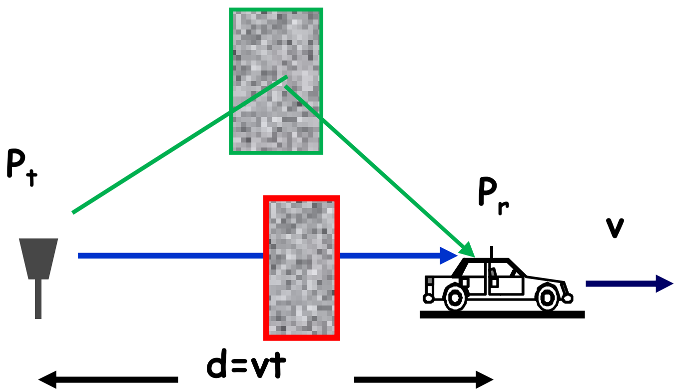
Wireless Communications

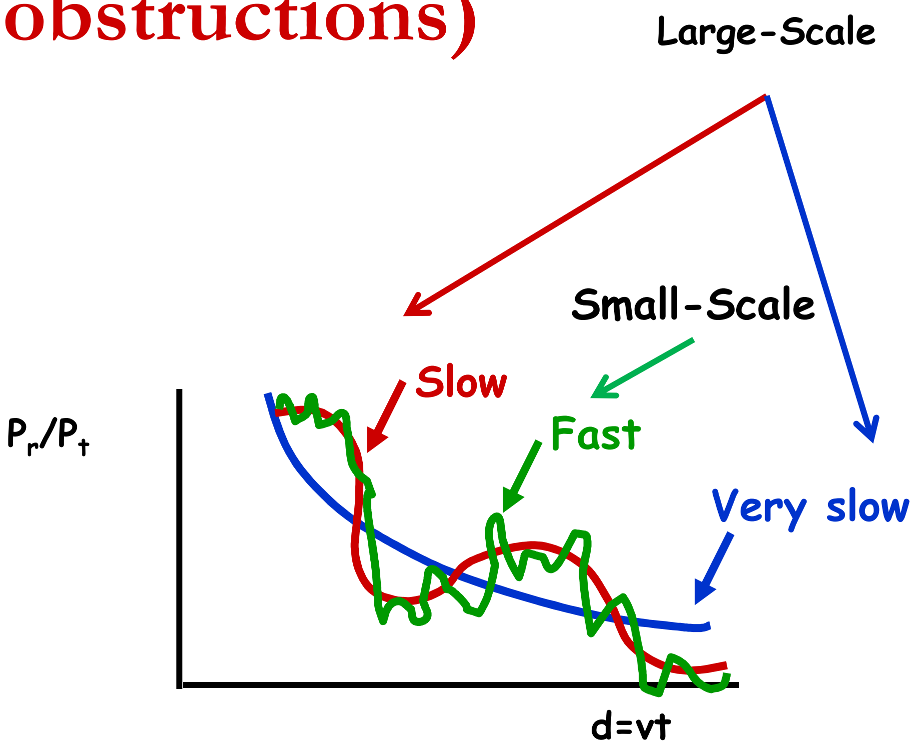
Lecture 4

Small-Scale Fading

Propagation Characteristics

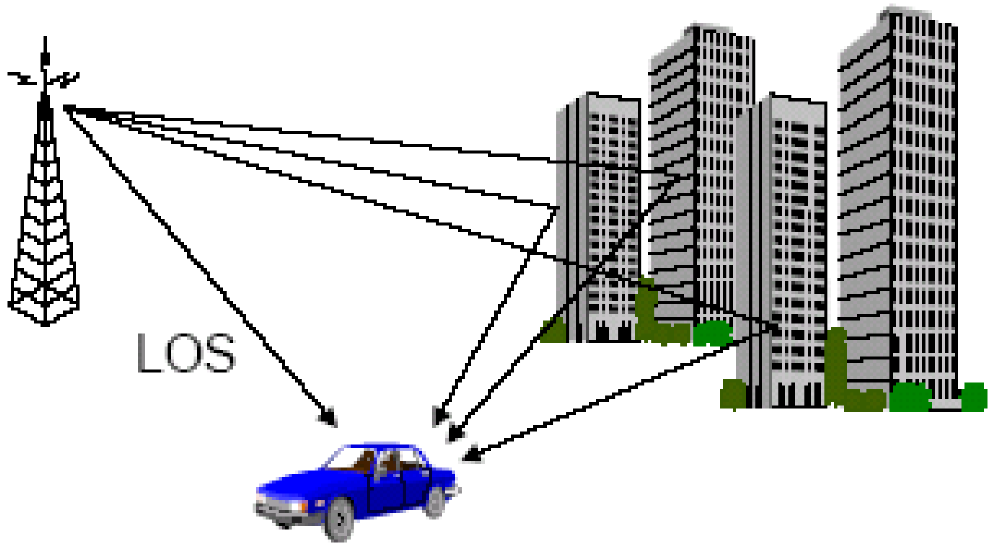
- Path Loss (includes average shadowing)
- Shadowing (due to obstructions)
- Multipath Fading

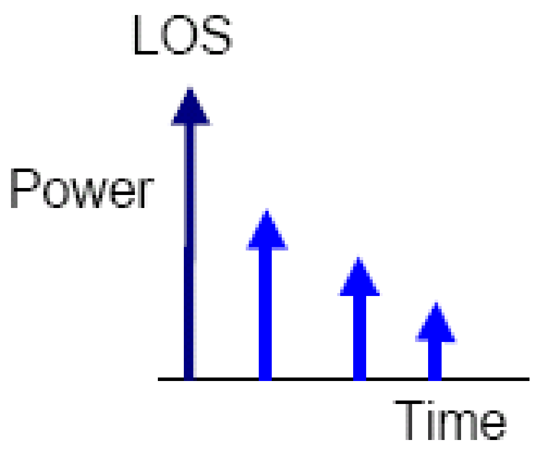




Multipath Propagation

- Multiple waves arrive at the receiver
  - Delay spread
  - Doppler shift
  - Angle spread
- Influencing factors
  - Speed of the mobile
  - Speed of surrounding objects
  - Signal bandwidth





