LECTURE # 4

In this lecture you will learn about:

- Numerical on Parallelogram law
- Numerical on Sine and Cosine Law

.

Course Name:

"Applied Mechanics"

Course Code: CT-144

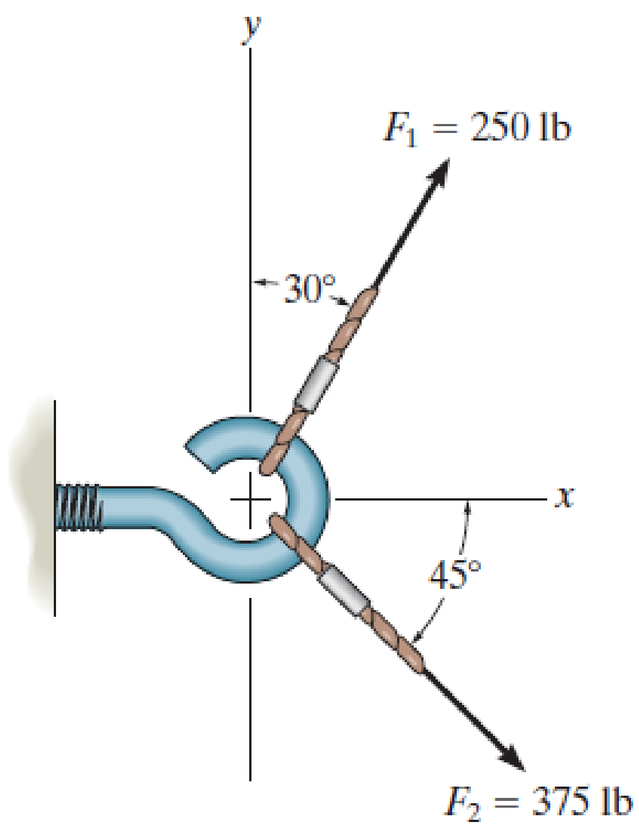
Credit Hours: 3

Semester: Summer 2020

Prepared By: Engr. Khurshid Alam

PROBLEM # 1

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive *x* axis.



Problem

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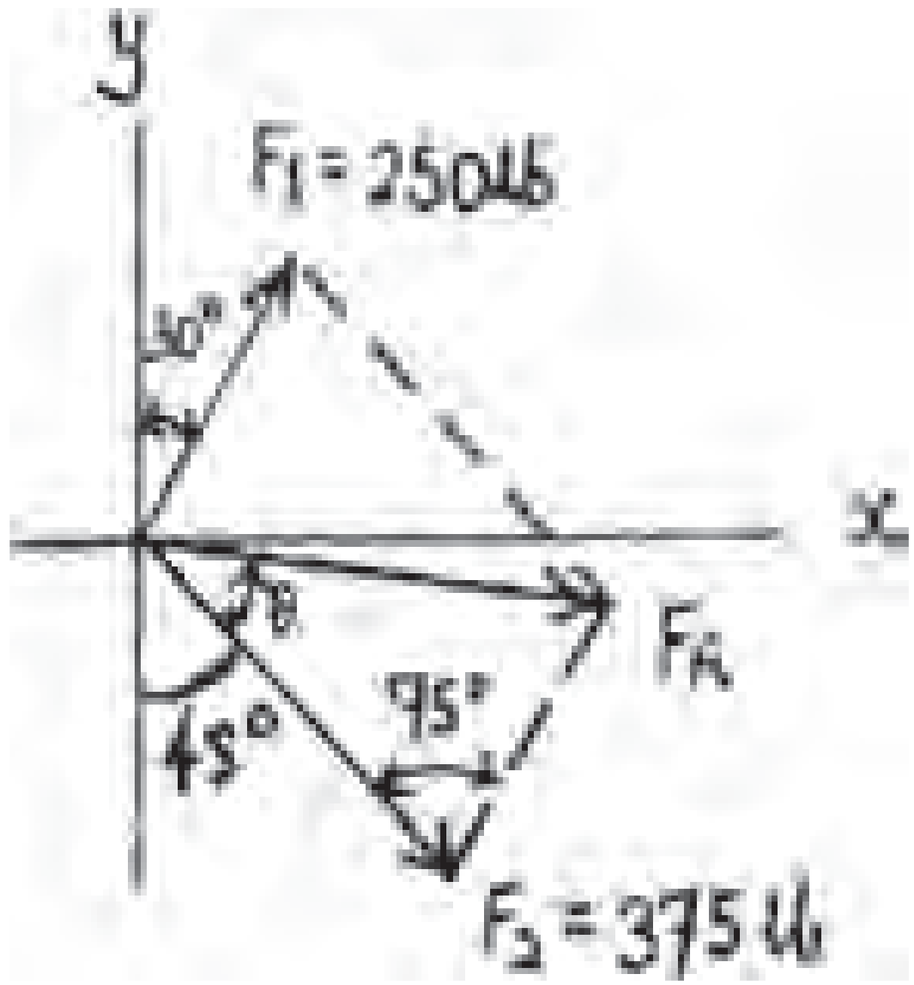
SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb}$$

