LECTURE # 5

In this lecture you will learn about:

- Equilibrium of Rigid Bodies.
- Modelling the Action of Forces in Two-Dimensional Analysis.
- Equations of Equilibrium in Two-Dimensional Case.
- Two Force Members.
- Three Force Members.
- Free Body Diagram
- Sample Free Body Diagrams.

Course Name:

"Applied Mechanics"

Course Code: CT-144

Credit Hours: 3

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EQUILIBRIUM OF RIGID BODIES

When a rigid body is in ***equilibrium***, both the resultant force and the resultant couple must be zero.

$$\vec{R} = \Sigma \vec{F} = 0$$

$$R_x \vec{i} + R_y \vec{j} + R_z \vec{k} = 0$$

$$\Sigma \vec{M} = 0$$

$$\Sigma M_x \vec{i} + \Sigma M_y \vec{j} + \Sigma M_z \vec{k} = 0$$

EQUILIBRIUM OF RIGID BODIES

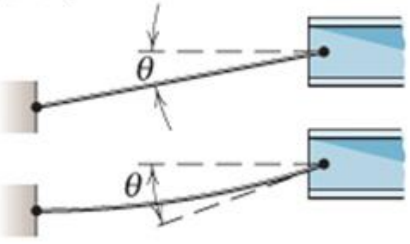
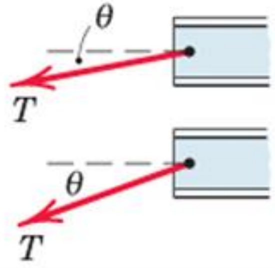
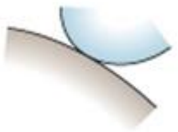
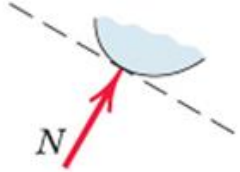
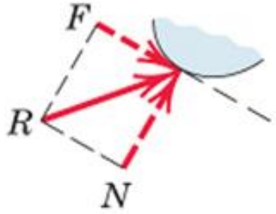
- Forces and moments acting on a rigid body could be *external* forces/moments or *internal* forces/moments.
- Forces acting from one body to another by direct physical contact or from the Earth are examples of external forces.
- Fluid pressure acting to the wall of a water tank or a force exerted by the tire of a truck to the road is all external forces.

EQUILIBRIUM OF RIGID BODIES

In rigid bodies subjected to two dimensional force systems, the forces exerted from supports and connection elements are shown in the free body diagram as follows:

It should be kept in mind that reaction will occur along the direction in which the motion of the body is restricted.