Multipath Power Delay Profile

Small-Scale Fading

- Multipath signals have **randomly** distributed amplitudes, phases, & direction of arrival
  - Vector summation ($A \angle \theta$) @ Rx of multipath leads to **constructive/destructive interference** as mobile Rx moves in space (time)
  - RSS can vary by **20–30 dB** over distances of only $\lambda / 4$!
  - $\lambda / 4 \rightarrow$ ~ 5–10 cm or 3–5 msec (for $v = 40$ mph)
  - Fading occurs about RSS predicted from large-scale path loss models
- Even **fixed** Tx/Rx wireless links can experience fading due to motion of objects (cars, people, trees, etc.) in surrounding environment

Mobile Radio Channel

- Physical Factors Influencing Fading in Mobile Radio Channel (MRC)

1) Multipath Propagation

- Number **and** strength of multipath signals
- Time delay of signal arrival
  - » Large path differences → large delay
- Urban area w/ many buildings distributed over large spatial scale
  - » Large # of strong multipath signals with a **few** having large time delay
- Suburb with nearby office park or shopping mall
  - » Moderate # of strong multipath signals with small → moderate delay times
- Rural → few multipath signals (LOS + ground reflection)

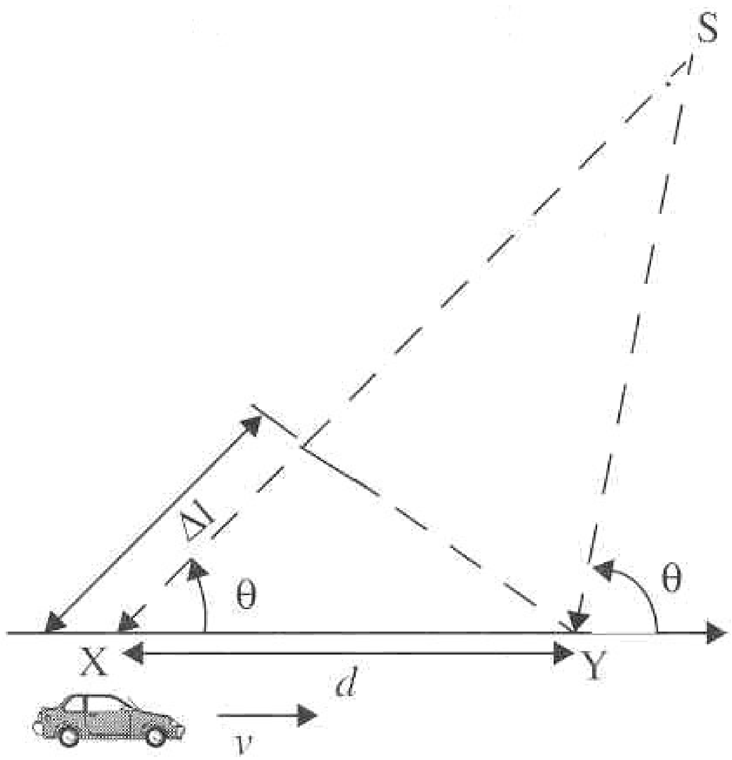
Mobile Radio Channel

- Physical Factors Influencing Fading in Mobile Radio Channel (MRC)

2) Speed of Mobile

- Relative motion between base station & mobile causes random frequency modulation due to Doppler shift (f_d)
  - » $f_d = (v/\lambda)\cos\theta$
- Multipath signals will have different f_d's for constant v b/c of **random** arrival directions (θ)!!

Doppler Shift



$f_d = (v/\lambda)\cos\theta$ where

v : velocity (m/s)
λ : wavelength (m)
θ : angle between mobile direction and arrival direction of RF energy

+ shift $\rightarrow$ mobile moving toward S
– shift $\rightarrow$ mobile moving away from S

Mobile Radio Channel

- Physical Factors Influencing Fading in Mobile Radio Channel (MRC)

  3) Speed of Surrounding Objects

  - Also generates multipath signals with Doppler shift
  - Dominates small-scale fading if speed of objects > mobile speed
    - Otherwise ignored

Mobile Radio Channel

- Physical Factors Influencing Fading in Mobile Radio Channel (MRC)

  4) Tx Signal Bandwidth (B_s)

  - MRC modeled as filter w/ specific bandwidth (BW)
  - Relationship between signal BW & MRC BW will determine:

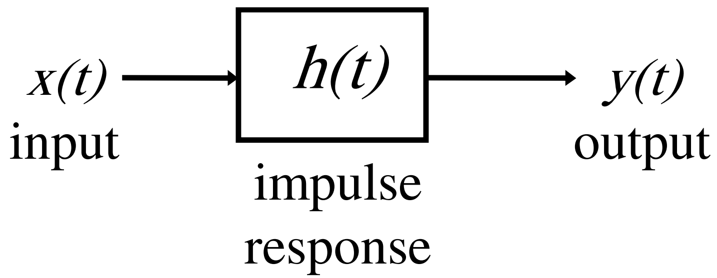
    a) if small-scale fading is significant
    b) if time distortion of signal leads to inter-symbol interference (ISI)

MRC Impulse Response

- Model MRC as linear filter with **<u>** time-varying **</u>** impulse response
- Filter model due to vector summation of random amplitudes & phases of multipath signals
- Time variation due to mobile motion $\rightarrow$ time delay of multipath signals varies with location of Rx!
- MRC has fundamental bandwidth limitation $\rightarrow$ model as BPF