$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

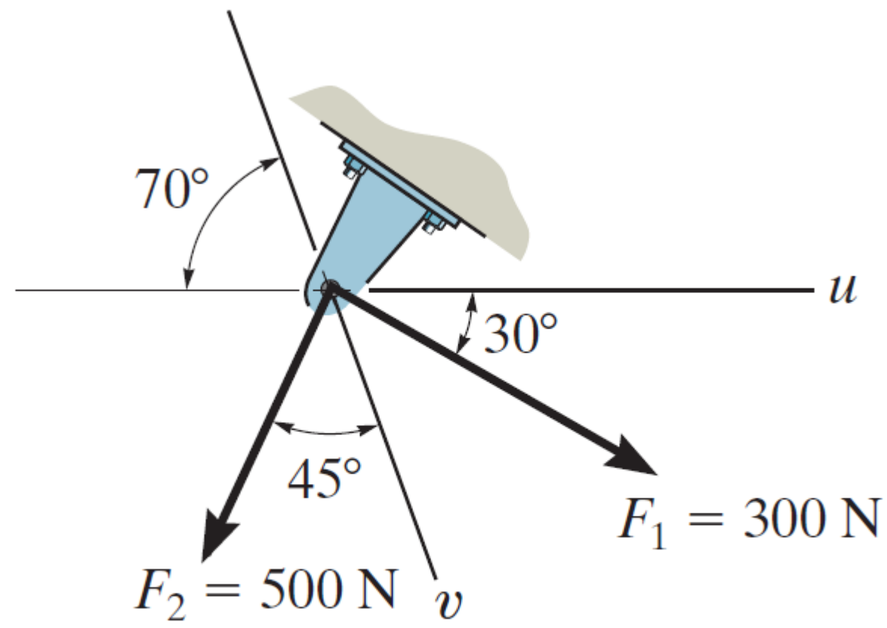
$$\theta = 37.89^\circ$$

$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$$



PROBLEM # 2

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



Problem

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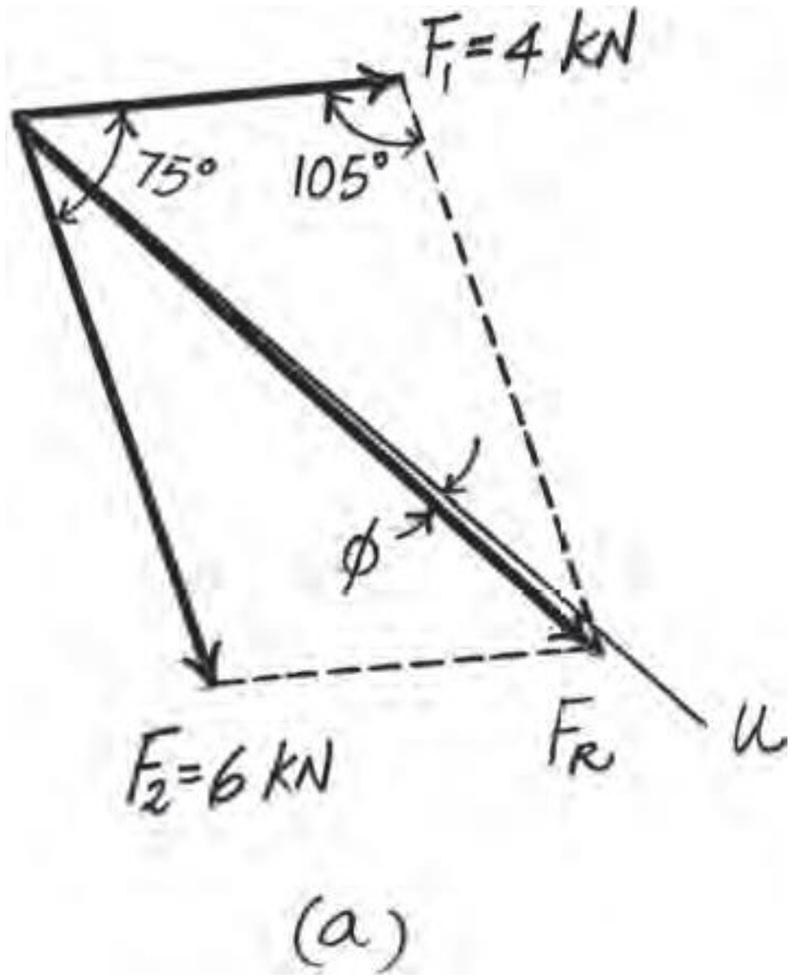
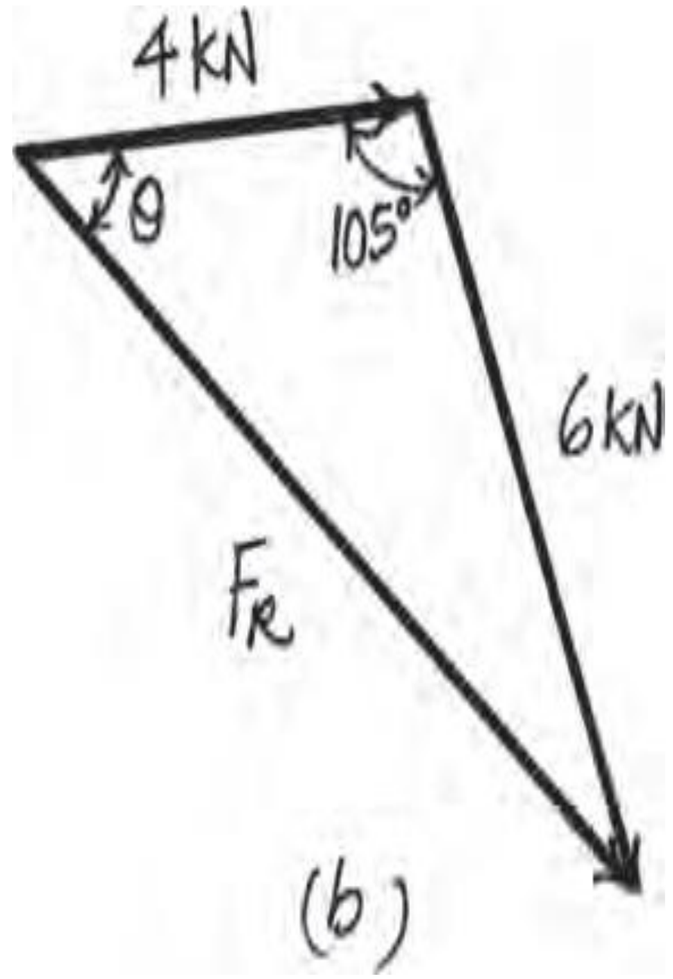
SOLUTION





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SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying Law of cosines by referring to Fig. *b*,

$$F_R = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 105°} = 8.026\ \text{kN} = 8.03\ \text{kN}$$

Using this result to apply Law of sines, Fig. *b*,

$$\frac{\sin\theta}{6} = \frac{\sin 105°}{8.026}; \qquad \theta = 46.22°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured clockwise from the positive u axis is

