Lecture # 4 (B)

Water Flow In Open Channels

In this lecture you will learn about:

- Energy Principles in Open Channels.
- Non Uniform Flow in Open Channels.
- Hydraulic Jump.

Course Name

“Irrigation And Hydraulic Structures”

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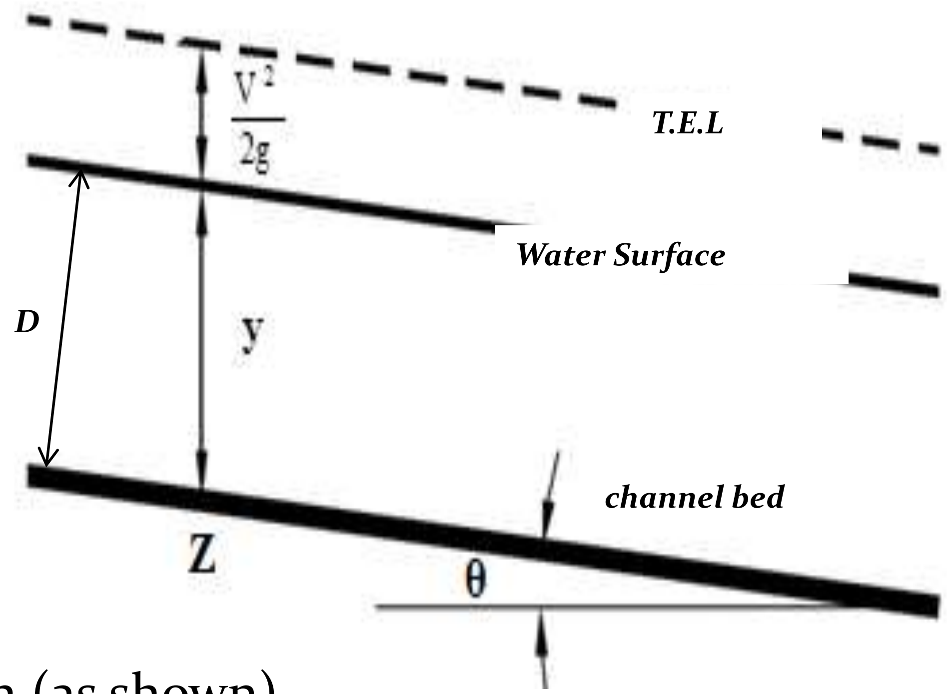
Energy Principles in Open Channel Flow

Referring to the figure shown, the total energy of a flowing liquid per unit weight is given by

$$Total\ Energy = Z + y + \frac{V^2}{2g}$$



Where:
Z = *height of the bottom of* channel above datum,
y = *depth of liquid,*
V = *mean velocity of flow.*

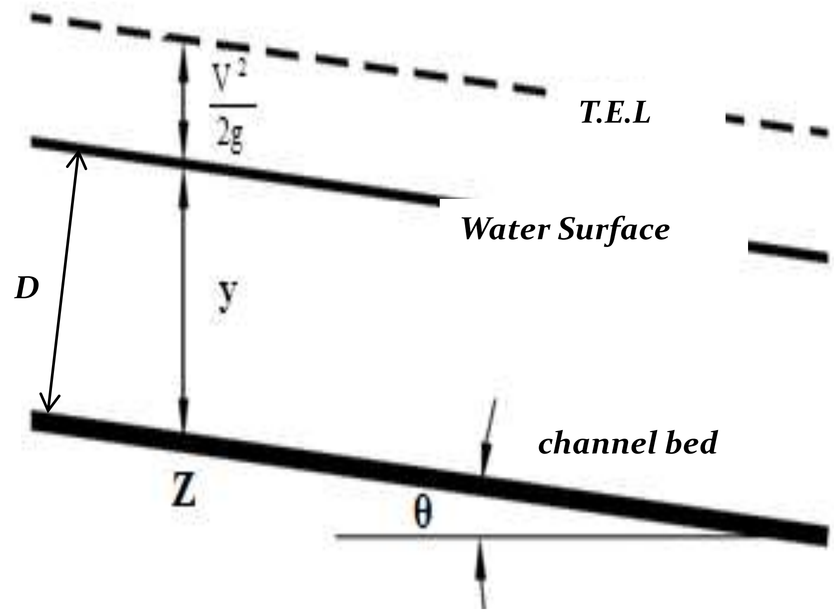
If the channel bed is taken as the datum (as shown), then the total energy per unit weight will be.
This energy is known as specific energy, Es. Specific energy of a flowing liquid in a channel is defined as energy per unit weight of the liquid measured from the channel bed as datum

$$E_{specific} = y + \frac{V^2}{2g}$$

Energy Principles in Open Channel Flow

The specific energy of a flowing liquid can be re-written in the form:

$$E_s = y + \frac{V^2}{2g} = E_p + E_k$$



where

$$E_p = \text{potential energy of flow} = y$$

$$E_k = \text{kinetic energy of flow} = \frac{V^2}{2g}$$

Since the velocity of flow,

$$V = \frac{Q}{A}, \qquad \text{then}$$

$$E_s = y + \frac{Q^2}{2gA^2}$$

which is valid for *any cross section*.

Energy Principles in Open Channel Flow

Specific Energy Curve (rectangular channel)

It is defined as the curve which shows the variation of specific energy (Es) with depth of flow y. It can be obtained as follows:

Let us consider a rectangular channel in which a constant discharge is taking place.

If q = discharge per unit width = $\frac{Q}{B}$ = constant (since Q *and* B are constants),