MRC Impulse Response

- Linear filter theory



$$y(t) = x(t) \otimes h(t)$$

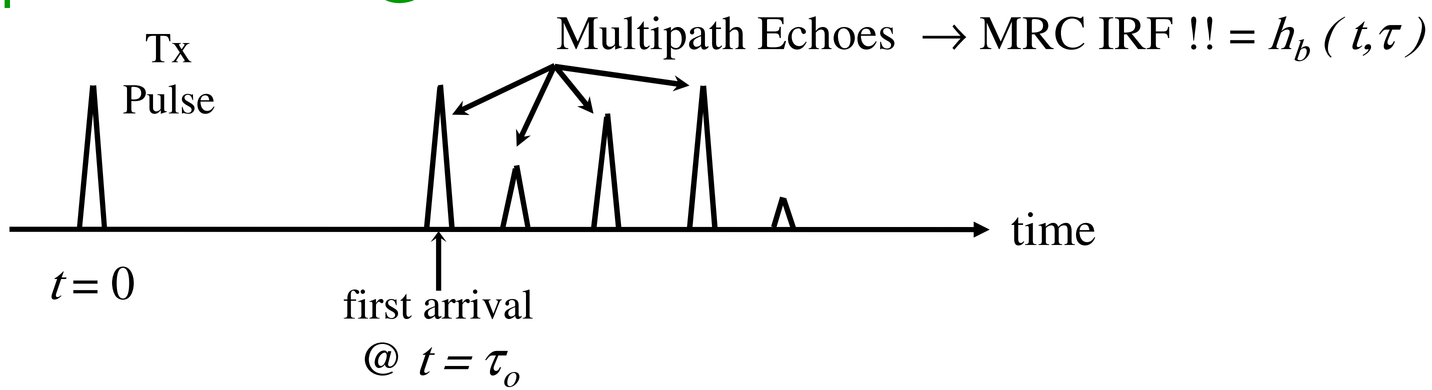
or

$$Y(f) = X(f) \cdot H(f)$$

- How is unknown $h(t)$ determined?
  - Let $x(t) = \delta(t) \rightarrow$ delta or **impulse** input then
  - $y(t) = h(t) \rightarrow$ impulse response function (IRF)
  - IRF for standard filter theory is the same regardless of when it is measured $\rightarrow$ **time invariant!**

MRC Impulse Response

- How is IRF of MRC determined?
  - "Channel sounding" → Radar → Fig. 5.6, pg. 192
  - Transmit short time duration pulse (wide BW) and record multipath echoes @ Rx



  - » Short duration Tx pulse ≈ unit impulse $\delta(t)$!
  - » Define **excess delay** time as τ where $t > \tau_o$

