LECTURE # 5

In this lecture you will learn about:

- Degree of Freedom
- Single Degree of Freedom
- Multiple Degree of Freedom
- Damping
- E.O.M

Course Name:

"Introduction To Earthquake Engineering"

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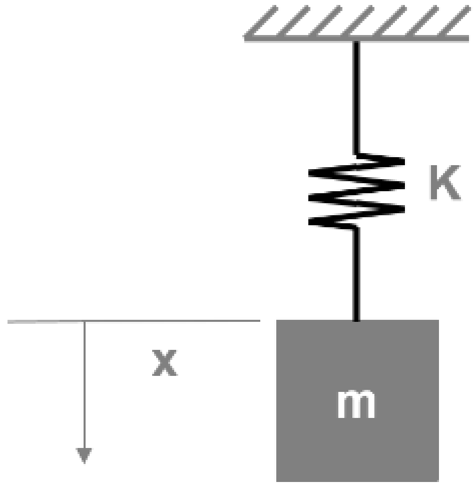
DEGREE OF FREEDOM

- Degrees of freedom (DOF) of a system is defined as the number of independent variables required to completely determine the positions of all parts of a system at any instant of time.

- It is defined as minimum number of parameters used to define a system.

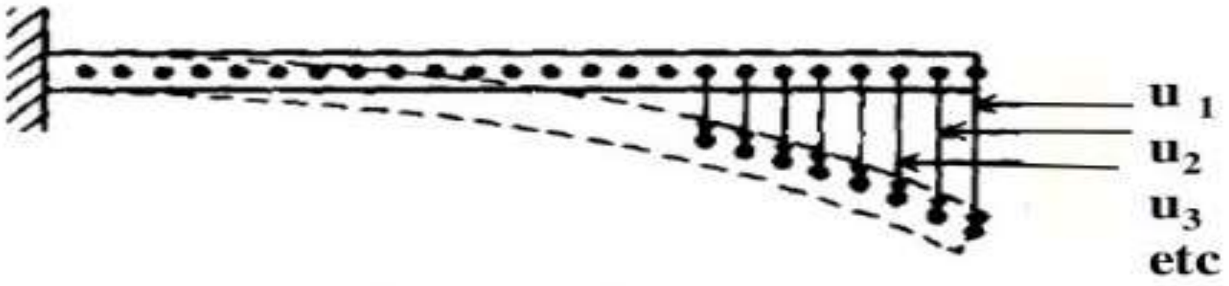


CONTINOUS VS DISCRETE SYSTEMS

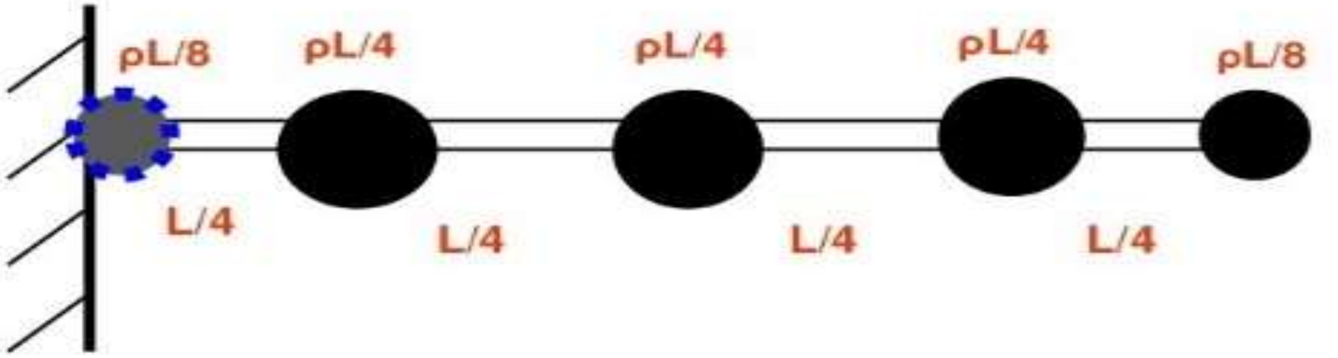
Some systems, especially those involving continuous elastic members, have an infinite number of DOF. As an example of this is a cantilever beam with self-weight only (see next slide). This beam has infinites mass points and need infinites number of displacements to draw its deflected shape and thus has an infinite DOF. Systems with infinite DOF are called **Continuous or Distributed systems.**

Systems with a finite number of degree of freedom are called **Discrete or Lumped mass parameter systems**.