$$\phi = 46.22° - 45° = 1.22°$$

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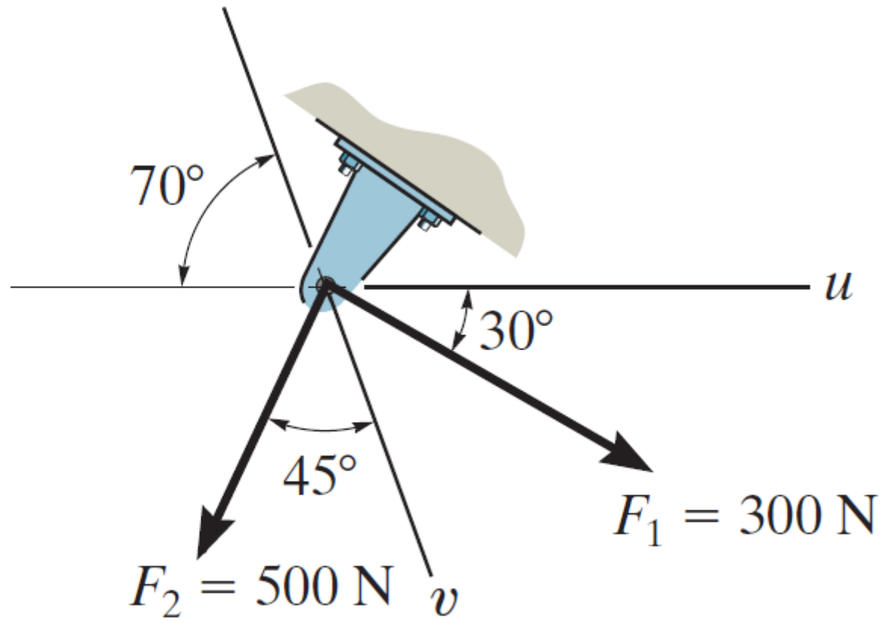
PROBLEM # 3

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



Problem

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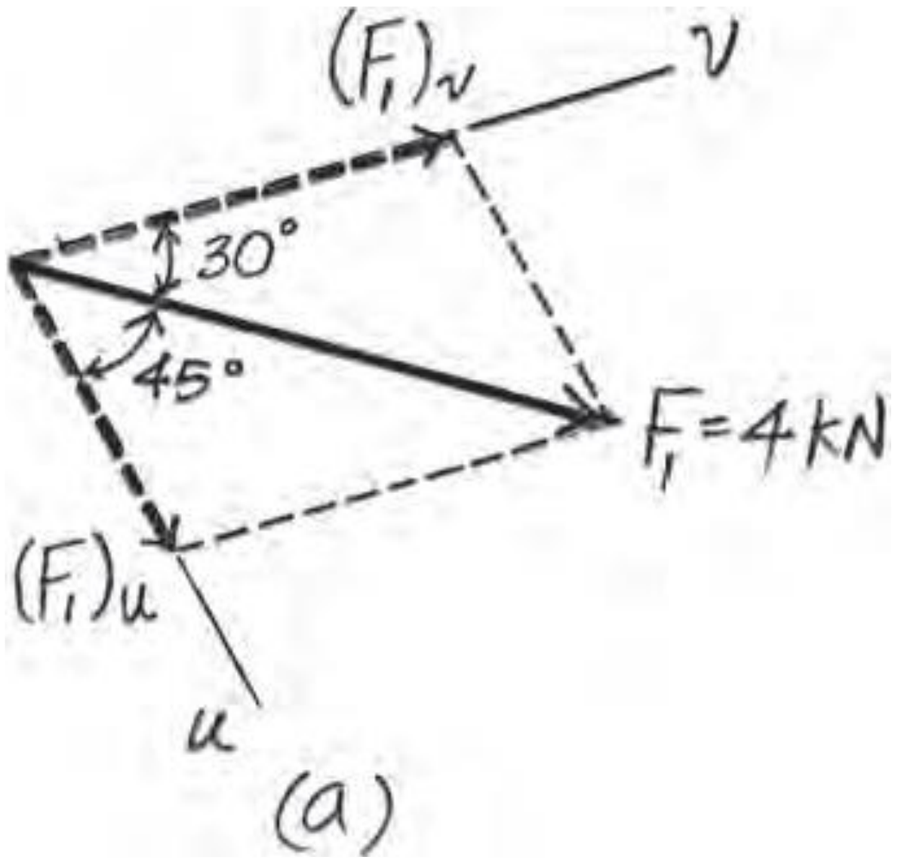
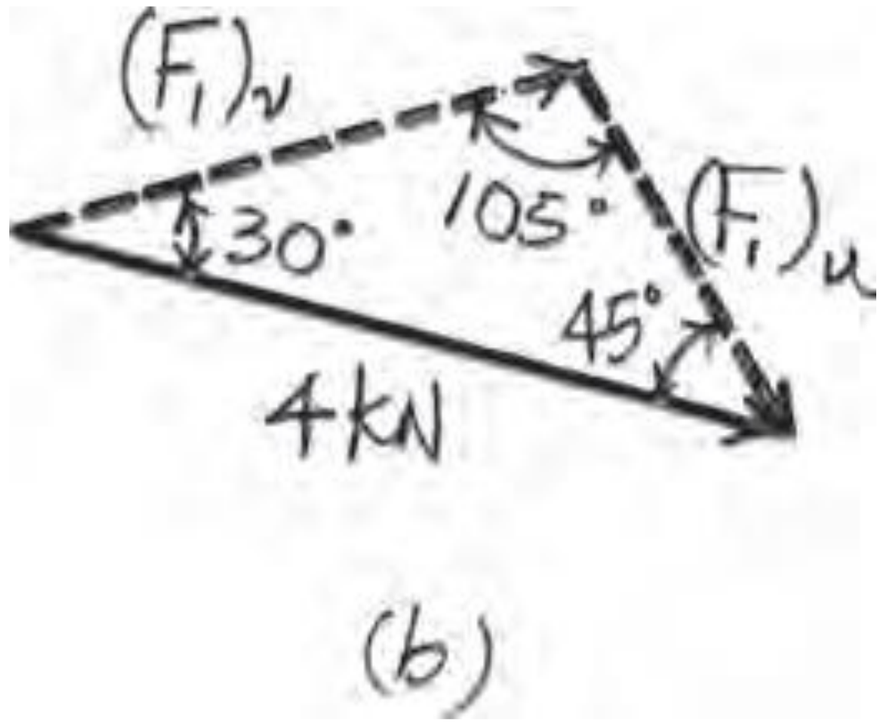
SOLUTION