Channel Sounding

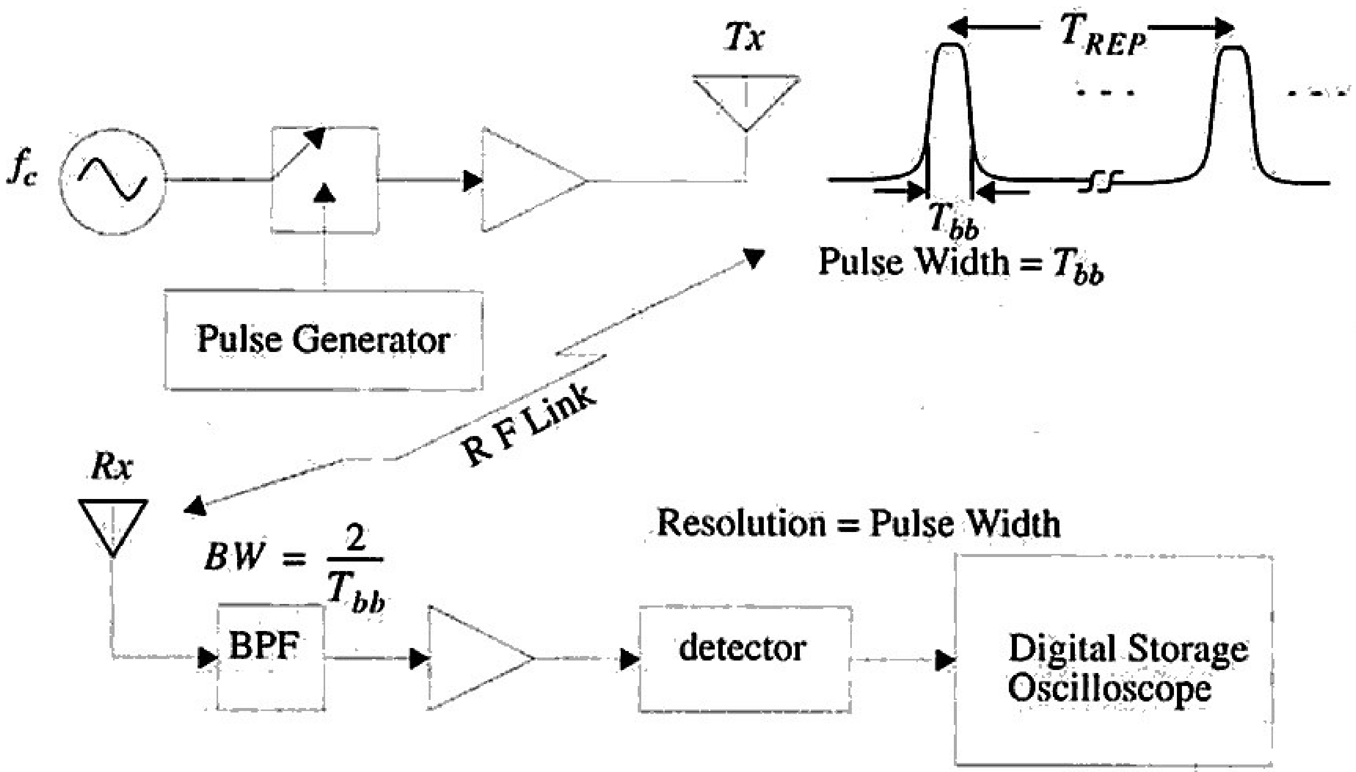
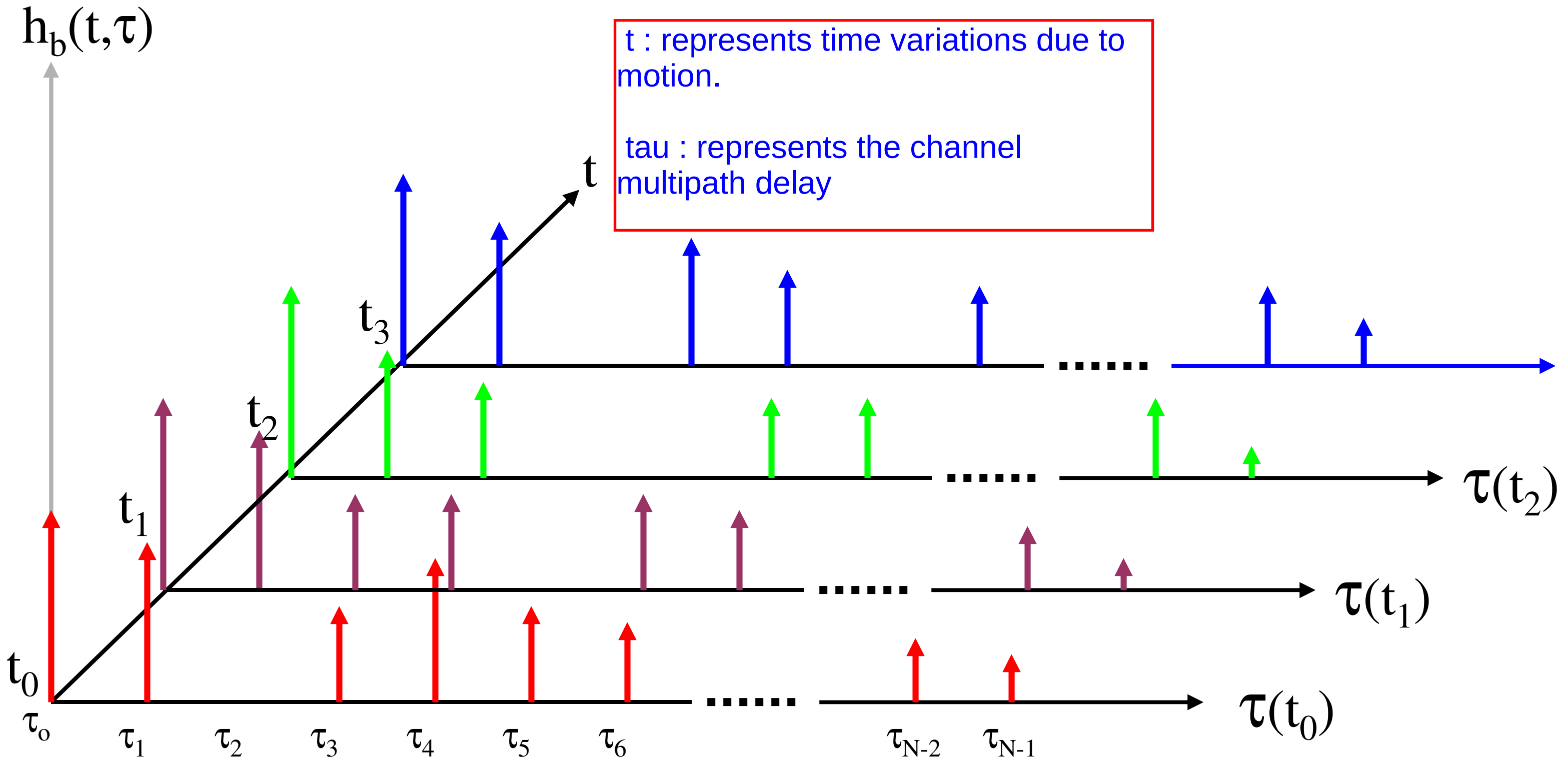


Figure 5.6 Direct RF channel impulse response measurement system.

MRC Impulse Response

- Amplitude **and** delay time of multipath returns **change** as mobile moves ➔ MRC is time **variant**

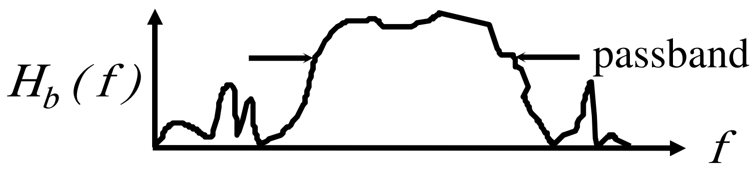


MRC Impulse Response

- Model multipath returns as unit impulses

$$h_b(t) = \sum_{i=0}^{N-1} a_i(t,\tau)\ \exp[j\theta_i(t,\tau)]\ \delta(\tau - \tau_i(t))$$

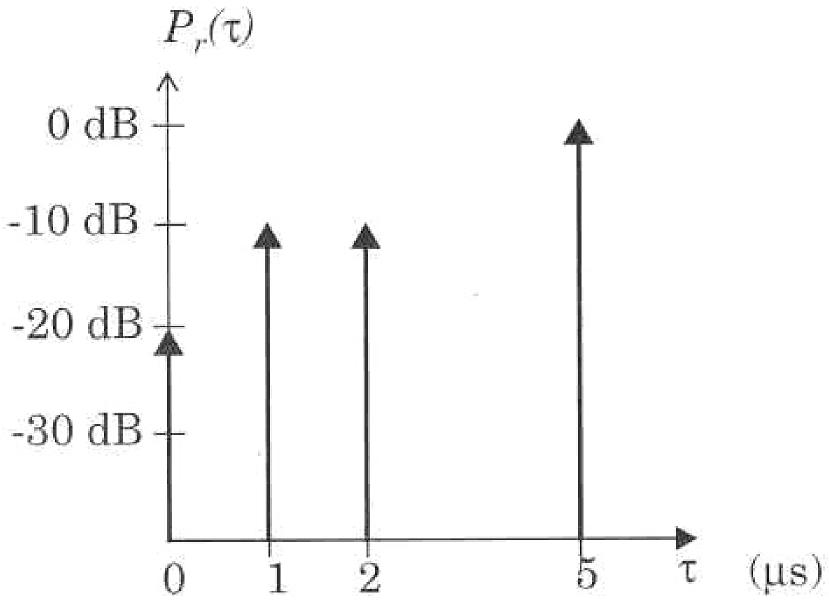
  - $a_i \angle\ \theta_i$ = amplitude & phase of multipath signals (δ)
  - N = # of multipath components
- FFT of IRF gives spectral characteristics of channel $\rightarrow$ frequency response!