CONTINOUS VS DISCRETE SYSTEMS



Continuous or distributed system



Corresponding lumped mass system of the above given cantilever beam with DOF= 4

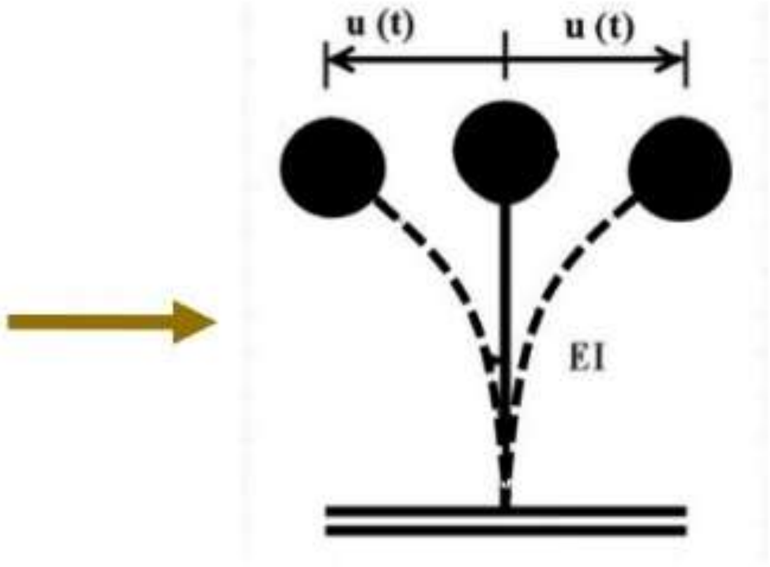
ρ = Mass per unit length

SINGLE DEGREE OF FREEDOM (SDOF) SYSTEM

- In a single degree of freedom system, the deformation of the entire structure can be described by a single number equal to the displacement of a point from an at-rest position.
- Single Degree of freedom systems do not normally exist in real life. We live in a three-dimensional world and all mass is distributed resulting in systems that have an infinite number of degrees of freedom. There are, however, instances where a structure may be approximated as a single degree of freedom system.
- The study of SDOF systems is an integral step in understanding the responses of more complicated and realistic systems.

IDEALIZATION OF A STRUCTURAL SYSTEM AS SDOF SYSTEM

This 3-dimensional water tower may be considered as a single degree of freedom system when one considers vibration in one horizontal direction only