MODELLING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant

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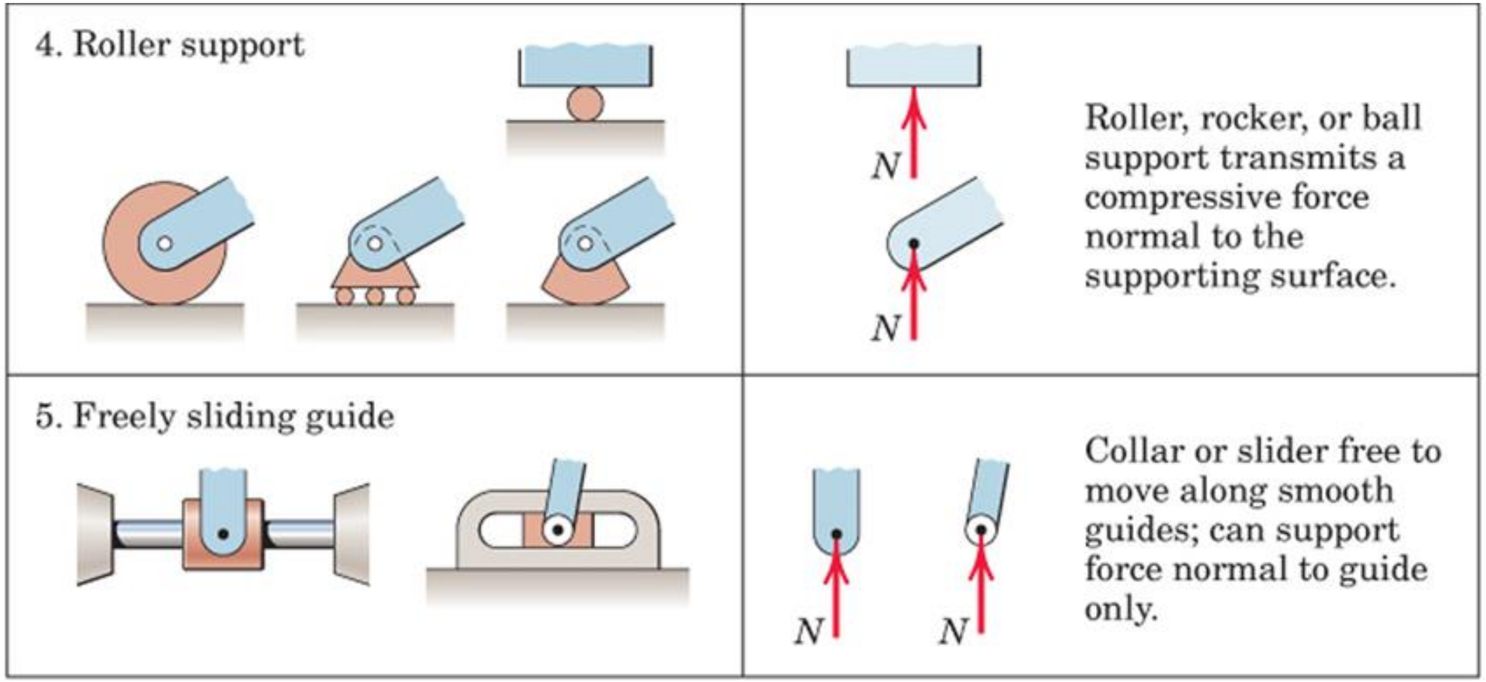
MODELLING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS



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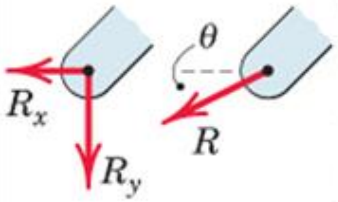
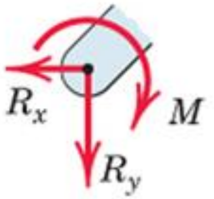
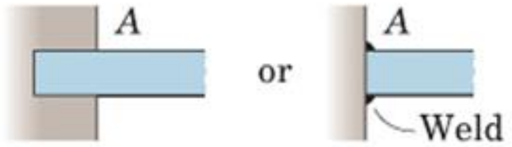
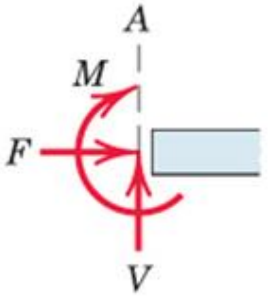
MODELLING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