  - MRC filter passband $\rightarrow$ "Channel BW" or Coherence BW = B_c

Multipath Channel Parameters

- Derived from multipath power delay profiles
  - $P(\tau_k)$: **relative** power amplitudes of multipath signals
  - Use ensemble average of many profiles in small localized area → typically 2–6 m → obtain **average** small-scale response



Multipath Channel Parameters

- **Time Dispersion Parameters**
  - Mean excess delay → $\bar{\tau} = \dfrac{\sum_k a_k^2 \, \tau_k}{\sum_k a_k^2} = \dfrac{\sum_k P(\tau_k) \, \tau_k}{\sum_k P(\tau_k)}$
  - **RMS delay spread** → $\sigma_\tau = \sqrt{\overline{\tau^2} - \left(\bar{\tau}\right)^2}$ and $\overline{\tau^2} = \dfrac{\sum_k P(\tau_k) \, \tau_k^2}{\sum_k P(\tau_k)}$
    - » Typical values:
      - ➢ Outdoor channel ~ 2–5 µsec
      - ➢ Indoor channel ~ 20–100 nsec
  - Maximum excess delay (X dB): excess delay value during which multipath power levels fall X dB **below** the maximum power level
    - » Worst case delay value
  - "Excess delay" : all values computed **relative** to time of first signal arrival t_o (see figure on slide #10)

Time Dispersion Parameters

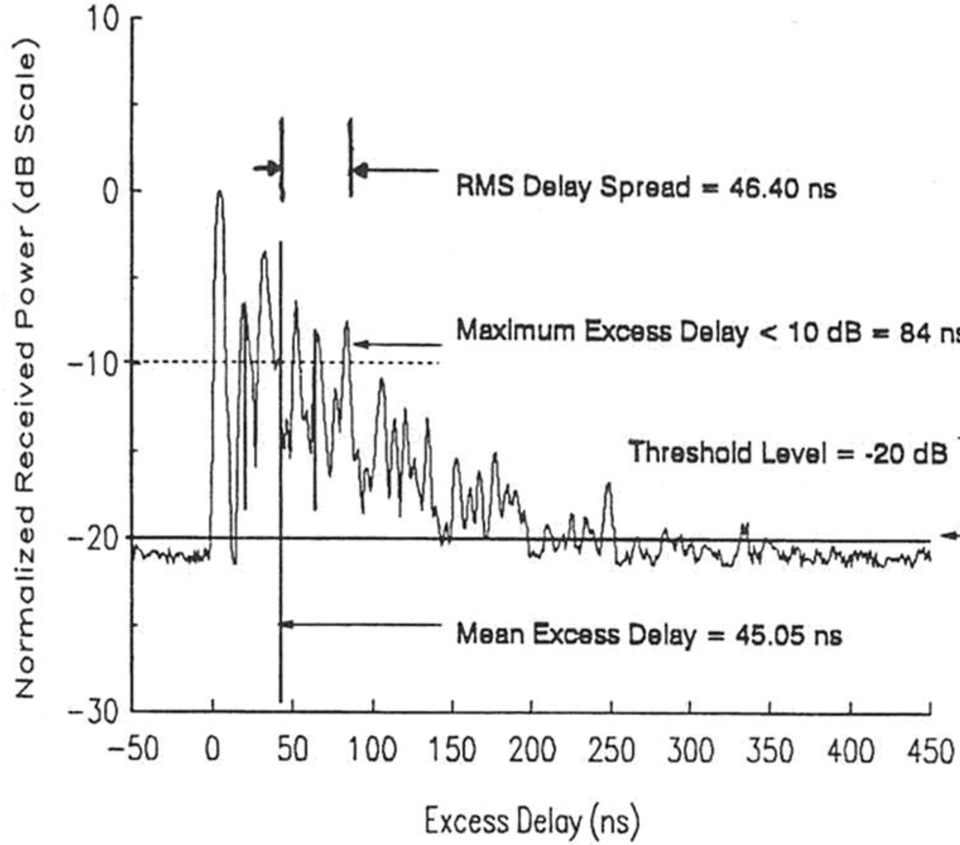


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

Time Dispersion Parameters

Table 5.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10–25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200–310 ns	Averaged typical case	[Cox72]
Suburban	910	1960–2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10–50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70–94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

MRC Parameters

- How do we characterize a time-varying MRC?
  - Statistical analyses must be used
- Four Key Characteristics of a Mobile Radio Channel (MRC)
  - Time Dispersion Characteristics
    1) RMS Multipath Delay Spread (σ_τ)
    2) Coherence BW or Channel BW (B_c)
  - Frequency Dispersion Characteristics
    3) Doppler Spread (B_D)
    4) Coherent Time (T_c)

MRC Parameters

- RMS Delay Spread (σ_τ)
  - Calculated from multipath power delay profiles
  - Mean Excess Delay $\rightarrow$ average delay beyond first return **weighted** by return power
  - RMS Delay Spread $\rightarrow$ 1 standard deviation (SD) of delay values **about** the mean excess delay
  - Statistical measures of propagation delay of **interfering** signals
    - » Desire small σ_τ
    - » Typical values:
      - ➢ outdoor channel ~ 1–5 μsec
      - ➢ indoor channel ~ 20–100 nsec

MRC Parameters

RMS Delay Spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \left(\overline{\tau}\right)^2}$$