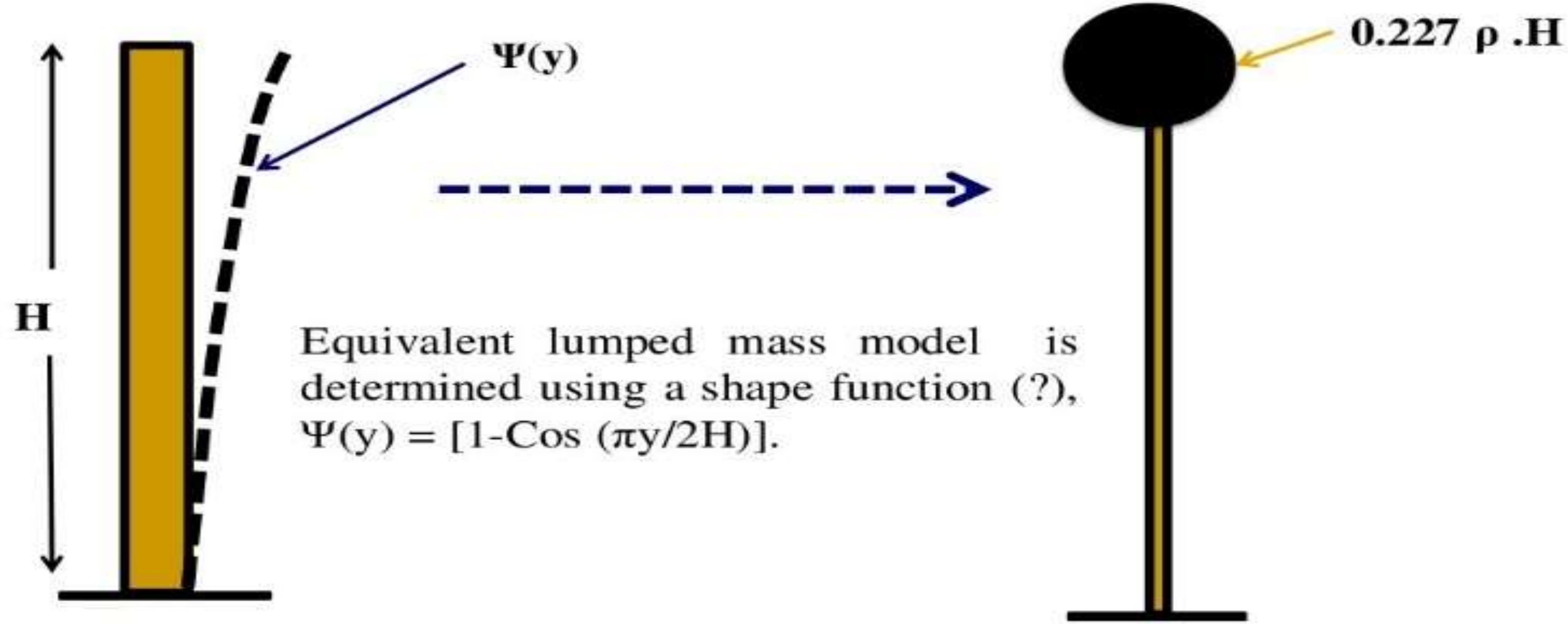
SDOF model of water tank

IDEALIZATION OF A STRUCTURAL SYSTEM AS SDOF SYSTEM

- The structural system of water tank may be simplified by assuming that the column has negligible mass along its length. This is reasonable, assuming that the tube is hollow and that the mass of the tube is insignificant when compared with the mass of the water tank and water at the top.

- This means that we can consider that the tank is a point mass

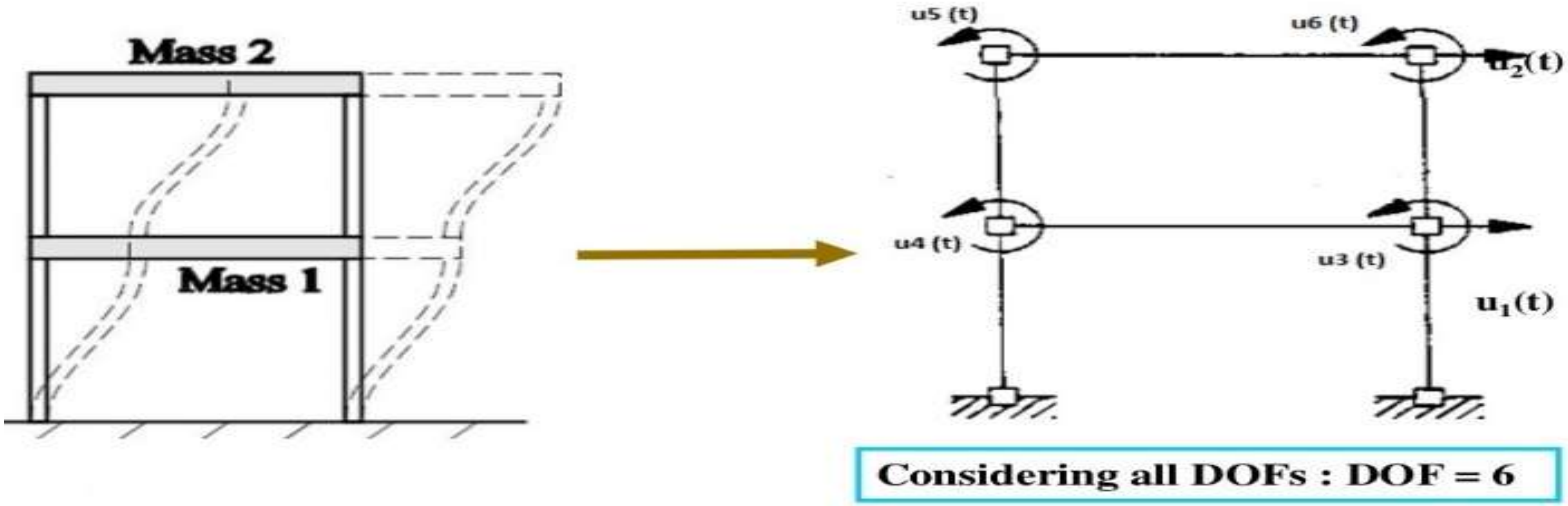
EQUIVALENT LUMPED MASS SDOF SYSTEM OF A CANTILEVER WALL WITH UNIFORM X-SECTIONAL AREA