then

Velocity of flow,

$$V = \frac{Q}{A} = \frac{Q}{B \times y} = \frac{q}{y}$$

But $E_s = y + \frac{V^2}{2g} = E_p + E_k$

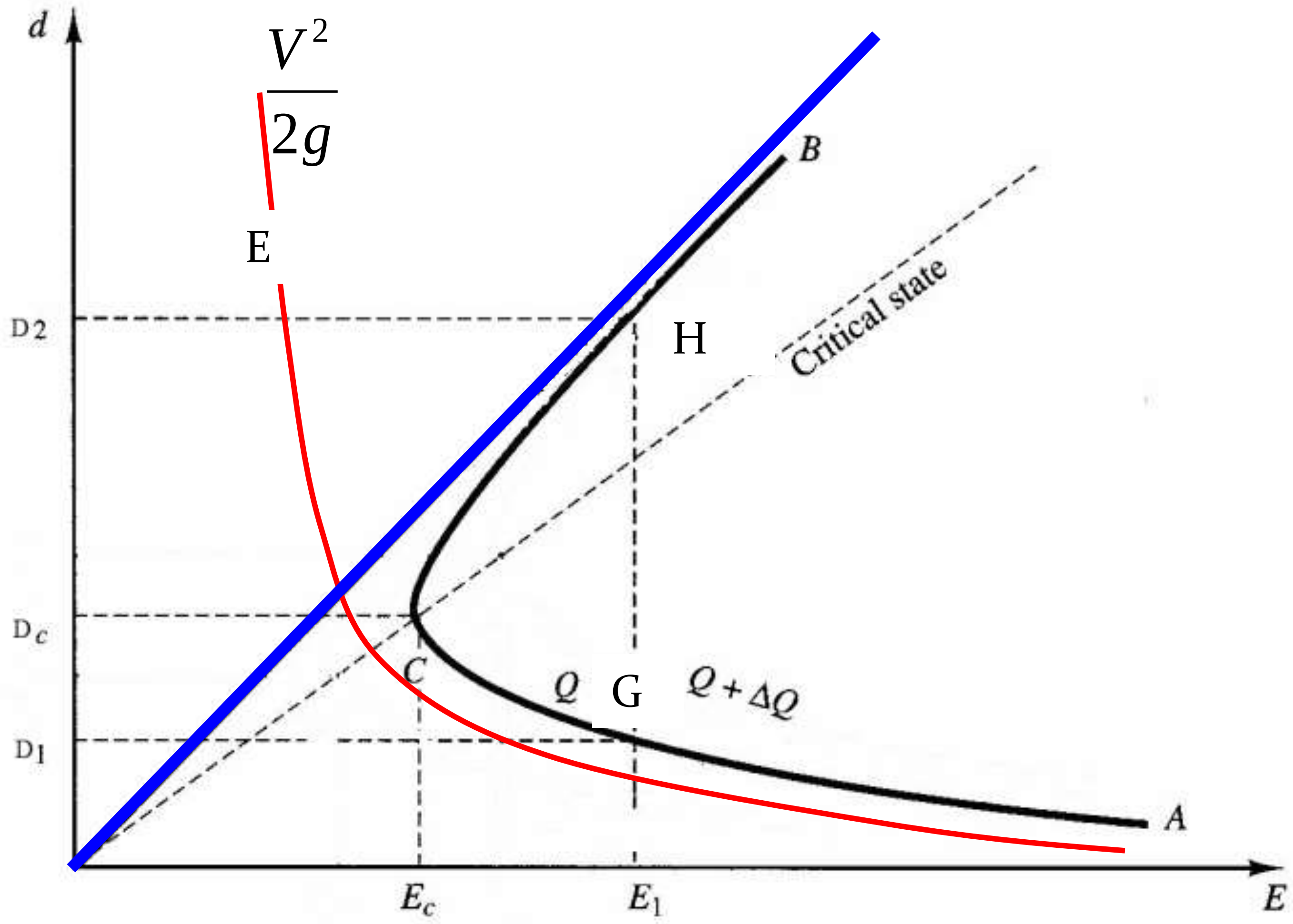
$$E_s = y + \frac{q^2}{2gy^2} = E_p + E_k$$

Or $E_{specific} = y + \frac{V^2}{2g}$ $E_s = y + \frac{Q^2}{2gA^2}$

Energy Principles in Open Channel Flow

Specific Energy Curve (rectangular channel)

The graph between specific energy (x-axis) and depth (yaxis) may plotted.



Energy Principles in Open Channel Flow

Specific Energy Curve (rectangular channel)

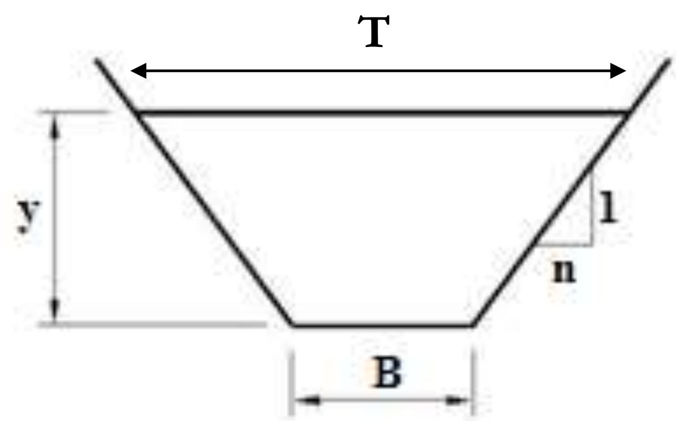
Referring to the diagram above, the following features can be observed:

1. The depth of flow at point C is referred to as ***critical depth, yc. It is defined as that*** depth of flow of liquid at which the specific energy is minimum, E_{min}, *i.e.;* E_{min} *@ yc . The flow that corresponds to this point is called* ***critical flow (Fr*** = 1.0).
2. For values of *Es greater than* E_{min} *, there are two corresponding depths. One depth* is greater than the critical depth and the other is smaller then the critical depth, for example ; *Es1 @* y_1 *and* y_2 These two depths for a given specific energy are called the ***alternate depths.***
3. If the flow depth $y > y_c$ *, the flow is said to be* ***sub-critical (Fr < 1.0). In this*** case *Es increases as y increases.*
4. If the flow depth $y < y_c$ *, the flow is said to be super-**critical (Fr > 1.0). In this*** case *Es increases as y increases.*

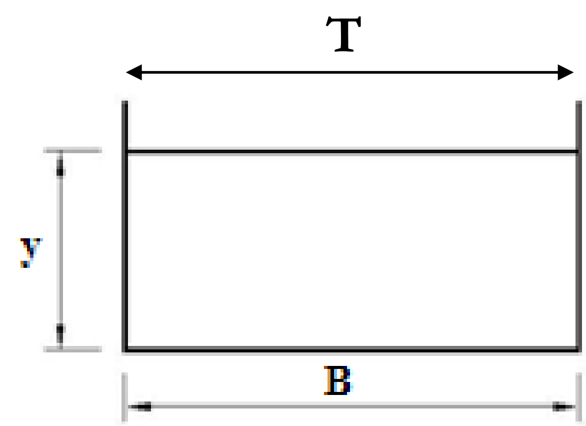
Froude Number (Fr)

$$F_r = \frac{V}{\sqrt{g D_h}}$$

$$D_h = \frac{\text{Area of Flow (Wetted Area)}}{\text{Water Surface Width}} = \frac{A}{T}$$

