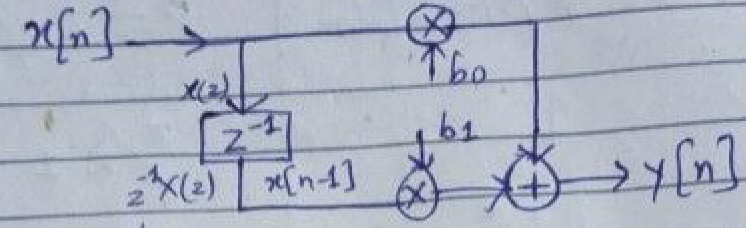
$$x[n-1] \overset{Z}{\longleftrightarrow} z^{-1} X(z)$$

$$x[n-n_0] \overset{Z}{\longleftrightarrow} z^{-n_0} X(z)$$

Z-transform as an operator :-

$$y[n] = z^{-1}\{x[n]\} = x[n-1]$$

$$Y(z) = z^{-1} X(z)$$



We can write as →

$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z)$$
$$y[n] = b_0 x[n] + b_1 x[n-1]$$

Convolution →

$$Y(z) = H(z) \cdot X(z)$$

Q $x[n] = 2\delta[n] - 3\delta[n-2] + 4\delta[n-3]$

$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

Find $y[n]$

$$X(z) = 2 - 3z^{-2} + 4z^{-3}$$
$$H(z) = 1 + 2z^{-1} + z^{-2}$$

Now,

$$Y(z) = H(z) * X(z)$$
$$= (2 - 3z^{-2} + 4z^{-3})(1 + 2z^{-1} + z^{-2})$$
$$= 2 + 4z^{-1} + 2z^{-2} - 3z^{-2} - 6z^{-3} - 3z^{-4}$$
$$+ 4z^{-3} + 8z^{-4} + 4z^{-5}$$

$$Y(z) = 2 + 4z^{-1} - z^{-2} - 2z^{-3} + 5z^{-4} + 4z^{-5}$$

To find $y[n]$, use the delay property

$$y[n] = 2\delta[n] + 4\delta[n-1] - \delta[n-2] - 2\delta[n-3]$$
$$+ 5\delta[n-4] + 4\delta[n-5]$$

Assign Q $x[n] = 4 - 3\delta[n-1] + 4\delta[n-2]$

$h[n] = 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3]$

Time representation	Z-transform
1	$z/z-1$
$(-1)^k$	$z/z+1$
k	$z/(z-1)^2$
k^2	$z(z+1)/(z-1)^3$
e^{ak}	$z/z-e^a$
a^k	$z/z-a$
ka^k	$za/(z-a)^2$

$$X(z) = \frac{2z^2+z}{z^2-1.5z+0.5}$$

$$X(z) = \frac{z(2z+1)}{z^2-1.5z+0.5}$$

$$\frac{X(z)}{(z)} = \frac{2z+1}{z^2-1.5z+0.5}$$

$$\frac{2z+1}{z^2-1.5z+0.5} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)} \rightarrow$$

Or $2z+1 = A(z-0.5)+B(z-1$

Put $z = 0.5$

$$2(0.5)+1 = A(\cancel{0.5-0.5})+B(0.5-1)$$

$$2 = (-0.5)B$$

$$B = -4$$

Put $z = 1$

$$2(1)+1 = A(1-0.5)+\cancel{B(1-1)}$$

$$2+1 = A(+0.5)$$

$$3 = A(0.5)$$

$$A = 6$$

Now put A, B in (*)

$$\frac{2z+1}{(z-1)(z-0.5)} = \frac{6}{(z-1)} - \frac{4}{(z-0.5)}$$

$$X(z) = 6\frac{z}{z-1} - 4\frac{z}{z-0.5}$$

~~X(z)~~ is inverse z-transfolm

$$x[n] = 6u[n] - 4(0.5)^k$$