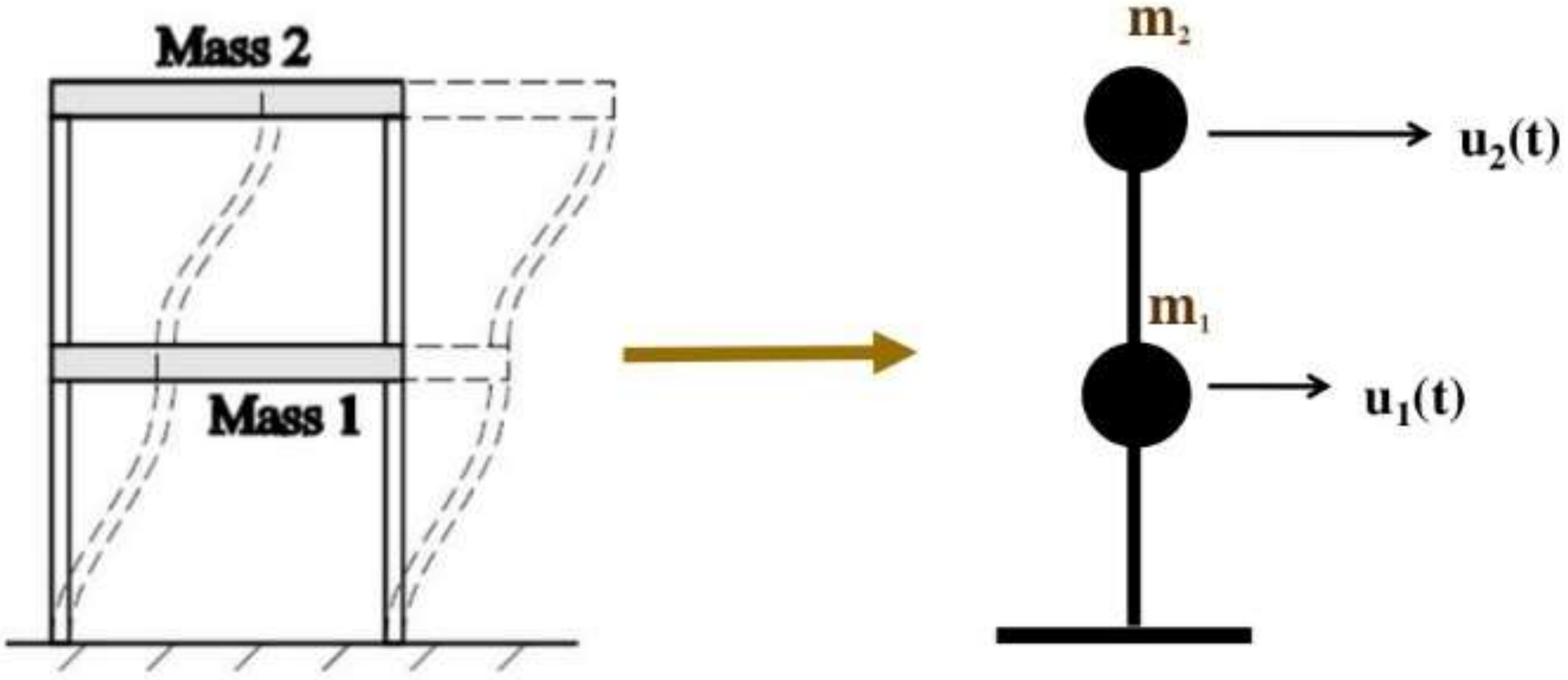
ρ = Mass per unit height, H= total height, y= Any distance along height and k = lateral stiffness of cantilever member = EI/H^3

MULTIPLE DEGREE-OF-FREEDOM (MDOF) SYSTEM

In a Multi degree of freedom system, the deformation of the entire structure cannot be described by a single displacement. More than one displacement coordinates are required to completely specify the displaced shape.