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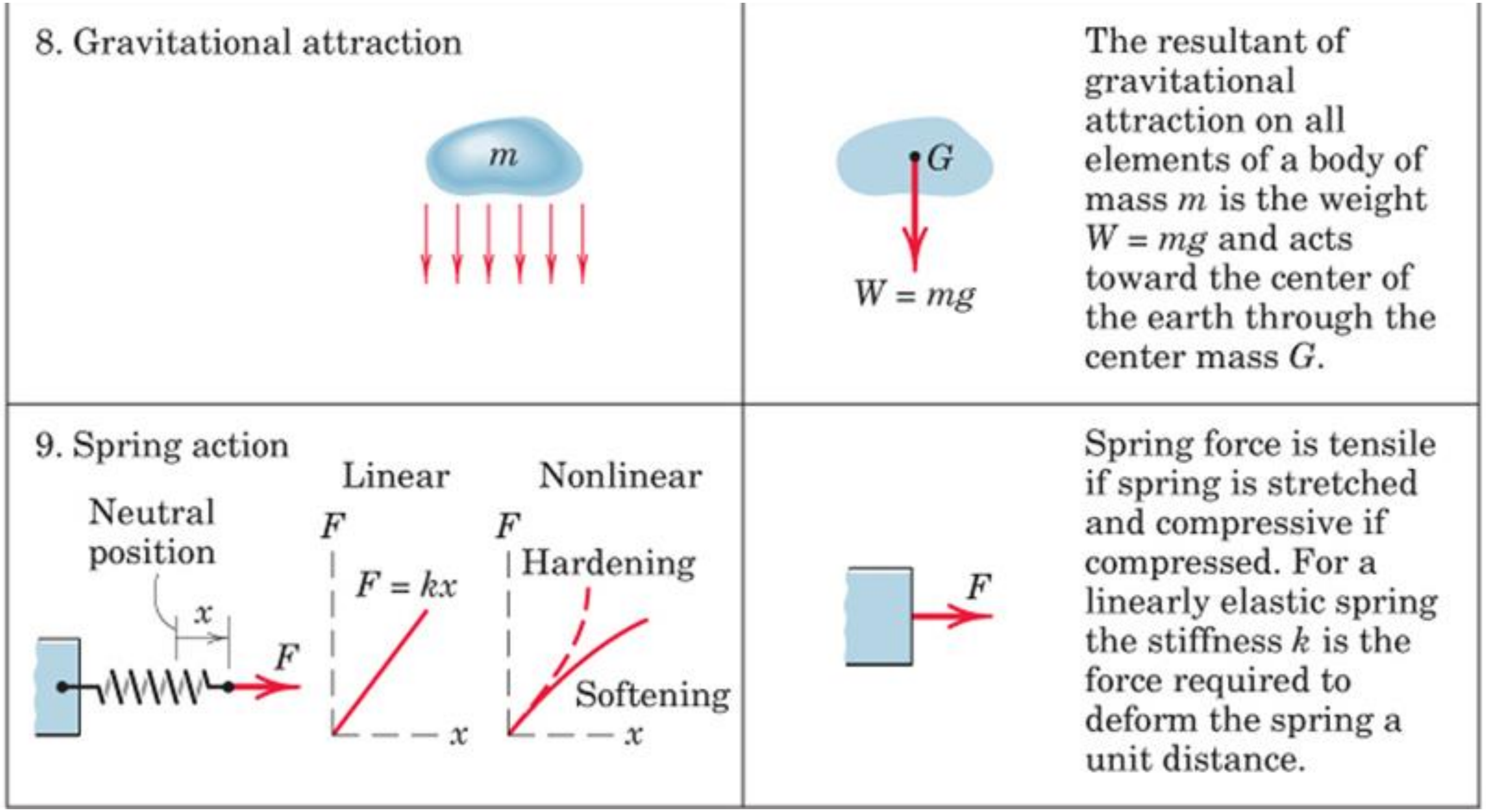
MODELLING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS



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EQUATIONS OF EQUILIBRIUM IN TWO DIMENSIONAL CASE

If all the forces acting on the rigid body are planar and all the couples are perpendicular to the plane of the body, equations of equilibrium become two dimensional.

$$\vec{R} = \Sigma\vec{F} = R_x\vec{i} + R_y\vec{j} = 0 \qquad R_x = \Sigma F_x = 0 \qquad R_y = \Sigma F_y = 0$$

$$\Sigma\vec{M} = \Sigma M_z\vec{k} = 0$$

Or in scalar form,

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

At most three unknowns can be determined.

ALTERNATIVE EQUATIONS OF EQUILIBRIUM

In two dimensional problems, in alternative to the above set of equations, two more sets of equations can be employed in the solution of problems.

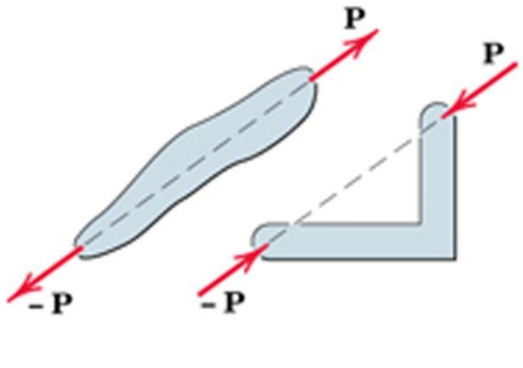
$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