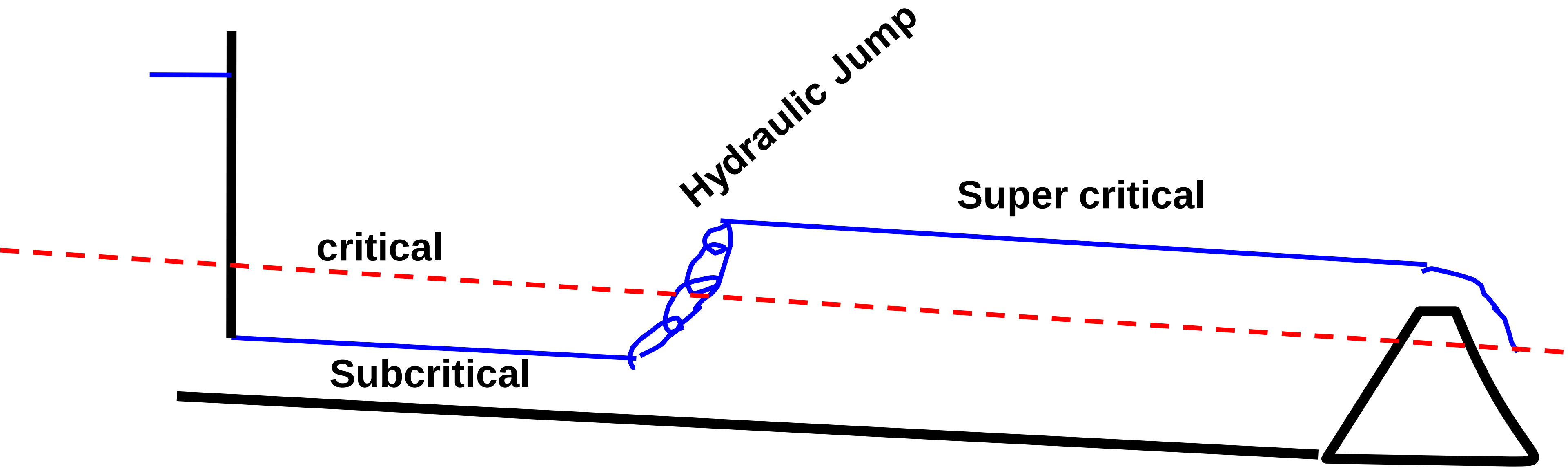


$$F_r^{\,2} = \frac{Q^2 T}{A^3 g}$$



Fr	Flow
1 > Fr	Sub-critical
1 = Fr	Critical
1 < Fr	Supercritical

Critical Flow



Energy Principles in Open Channel Flow

Rectangular Channel

For rectangular section

$$F_r = \frac{V}{\sqrt{gD_h}} = \frac{V}{\sqrt{gD}}$$

At critical Flow

a) Critical depth, *yc , is defined as that depth of flow of liquid at which the specific* energy is minimum, *Emin,*

$$F_r = 1 = \frac{V}{\sqrt{gD_h}}$$

$$y_c = \left(\frac{Q^2}{B^2 g}\right)^{\frac{1}{3}}$$

q=Q/B

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

b) Critical velocity, Vc , is the velocity of flow at critical depth.

$$V_c = \sqrt{g \times y_c}$$

Energy Principles in Open Channel Flow

Rectangular Channel

c) Critical, Sub-critical, and Super-critical Flows:

Critical flow is defined as the flow at which the specific energy is minimum or the flow that corresponds to critical depth. Refer to point *C in above figure,* E_{min} @ y_c.

$$V_c = \sqrt{g \times y_c}$$

$$\frac{V_c}{\sqrt{g \times y_c}} = 1$$ and $$Fr = \frac{V}{\sqrt{gD}}$$ therefore for critical flow *Fr* = 1.0

If the depth flow *y* > *yc* , *the flow is said to be* ***sub-critical. In this case Es increases*** as *y increases. For this type of flow, Fr* < 1.0 .

If the depth flow *y* < *yc* , *the flow is said to be* ***super-critical. In this case Es increases*** as *y decreases. For this type of flow, Fr* > 1.0 .

Energy Principles in Open Channel Flow

Rectangular Channel

d) Minimum Specific Energy in terms of critical depth:

At (Emin , yc) ,

$$E_{\min} = y_c + \frac{q^2}{2 g y_c^2} ,$$

$$E_{\min} = y_c + \frac{y_c}{2} ,$$

$$E_{\min} = \frac{3 y_c}{2}$$

$$y_c = \frac{2 E_{\min}}{3}$$

Energy Principles in Open Channel Flow

Other Sections

at critical flow Fr =1 where: $F_r^{\,2} = \dfrac{Q^2 T}{A^3 g} = 1$

$$E_c = \frac{3}{2} D_c$$

Rectangular section

$$E_c = \frac{(3B + 5nD_c)D_c}{2(B + 2nD_c)}$$

Trapezoidal section

$$E_c = \frac{d}{2}(1 - \cos\alpha) + \frac{d}{16}\frac{(2\alpha - \sin 2\alpha)}{\sin\alpha}$$

Circular section

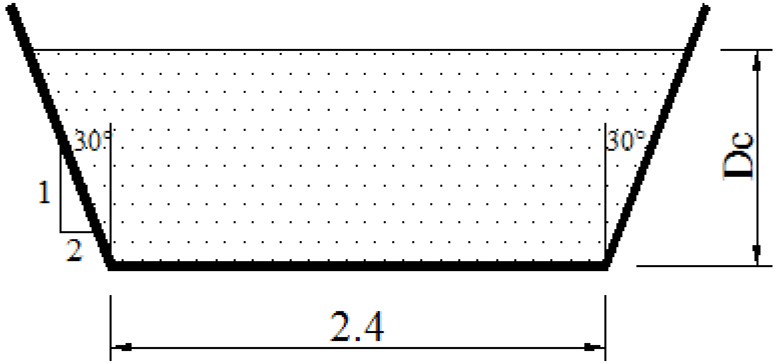
$$E_c = \frac{5}{4} D_c$$

Triangle section

Example 1

Determine the critical depth if the flow is 1.33m3/s. the channel width is 2.4m

$$\frac{Q^2 T}{A^3 g} = 1$$



$$\frac{Q^2 (B + 2 \times nD_c)}{((B + nD_c)D_c)^3 g} = \frac{1.33^2 \left(2.4 + 2 \times \frac{1}{\sqrt{3}} D_c\right)}{\left(\left(2.4 + \frac{1}{\sqrt{3}} D_c\right) D_c\right)^3 \times 9.81} = 1$$

$$D_c = 0.31\,\text{m}$$

Example 2

Rectangular channel , $Q=25m^3/s$, bed slope =0.006, determine the channel width with critical flow using manning n=0.016

$$y_c = \left(\frac{Q^2}{B^2 g}\right)^{\frac{1}{3}} = \left(\frac{25^2}{B^2 \times 9.81}\right)^{\frac{1}{3}} = \left(\frac{4}{B^2}\right)^{\frac{1}{3}} = \frac{4}{B^{\frac{2}{3}}}$$

$$V = \frac{1}{n} R^{2/3} \sqrt{S}$$

$$R = \frac{A}{P} = \frac{D_c B}{2D_c + B}$$

$$\frac{25}{D_c B} = \frac{1}{0.016}\left(\frac{D_c B}{2D_c + B}\right)^{2/3} \sqrt{0.006}$$