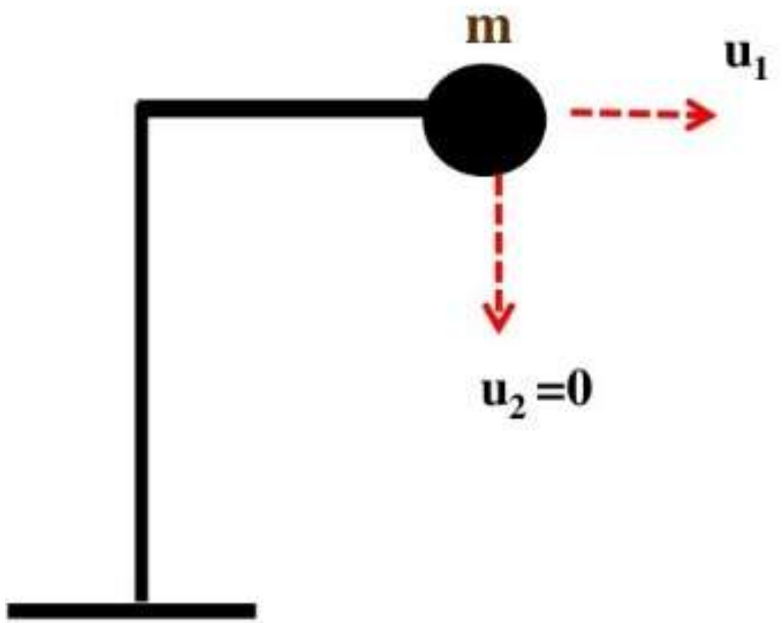
MULTIPLE DEGREE-OF-FREEDOM (MDOF) SYSTEM



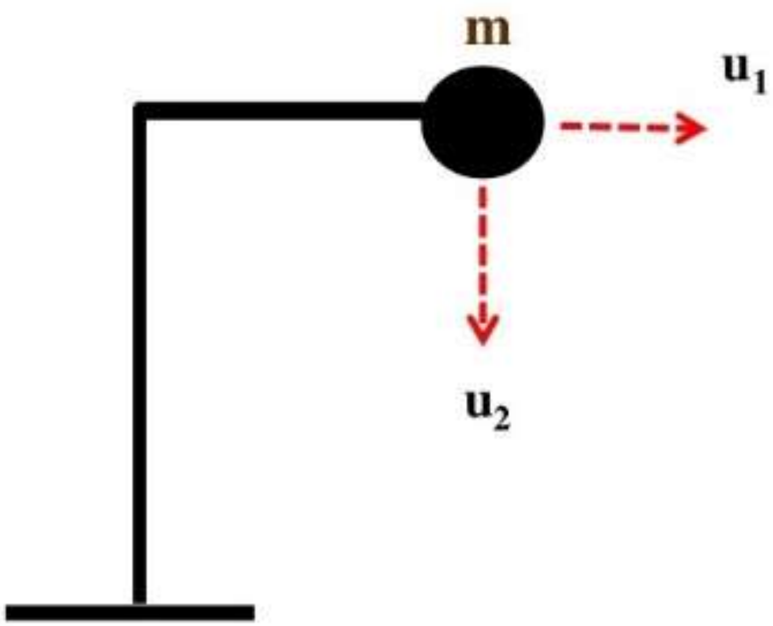
Lumped mass model of building (DOF=2). $u_3(t)$ to $u_6(t)$, as shown on previous slide, is eliminated by lumping the masses at mid length of beam.

MULTIPLE DEGREE-OF-FREEDOM (MDOF) SYSTEM

What is the DOF for this system…?