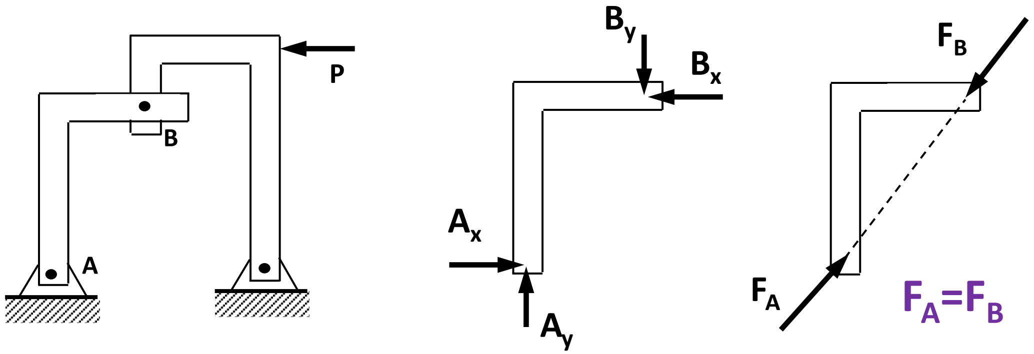
Points A, B and C in the latter set cannot lie along the same line, if they do, trivial equations will be obtained.

TWO-FORCE MEMBER

Members which are subjected to only two forces are named as “***two force members***”. Forces acting on these members are equal in magnitude, opposite in direction and are directed along the line joining the two points where the forces are applied



Two-force members



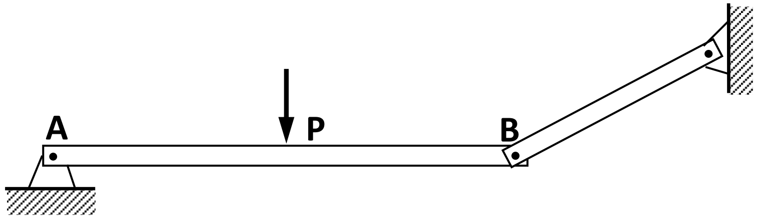
Weight is neglected. If weight is considered, the member will not be a two force member!

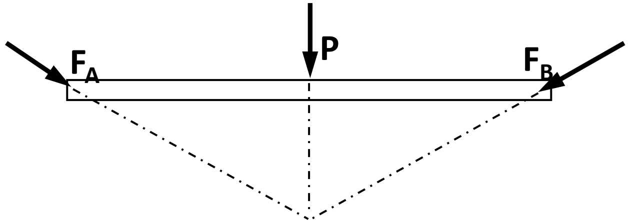
Examples of Two Force Members

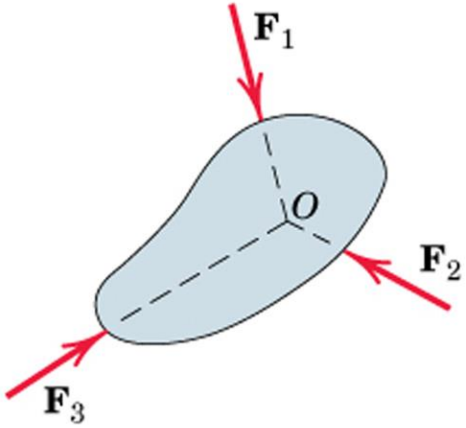
THREE-FORCE MEMBER

In rigid bodies acted on by only three forces, the lines of action of the forces must be concurrent; otherwise the body will rotate about the intersection point of the two forces due to the third force which is not concurrent. If the forces acting on the body are parallel, then the point of concurrency is assumed to be in infinity.