Mean Excess Delay

$$\overline{\tau} = \frac{\sum_k P(\tau_k)\,\tau_k}{\sum_k P(\tau_k)}$$

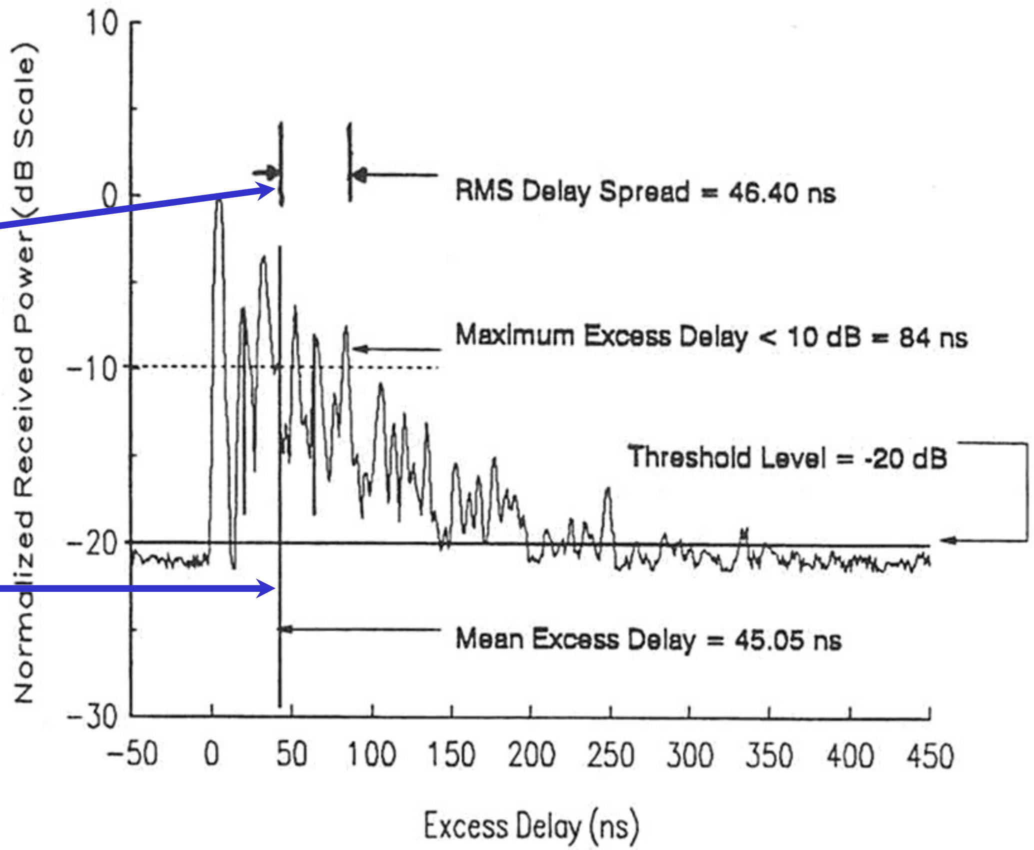


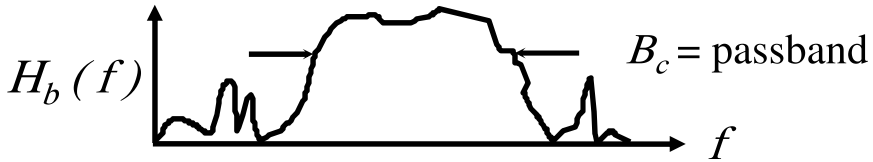
Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

MRC Parameters

- Coherence BW or Channel BW (B_c)
  - Directly related to RMS Delay Spread
  - FFT of multipath delay profile shows frequency (spectral) characteristics of MRC
  - B_c : statistical measure of frequency range where MRC response is **flat**
    - » "Flat" = passes all frequencies with ≈ equal gain & linear phase
    - » Amplitudes of different frequency components correlated
      - ➢ 0.5 correlation → $B_c \approx 1/5\ \sigma_\tau$
      - ➢ 0.9 correlation → $B_c \approx 1/50\ \sigma_\tau$ (worst case/conservative)

MRC Parameters

- Coherence BW or Channel BW (B_c)
  - FFT of multipath delay (channel IRF) gives frequency characteristics of channel
  - MRC filter passband $\rightarrow$ Channel or Coherence BW = B_c



  - ***MRC has bandlimited BPF response $\rightarrow$ multipath signals are the **direct** cause***
  - MRC is fundamentally a bandlimited channel
    - » Tx signals with BW > B_c will be distorted in frequency **and** time domain!!

MRC Parameters

- Coherence BW or Channel BW (B_c)
  - B_c and σ_τ are **related** quantities that characterize time-dispersive nature of MRC from multipath interference
    - » Frequency vs. Time domain perspective for **same** phenomena
  - B_c and σ_τ do **NOT** characterize time-varying nature of MRC due to motion of mobile and/or surrounding objects

MRC Parameters

- Doppler Spread (B_D) : measure of spectral broadening of Tx signal caused by motion → i.e. Doppler shift
  - B_D = max Doppler shift = $f_{max} = v_{max} / \lambda$ (~ B_D = 50-200 Hz)
  - If Tx signal $B_s >> B_D$ then **frequency** effects of Doppler spread are **NOT** important
  - Important for low bps (data rate) applications (e.g. paging)
- Coherence Time (T_c)
  - T_c : statistical measure of **time interval** over which MRC remains **invariant** → amplitude & phase of multipath signals ≈ constant
  - For digital communications → $T_c = 0.423 / B_D$
  - Provides physical measure of how **fast** the channel conditions change
    - » $T > T_c$ → changing channel → signal fluctuation/fading

Small Scale Fading

◆ Types of Small-Scale Fading

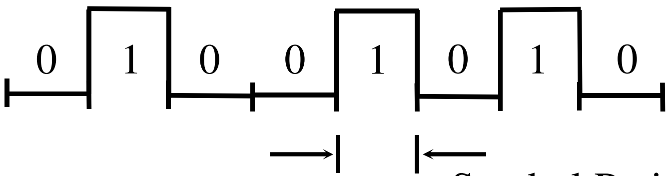
Fading can be caused by two **independent** MRC propagation mechanisms:

1) Time dispersion → multipath delay (B_c, σ_τ)
2) Frequency dispersion → Doppler spread (B_D, T_c)

Small Scale Fading

- Relationship between Tx signal parameters and channel parameters lead to <u>**four**</u> distinct effects

- Important <u>**digital**</u> Tx signal parameters → symbol period & signal BW



Symbol Period = T_s

Signal BW = $B_s \propto 1/T_s$

Small Scale Fading

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading

1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading

1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

Slow Fading

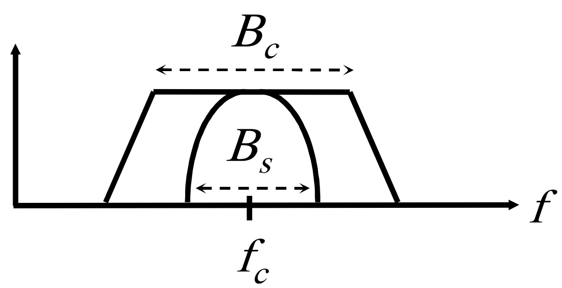
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Fading Types

1) Fading due to Multipath Delay

A) Flat Fading $\rightarrow B_s << B_c$ **or** $T_s >> \sigma_\tau$

- Signal BW << Channel BW



  - » Frequency domain perspective
  - » Common type of fading
  - » Spectral properties of Tx signal preserved
  - » Channel "gain" varies with time causing deep fades
  - » Fades ~ 20–30 dB $\rightarrow$ Rayleigh fading

Fading Types

1) Fading due to Multipath Delay

A) Flat Fading (continued) $T_s >> \sigma_\tau$???

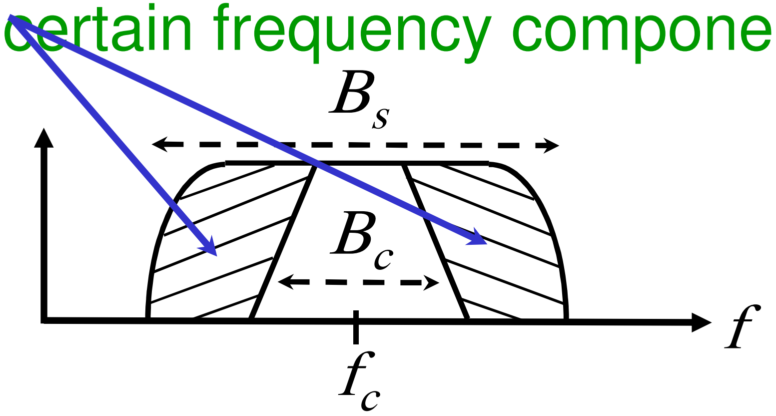
- Symbol Period >> RMS Delay Spread
  - » All multipath signals arrive at mobile Rx within ≈ **same** symbol period
    - ➢ No Inter-Symbol Interference (ISI)
  - » Interference **does** cause signal **amplitude** to vary from symbol to symbol
  - » Generally considered **desirable**!
    - ➢ Forward link → increase mobile Rx gain (AGC)
    - ➢ Reverse link → increase mobile Tx power (power control)

Fading Types

1) Fading due to Multipath Delay

B) Frequency Selective Fading $\rightarrow B_s > B_c$ **or** $T_s < \sigma_\tau$

- $B_s > B_c \rightarrow$ certain frequency components of signal attenuated



- Frequency distortion causes time domain shape of transmitted signal to be distorted as well

Fading Types

1) Fading due to Multipath Delay

B) Frequency Selective Fading (continued)

- $T_s < \sigma_\tau$??? $\rightarrow$ **delayed** versions of Tx signal arrive during **different** symbol periods
  - » e.g. LOS $\rightarrow$ "1" & multipath "0" (from prior symbol!)
  - » InterSymbol Interference $\rightarrow$ ISI
  - » **Very** undesirable $\rightarrow$ must compensate by "equalizing" Rx signal spectrum by providing more gain @ certain frequencies
- Very difficult to predict mobile Rx performance

Fading Types

2) Fading due to Doppler Spread

A) Fast Fading $\rightarrow$ $T_s > T_c$ or $B_s < B_D$

- $T_s > T_c$
  - » MRC changes within 1 symbol period
  - » Rapid amplitude fluctuations
- $B_s < B_D$
  - » Doppler shifts significantly alter spectral BW of TX signal
  - » Signal "spreading"
- Only occurs for low data rate applications $\rightarrow$ large T_s
- Uncommon in most digital communication system fades