DOF can be taken 1 when flexural stiffness of beam is too high as compared to column



DOF is 2 when we have a flexible beam

DAMPING

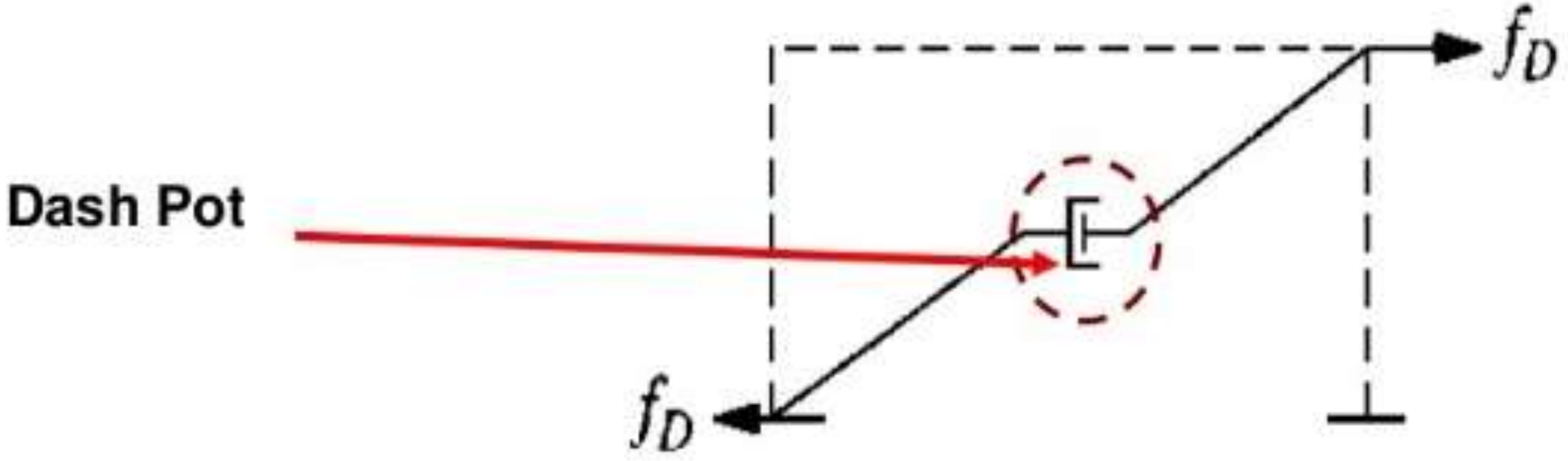
The degree of structural amplification of the ground motion at the base of the building is limited by structural damping. Therefore, damping is the ability of the structural system to dissipate the energy of the earthquake ground shaking. Since the building response is inversely proportional to damping. The more damping a building possesses, the sooner it will stop vibrating—which of course is highly desirable from the standpoint of earthquake performance.

DAMPING

There is no numerical method available for determining the damping. It is only obtained by experiments. In a structure, damping is due to internal friction and the absorption of energy by the building's structural and non-structural elements. Today, some of the more advanced techniques of earthquake resistant design and construction employ added damping devices like shock absorbers to increase artificially the intrinsic damping of a building and so improve its earthquake performance.

DAMPING

Simple dashpots as shown schematically in below given figure exert a force f_D whose magnitude is proportional to the velocity of the vibrating mass