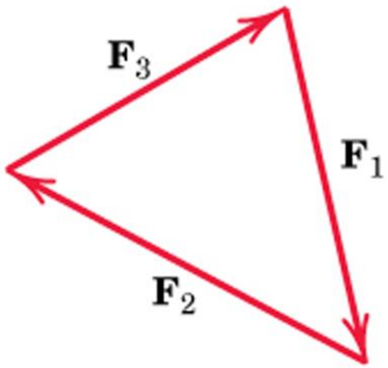
THREE-FORCE MEMBER







(*a*) Three-force member



(*b*) Closed polygon satisfies $\Sigma \mathbf{F} = \mathbf{0}$

FREE BODY DIAGRAM

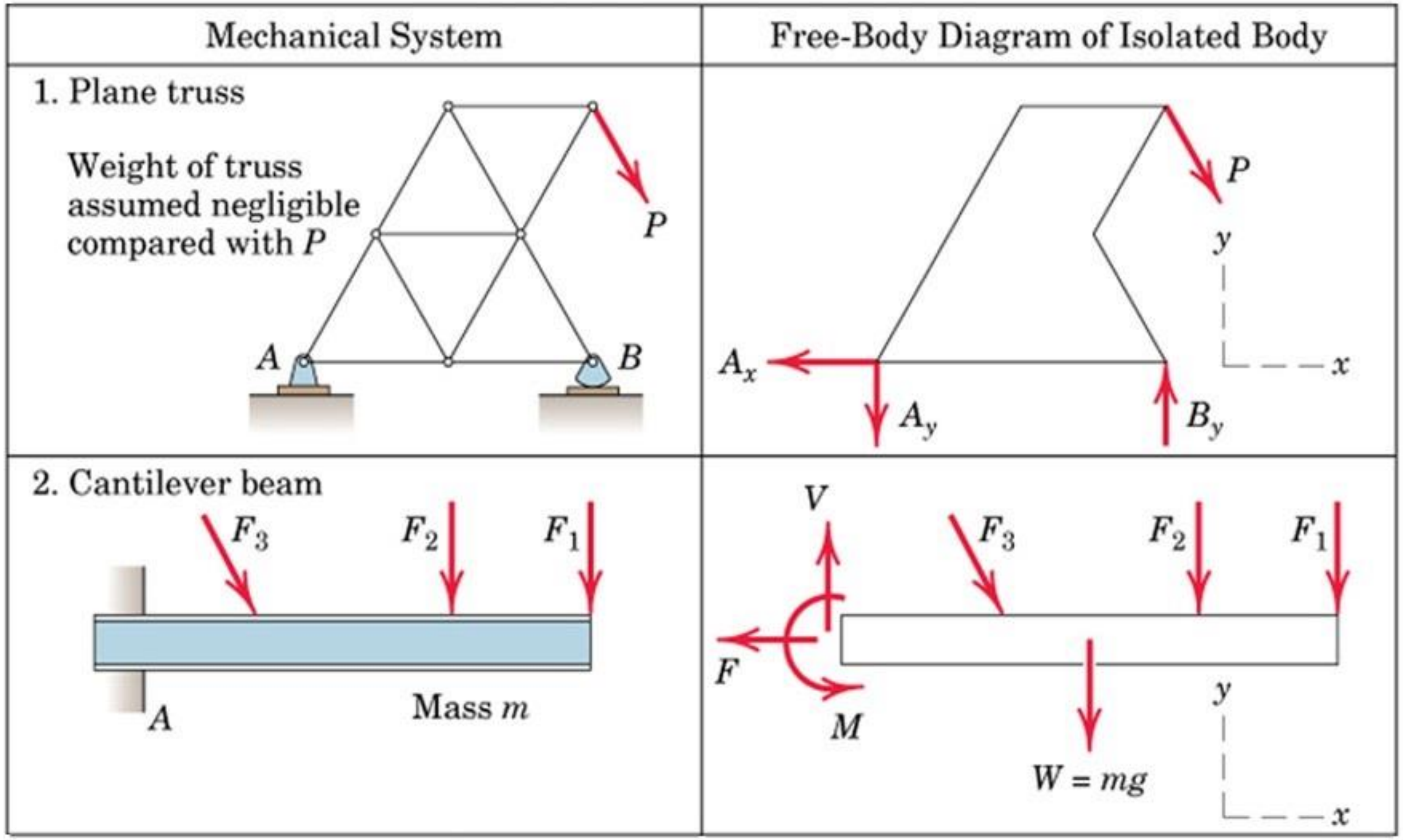
The procedure for drawing a free body diagram which isolates a body or system consists of the following steps:

- If there exists, identify the two force members in the problem.
- Decide which system to isolate.
- Isolate the chosen system by drawing a diagram which represents its ***complete external boundary***.