Example 2 cont.

$$\frac{25}{\frac{4}{B^{2/3}}B} = \frac{1}{0.016}\left(\frac{\left(\frac{4}{B^{2/3}}\right)B}{2\left(\frac{4}{B^{2/3}}\right)+B}\right)^{2/3} \times \sqrt{0.006}$$

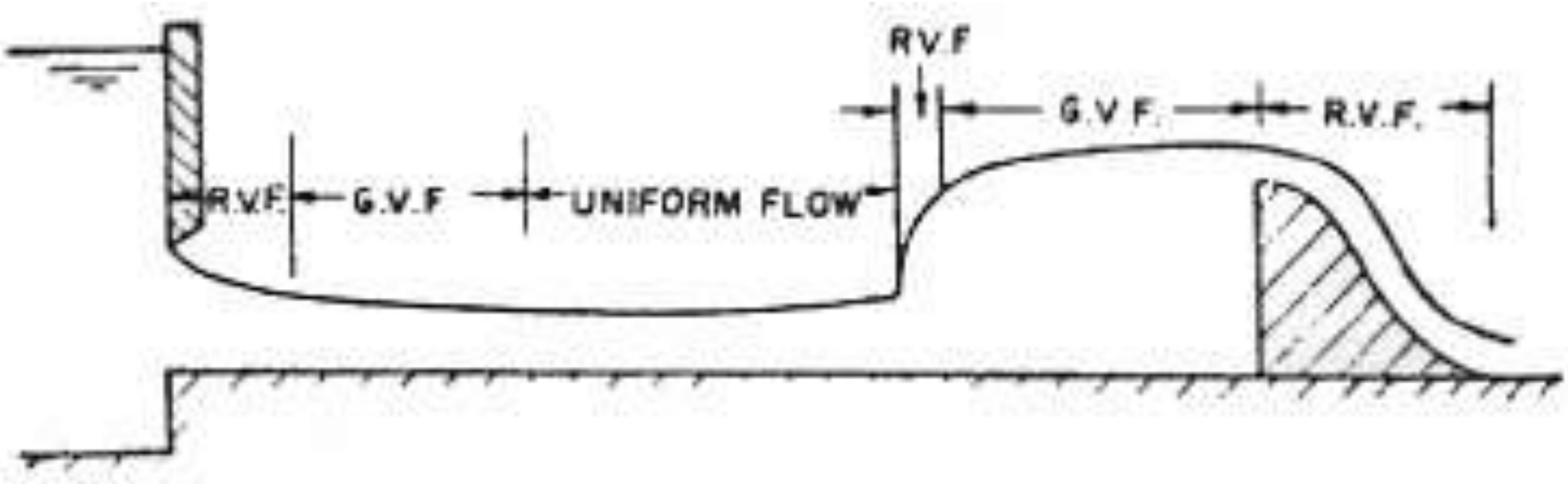
$$\frac{25}{4B^{1/3}} = \frac{1}{0.016}\left(\frac{4B}{8+B^{5/3}}\right)^{2/3} \times \sqrt{0.006}$$

$$B = 3m$$

Non-uniform Flow in Open Channels:

- Non-uniform flow is a flow for which the depth of flow is aried. This varied flow can be either ***Gradually varied flow (GVF) or Rapidly varied flow (RVF).***
- Such situations occur when control structures are used in the channel or when any obstruction is found in the channel
- Such situations may also occur at the free discharges and when a sharp change in the channel slope takes place.
- The most important elements, in non-uniform flow, that will be studied in this sectionare:
    - Classification of channel-bed slopes.
    - Classification of water surface profiles.
    - The dynamic equation of gradually varied flow.
    - Hydraulic jumps as examples of rapidly varied flow.

Non-uniform Flow in Open Channels:



Non-uniform Flow in Open Channels:

Classification of Channel-Bed Slopes

➢ **The slope of the channel bed can be classified as:**

1) *Critical Slope:* *the bottom slope of the channel is equal to the critical slope. In this* case $S_o = S_c$ or $y_n = y_c$.

2) *Mild Slope:* *the bottom slope of the channel is less than the critical slope. In this* case $S_o < S_c$ or $y_n > y_c$.

3) *Steep Slope:* *the bottom slope of the channel is greater than the critical slope. In this* case $S_o > S_c$ or $y_n < y_c$.

4) *Horizontal Slope:* *the bottom slope of the channel is equal to zero (horizontal bed).* In this case $S_o = 0.0$.

5) *Adverse Slope:* *the bottom slope of the channel rises in the direction of the flow* (slope is opposite to direction of flow). In this case S_o = *negative* .

The first letter of each slope type sometimes is used to indicate the slope of the bed. So the above slopes are abbreviated as *C, M, S, H, and A, respectively.*

Non-uniform Flow in Open Channels:

Classification of Channel-Bed Slopes

The figure below gives an example for the different types of channel slopes.