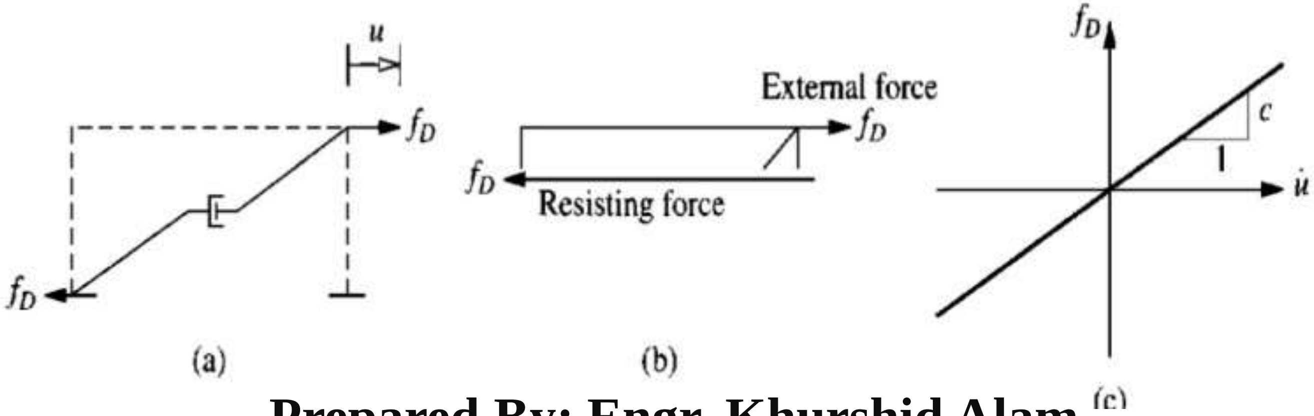


DAMPING

Figure **a** shows a linear viscous damper subjected to a force $\mathbf{f_D}$ along the DOF **u**. The internal force in the damper is equal and opposite to the external force $\mathbf{f_D}$ (Figure **b**). The damping force $\mathbf{f_D}$ is related to the velocity $\dot{u}$ across the linear viscous damper by:

$$\mathbf{f_D = c\dot{u}}$$

Where the constant **c** is the viscous damping coefficient



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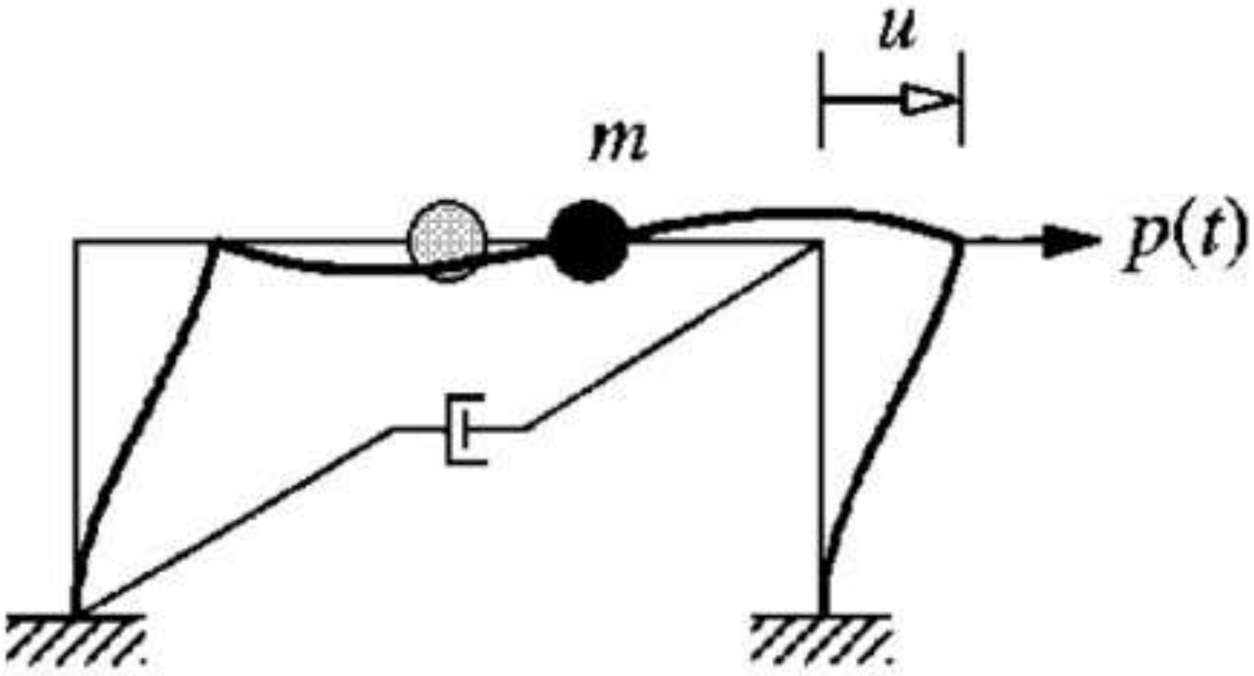
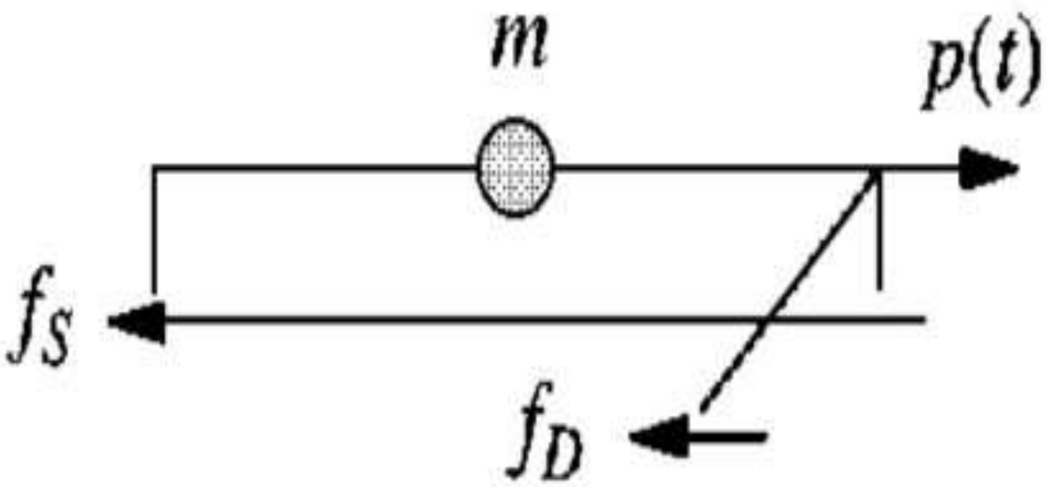
EQUATION OF MOTION (E.O.M) OF A SINGLE STORY FRAME UNDER EXTERNAL DYNAMIC FORCE