FREE BODY DIAGRAM

- If not given with the problem, select a coordinate system which appropriately suits with the given forces and/or dimensions.
- Identify ***all*** forces which act ***on*** the isolated system applied by removing the contacting or attracting bodies, and represent them in their proper positions on the diagram.
- Write the equations of equilibrium and solve for the unknowns.

FREE BODY DIAGRAM



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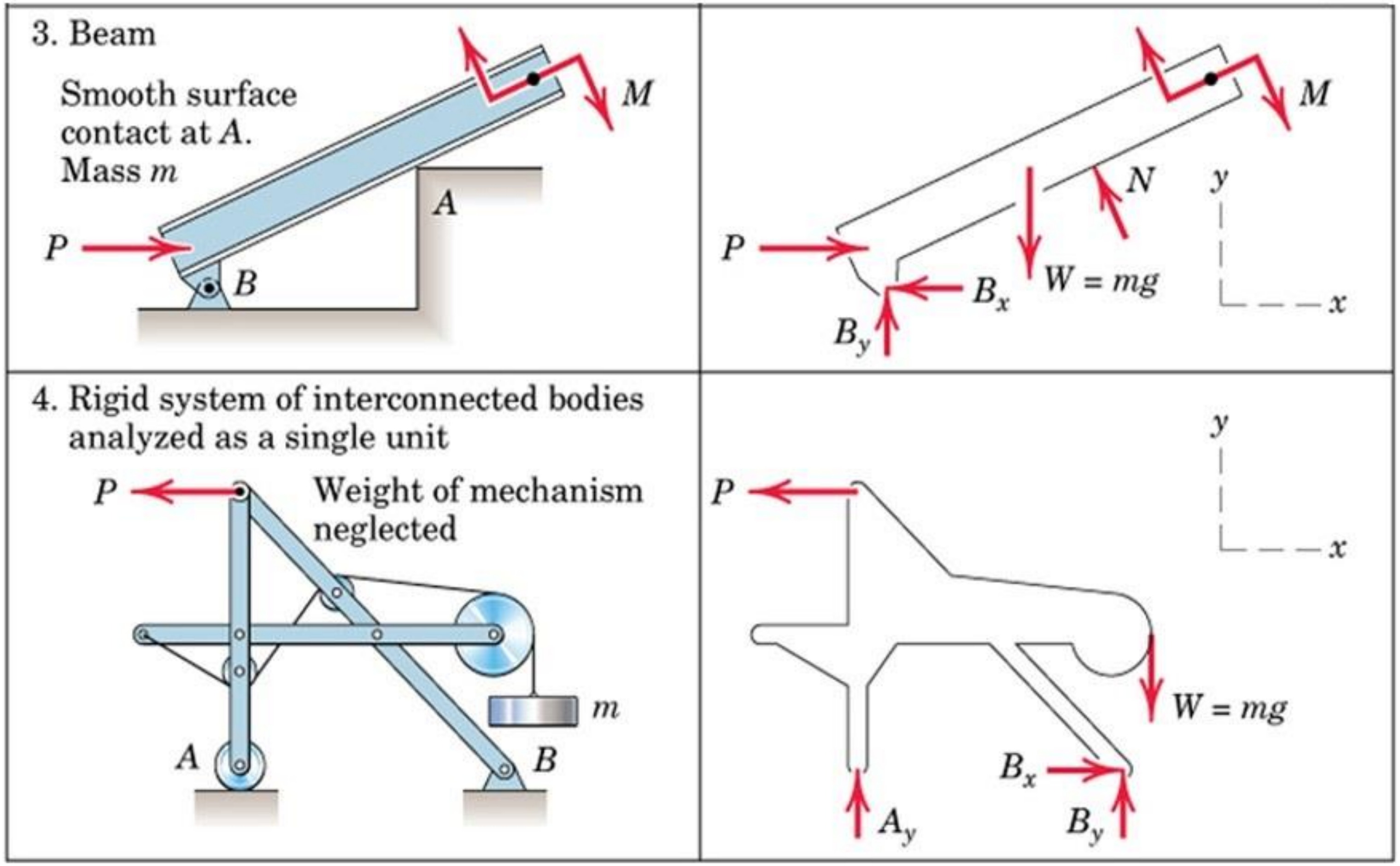
SAMPLE FREE-BODY DIAGRAMS