Non-uniform Flow in Open Channels:

Classification of Flow Profiles (water surface profiles):

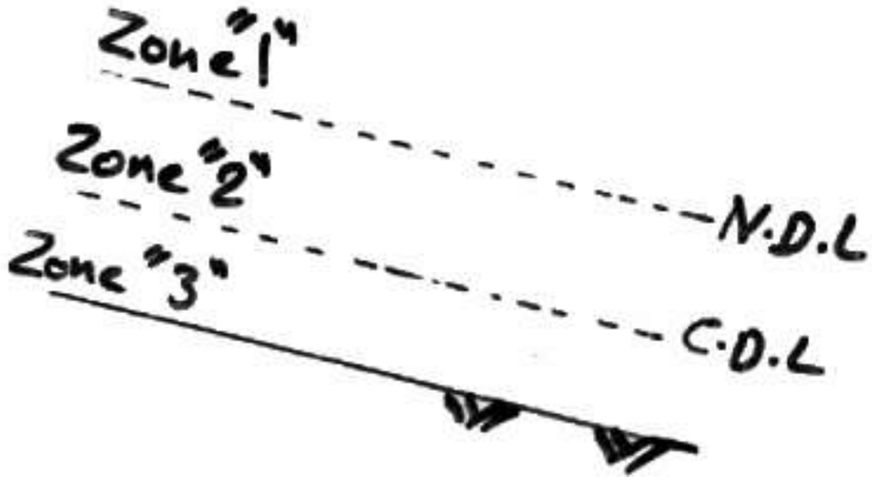
the following steps are followed to classify the flow profiles:

1) A line parallel to the channel bottom with a height of y_n is drawn and is designated as the normal depth line (N.D.L.)
2) A line parallel to the channel bottom with a height of y_c is drawn and is designated as the critical depth line (C.D.L.)
3) The vertical space in a longitudinal section is divided into 3 zones using the two lines drawn in steps 1 & 2 (see figure below):

⇒ Zone 1: it is the space above the two lines.

⇒ Zone 2: it is the space between the two lines.

⇒ Zone 3: it is the space below the two lines.



Non-uniform Flow in Open Channels:

Classification of Flow Profiles (water surface profiles):

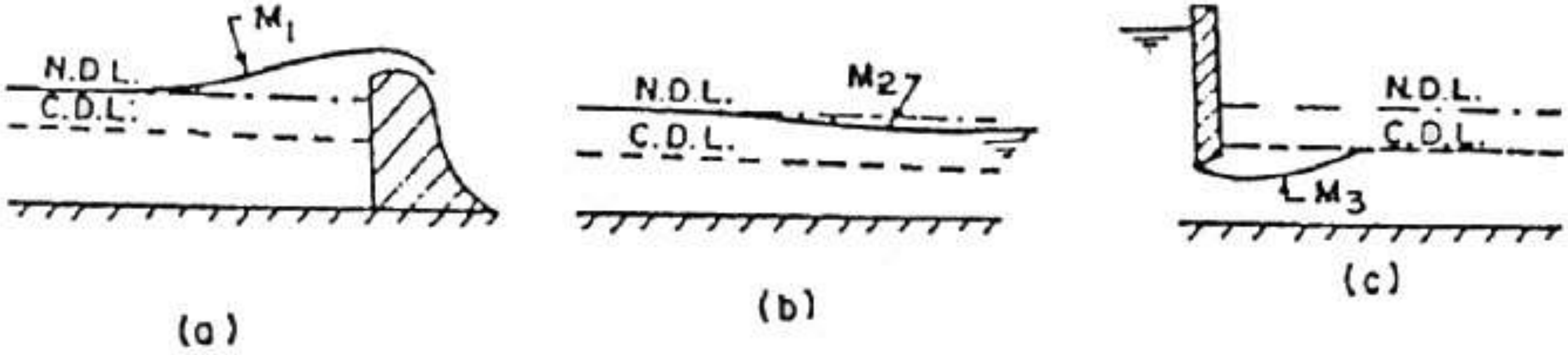
4) Depending upon the zone and the slope of the bed, the water profiles are classified into 13 types as follows:

(a) Mild slope curves M_1, M_2, M_3.
(b) Steep slope curves S_1, S_2, S_3.
(c) Critical slope curves C_1, C_2, C_3.
(d) Horizontal slope curves H_2, H_3.
(e) Averse slope curves A_2, A_3.

In all these curves, the letter indicates the slope type and the subscript indicates the zone. For example S_2 curve occurs in the zone 2 of the steep slope.

Non-uniform Flow in Open Channels:

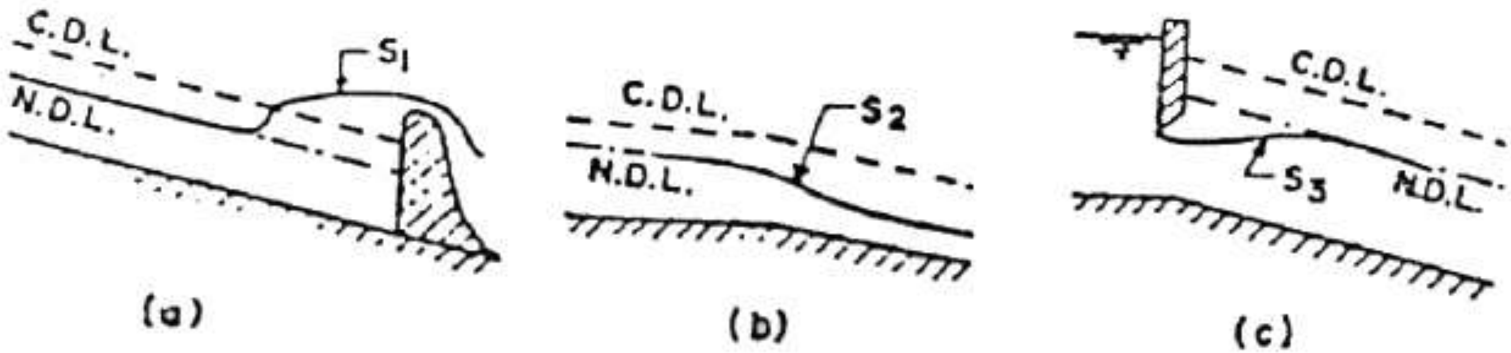
Classification of Flow Profiles (water surface profiles):



Flow profiles in Mild slope.

It can be noted that:

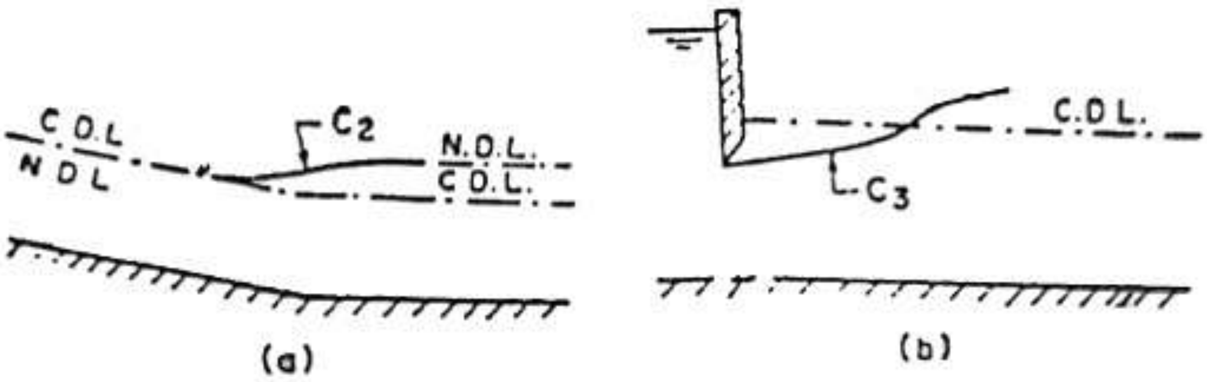
- M_1 is backwater curve,
- M_2 is drawdown curve,
- M_3 occurs when supercritical flow enters mild slope or when the slope of the bed changes from steep to mild.