Two commonly used vector mechanics based approaches are:

1. NEWTON'S SECOND LAW OF MOTION
2. D'ALEMBERT PRINCIPLE OF DYNAMIC EQUILIBRIUM

E.O.M USING NEWTON'S SECOND LAW OF MOTION



The Resultant force along x-axis

$p(t) - f_S - f_D$

Where;

f_S = Elastic resisting force; (also known elastic restoring force),

f_D= Damping resisting force

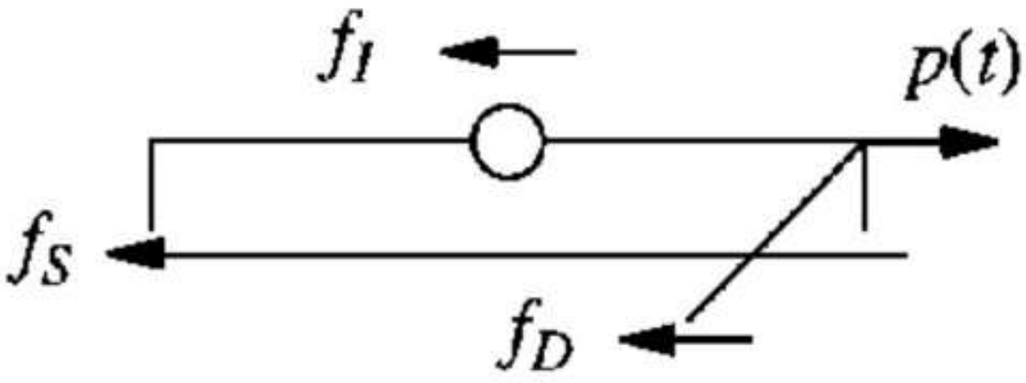
According to Newton's second law, the resulting force causing acceleration = $p(t) - f_S - f_D = m\ddot{u}$ or;

$f_S + f_D + m\ddot{u} = p(t)$; or;

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

E.O.M USING DYNAMIC EQUILIBRIUM

Using D' Alembert's Principle, a state of dynamic equilibrium can be defined by assuming that a fictitious **inertial force** f_I acts on the mass during motion.



Equilibrium along x-axis requires that:

-fS - fD – fI + p(t) = 0 or;

fS + fD + fI = p(t) or;

$$k u + c \dot{u} + m \ddot{u} = p\,(t)$$

Thank You