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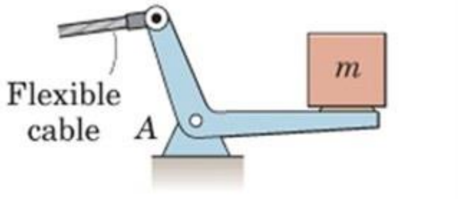
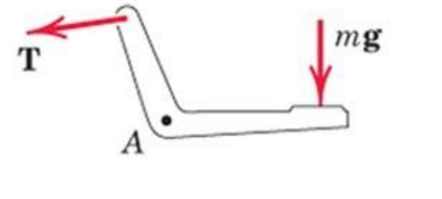
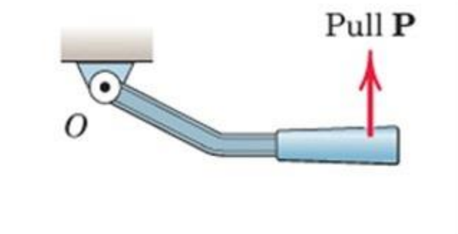
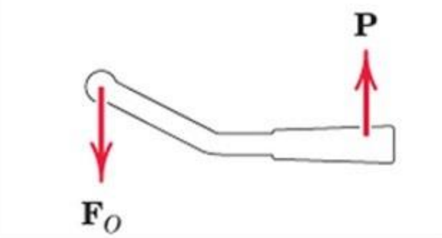
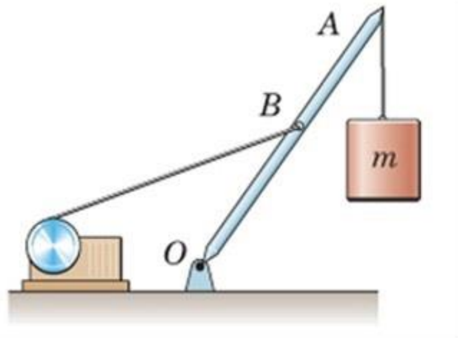
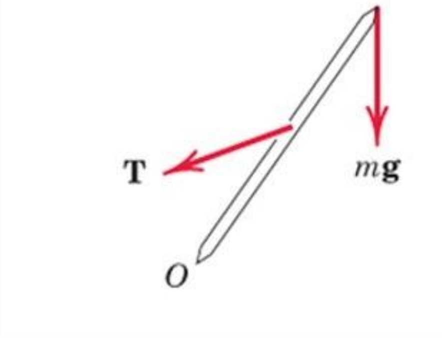
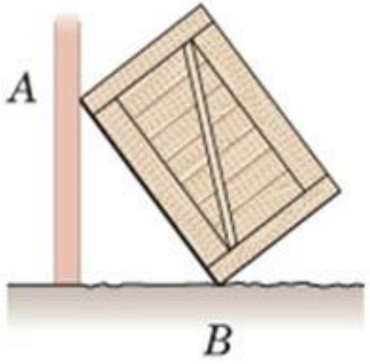
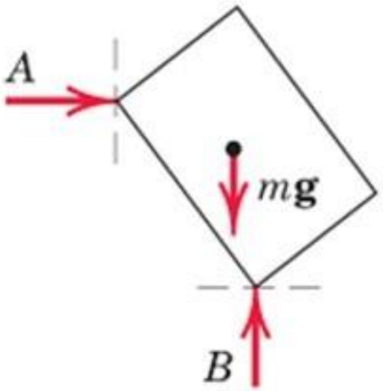
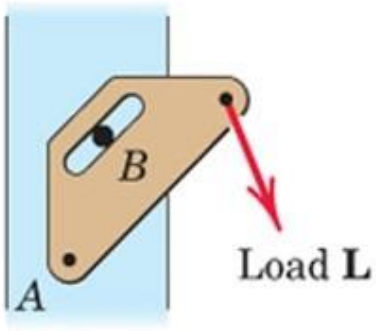
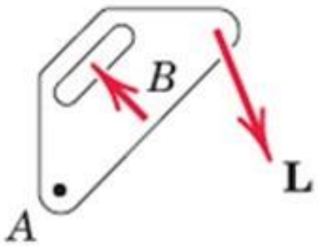
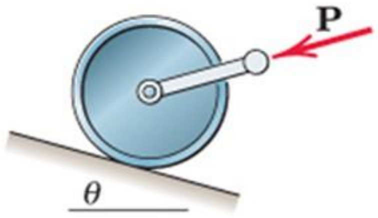
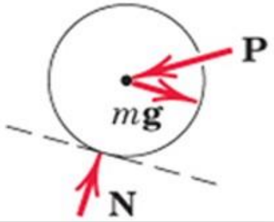
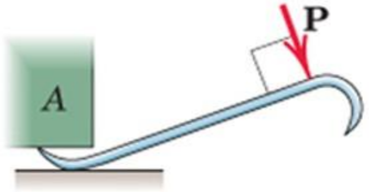
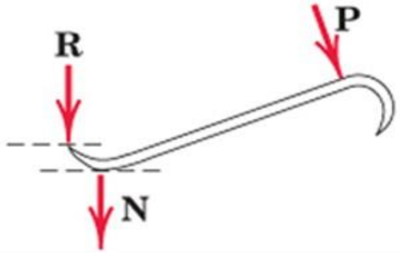
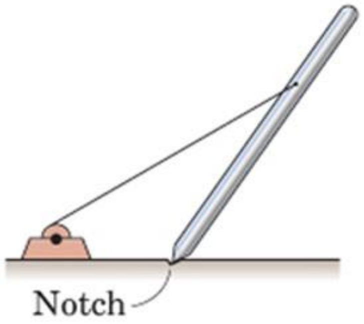
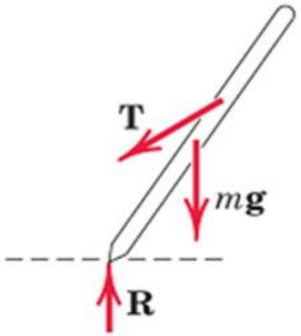
SAMPLE FREE-BODY DIAGRAMS