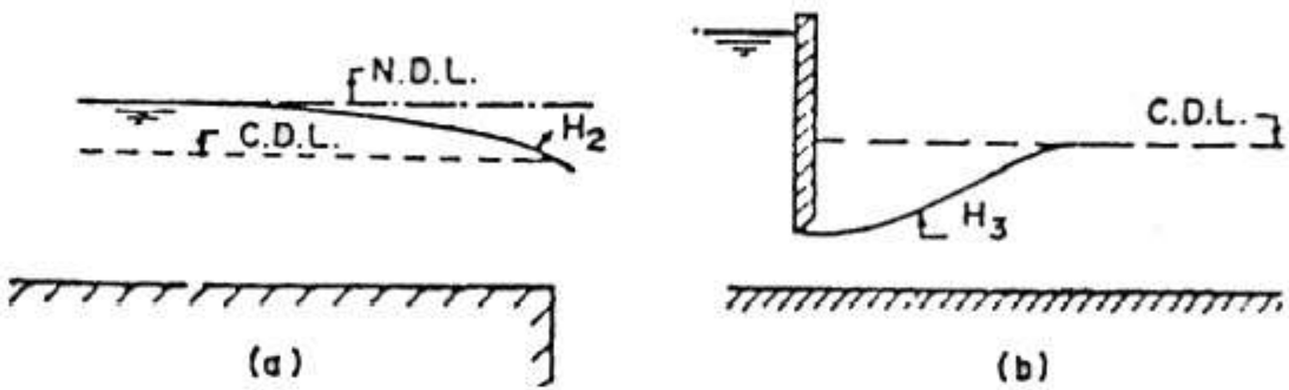
Flow profiles in steep slope.

Non-uniform Flow in Open Channels:

Classification of Flow Profiles (water surface profiles):



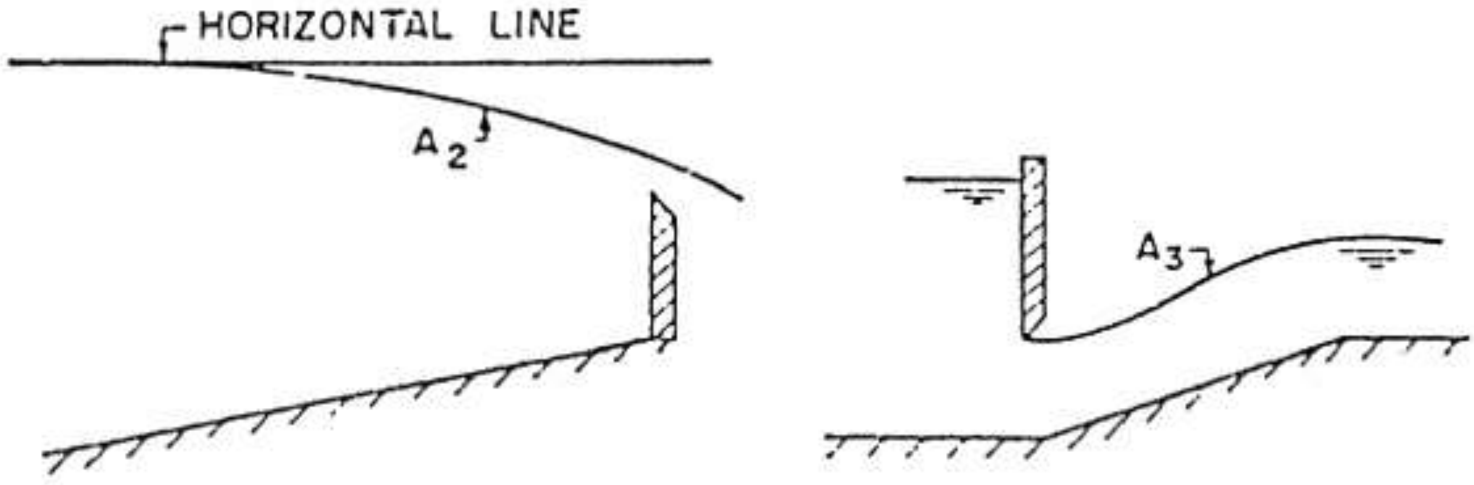
Flow profiles in critical slope.



Flow profiles in horizontal channels.

Non-uniform Flow in Open Channels:

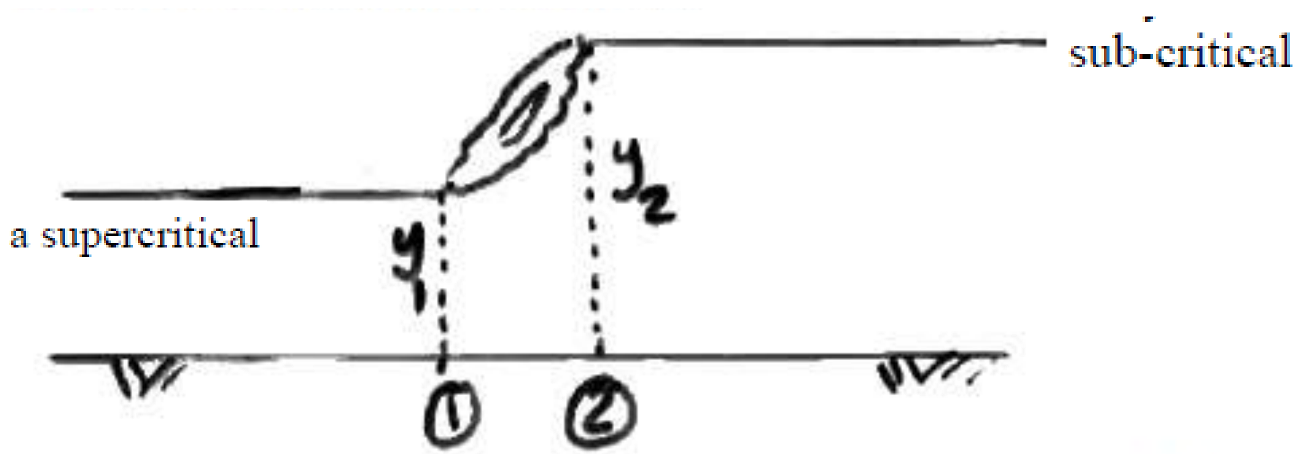
Classification of Flow Profiles (water surface profiles):



Flow Profiles in Adverse Slopes

Hydraulic Jump

A hydraulic jump occurs when flow changes from a supercritical flow (unstable) to a sub-critical flow (stable). There is a sudden rise in water level at the point where the hydraulic jump occurs. Rollers (eddies) of turbulent water form at this point. These rollers cause dissipation of energy.