Fading Types

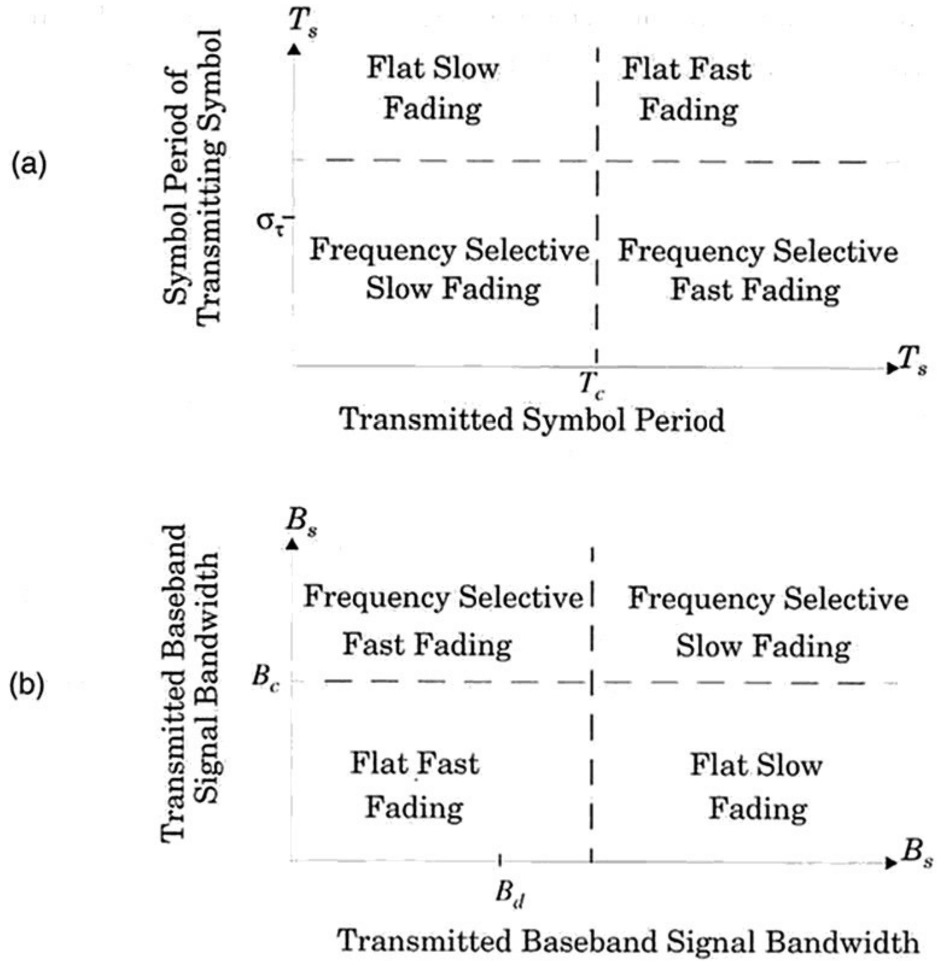
2) Fading due to Doppler Spread

A) Slow Fading $\rightarrow$ $T_s << T_c$ or $B_s >> B_D$

- MRC constant over many symbol periods
- Slow amplitude fluctuations
  - For v = 60 mph @ f_c = 2 GHz $\rightarrow$ B_D = 178 Hz
  - $B_s \approx$ 2 kHz >> B_D
- B_s almost always >> B_D for most applications

Fading Types

Flat/Slow Fading
is most desirable



Fading Signal Distributions

- Small-scale fading often referred to as "Rayleigh fading"
  - Random amplitude fluctuations follow a Rayleigh PDF
  - Applies for **flat** fading channel **only**
  - Rayleigh PDF

$$p(r)=\begin{cases}\dfrac{r}{\sigma^2}\exp\left(-\dfrac{r^2}{2\sigma^2}\right) & 0\le r\le\infty\\ 0 & r<0\end{cases}$$

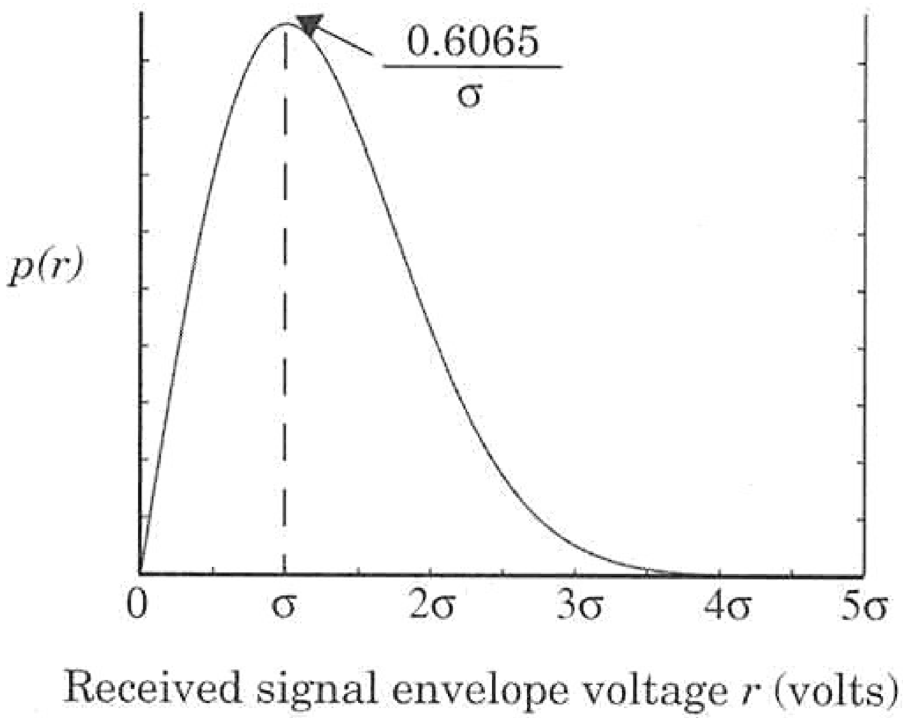
    - » σ: RMS value of Rx signal **before** detection (demodulation)
    - » Common model for Rx signal variation
      - ➢ Urban areas → heavy clutter → no LOS path

Raleigh PDF

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{r^2}{2\sigma^2}\right) & 0 \le r \le \infty \\ 0 & r < 0 \end{cases}$$

Probability that signal exceeds predefined threshold level R

$$\text{Prob}\,(r > R) \;=\; \int_R^\infty p(r)\,dr = \exp\left(\frac{-R^2}{2\,\sigma^2}\right)$$



Fading Signal Distributions

- Ricean PDF
  - One dominant signal component along with weaker multipath signals
  - Dominant signal → LOS path
    - » Suburban or rural areas with light clutter

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{\left(r^2 + A^2\right)}{2\sigma^2}\right) I_o\left(\dfrac{Ar}{\sigma^2}\right) & A \geq 0,\ r \geq 0 \\ 0 & r < 0 \end{cases}$$

A: Peak amplitude of dominant signal

I_0: Modified Bessel function

Ricean PDF

$$K = 10\log\left(\frac{A^2}{2\sigma^2}\right) \quad \text{dB}$$

For $A \to 0$ then $K \to -\infty$

Ricean PDF $\to$ Rayleigh PDF