	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		

SAMPLE FREE-BODY DIAGRAMS

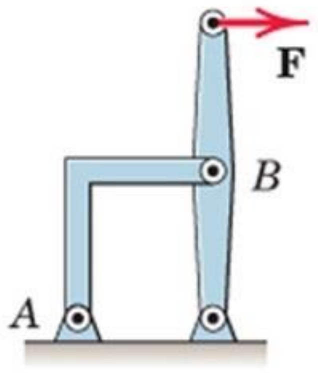
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

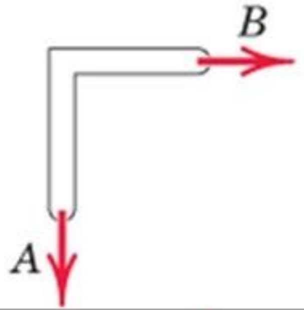
SAMPLE FREE-BODY DIAGRAMS

	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ .		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		

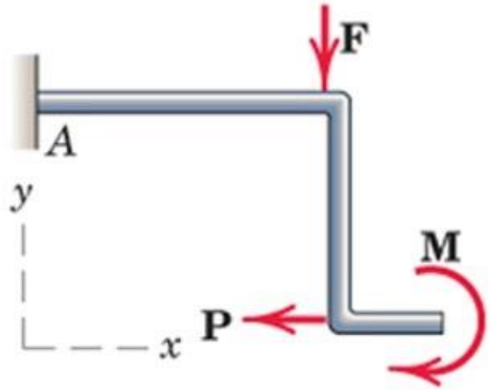
SAMPLE FREE-BODY DIAGRAMS

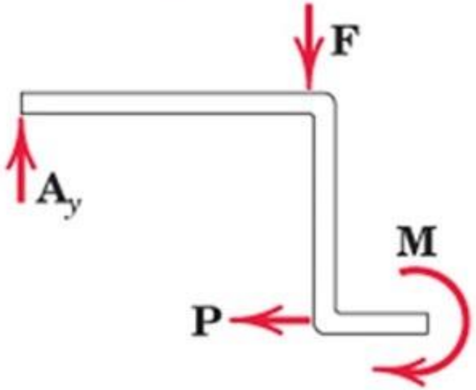
4. Supporting angle bracket for frame; pin joints.





5. Bent rod welded to support at *A* and subjected to two forces and couple.





Thank You