A hydraulic jump occurs in practice at the toe of a dam or below a sluice gate where the velocity is very high.

Hydraulic Jump

General Expression for Hydraulic Jump:

In the analysis of hydraulic jumps, the following assumptions are made:

(1) The length of hydraulic jump is small. Consequently, the loss of head due to friction is negligible.

(2) The flow is uniform and pressure distribution is due to hydrostatic before and after the jump.

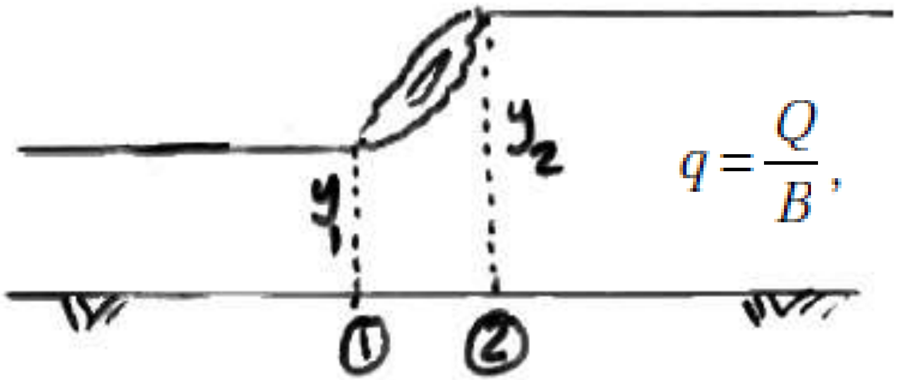
(3) The slope of the bed of the channel is very small, so that the component of the weight of the fluid in the direction of the flow is neglected.

Hydraulic Jump

Hydraulic Jump in Rectangular Channels

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \left(\frac{2q^2}{g y_1}\right)}$$

$$y_1 = -\frac{y_2}{2} + \sqrt{\left(\frac{y_2}{2}\right)^2 + \left(\frac{2q^2}{g y_2}\right)}$$



$$q = \frac{Q}{B},$$

- Ratio of conjugate depths:

But for Rectangular section $$y_c^3 = \frac{q^2}{g}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8q^2}{g y_1^3}}\right)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8q^2}{g y_2^3}}\right)$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\left(\frac{y_c}{y_1}\right)^3}\right)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\left(\frac{y_c}{y_2}\right)^3}\right)$$

Hydraulic Jump

Hydraulic Jump in Rectangular Channels

- **Ratio of conjugate depths:**

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8F_1^2}\right)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(-1 + \sqrt{1 + 8F_2^2}\right)$$

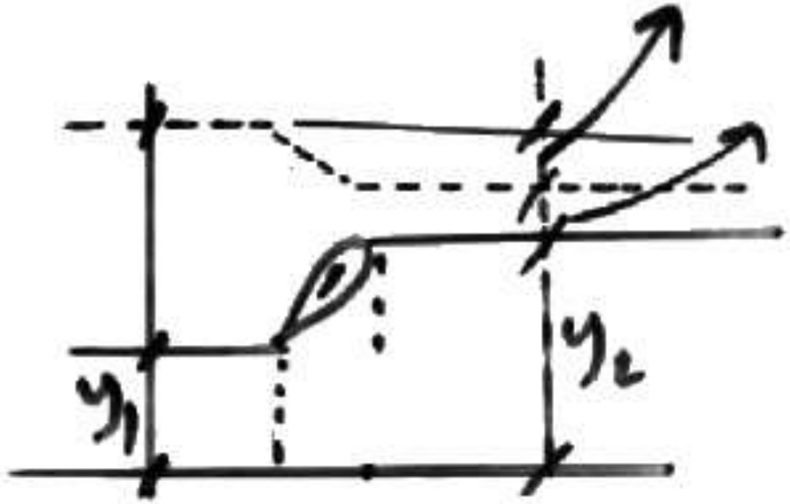
where $F_1 = \frac{V_1}{\sqrt{g y_1}}$ and $F_2 = \frac{V_2}{\sqrt{g y_2}}$

Hydraulic Jump in Rectangular Channels

- **Head Loss in a hydraulic jump** *(H_L)*:
  Due to the turbulent flow in hydraulic jump, a dissipation (loss) of energy occurs (see figures shown);

$$H_L = \Delta E = E_1 - E_2$$

where E = specific energy



For rectangular channels, specific energy is defined by equation (7.33) as

$$E_s = y + \frac{q^2}{2g\,y^2}$$

hence,

$$H_L = y_1 + \frac{q^2}{2g\,y_1^2} - \left(y_2 + \frac{q^2}{2g\,y_2^3} \right)$$

or

$$H_L = \frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2} \right] - (y_2 - y_1)$$

Hydraulic Jump in Rectangular Channels

or
$$H_L = \frac{q^2}{2g}\left(\frac{y_2^2 - y_1^2}{y_1^2\ y_2^3}\right) - (y_2 - y_1)$$

using equation (7.56b), we get
$$H_L = \frac{y_1\, y_2\,(y_1 + y_2)\,(y_2^2 - y_1^2)}{4 y_1^2\ y_2^2} - (y_2 - y_1)$$

after simplifying, we obtain
$$H_L = \frac{(y_2 - y_1)^3}{4\, y_1\, y_2}$$

Hydraulic Jump

- **Height of hydraulic jump** *(h_j)*:
  The difference of depths before and after the jump is known as the height of the jump,

$$h_j = y_2 - y_1 \qquad (7.62)$$

- **Length of hydraulic jump** *(L_j)*:
  The distance between the front face of the jump to a point on the downstream where the rollers (eddies) terminate and the flow becomes uniform is known as the length of the hydraulic jump. The length of the jump varies from 5 to 7 times its height. An average value is usually taken:

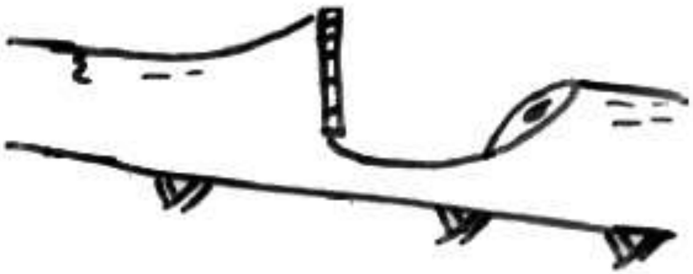
$$L_j \cong 6 h_j \qquad (7.63)$$

- **Location of hydraulic jump**:
  The most typical cases for the location of hydraulic jump are:

(One) Jump below a sluice gate.
(Two) Jump at the toe of a spillway.
(Three) Jump at a glacis.

(*glacis* is the name given to sloping floors provided in hydraulic structures.)

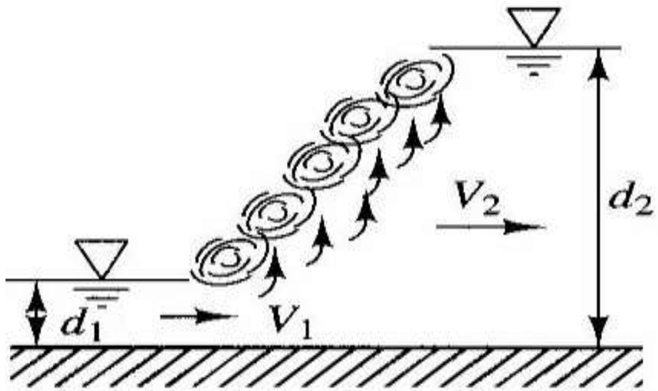
Hydraulic Jump



Recall that, generally, a hydraulic jump occurs when the flow changes from supercritical to subcritical flow.

Example 1

A 3-m wide rectangular channel carries 15 m³/s of water at a 0.7 m depth before entering a jump. Compute the downstrem water depth and the critical depth



$$q = \frac{15}{3} = 5\ \text{m}^3\text{/s.m}$$

$$d_c = \sqrt[3]{\frac{5^2}{9.81}} = 1.366m$$

$$V_1 = \frac{q}{d_1} = \frac{5}{0.7} = 7.14m/s$$

$$F_{r1} = \frac{V_1}{\sqrt{gd_1}} = \frac{7.14}{\sqrt{9.81 \times 0.7}} = 2.72$$

$$\frac{d_2}{0.7} = \frac{1}{2}\left(\sqrt{1+8(2.72)^2} - 1\right)$$

$$d_2 = 2.365m$$

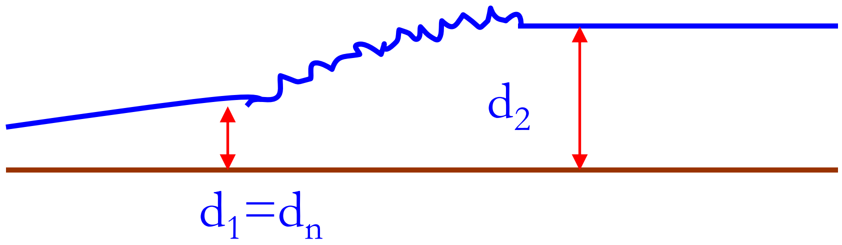
Example 2

A long, rectangular open channel 3 m wide carries a discharge of 15 m^3/sec. The channel slope is 0.004 and the Manning's coefficient is 0.01. At a certain point in the channel, flow reaches the normal depth.

(a) Determine the state of the flow. Is it supercritical or subcritical?

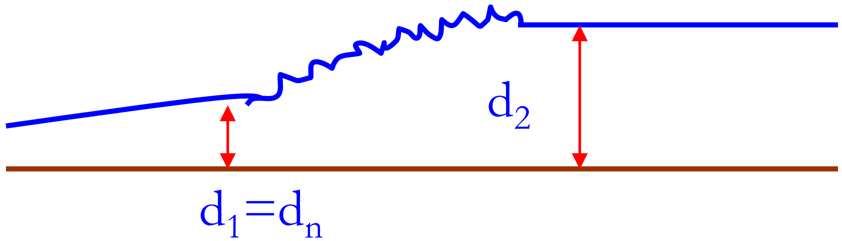
(b) If a hydraulic jump takes place at this depth, what is the sequent depth at the jump?

(c) Estimate the energy head loss through the jump.



d_n = Depth can calculated from manning equation

Hydraulic Jump



a) $$\frac{Q}{A}=\frac{1}{n}R_h^{2/3}\sqrt{S}$$

$$A=BD=3D$$

$$P=2D+B=2D+3$$

$$\frac{15}{3D}=\frac{1}{0.01}\left(\frac{3D}{2D+3}\right)^{\frac{2}{3}}\sqrt{0.004}$$

$$D=d_n=d_1=1.08m$$

$$V_1=\frac{15}{3d_1}=4.63m/s$$

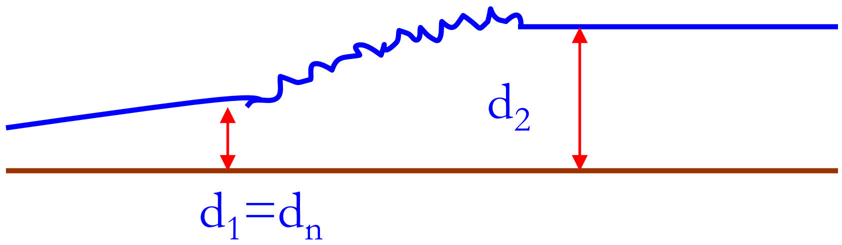
$$F_{r1}=\frac{V_1}{\sqrt{gd_1}}=\frac{4.63}{\sqrt{9.81\times1.08}}=1.42>1\rightarrow \text{the flow is supercritical}$$

b) $$\frac{d_2}{1.08}=\frac{1}{2}\left(\sqrt{1+8(1.42)^2}-1\right)$$

$$d_2=1.7m$$

Hydraulic Jump

c) $$\Delta E = \left(\frac{V_1^2}{2g} + d_1\right) - \left(\frac{V_2^2}{2g} + d_2\right)$$



$$V_2 = \frac{15}{3d_2} = \frac{15}{3(1.7)} = 2.94 m/s$$

$$\Delta E = \left(1.08 + \frac{4.63^2}{2g}\right) - \left(1.7 + \frac{2.94^2}{2g}\right) = 0.032 m$$

Thank You