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SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the sines law by referring to Fig. *b*.

$$\frac{(F_1)_v}{\sin 45°} = \frac{4}{\sin 105°}; \qquad (F_1)_v = 2.928\ \text{kN} = 2.93\ \text{kN}$$

$$\frac{(F_1)_u}{\sin 30°} = \frac{4}{\sin 105°}; \qquad (F_1)_u = 2.071\ \text{kN} = 2.07\ \text{kN}$$

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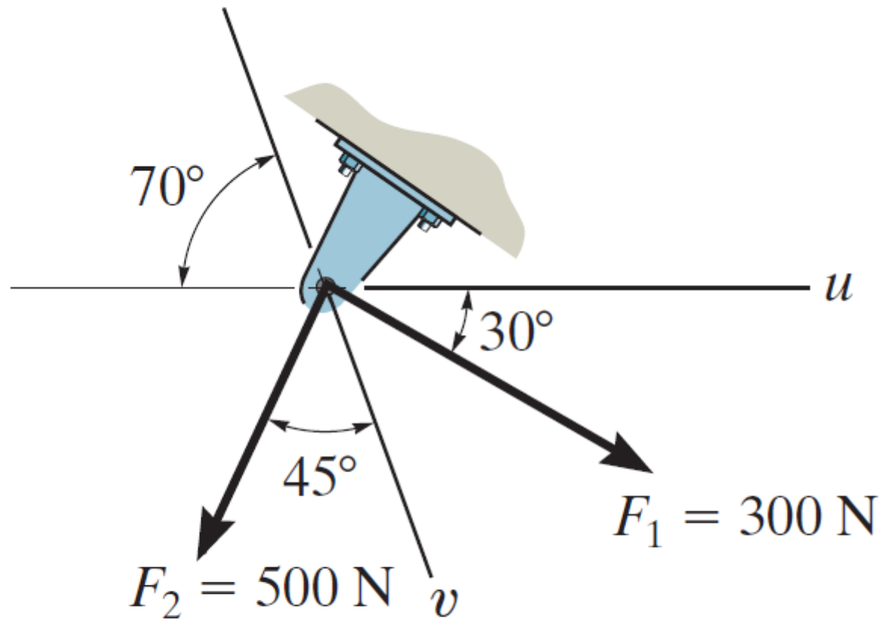
PROBLEM # 4

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



Problem

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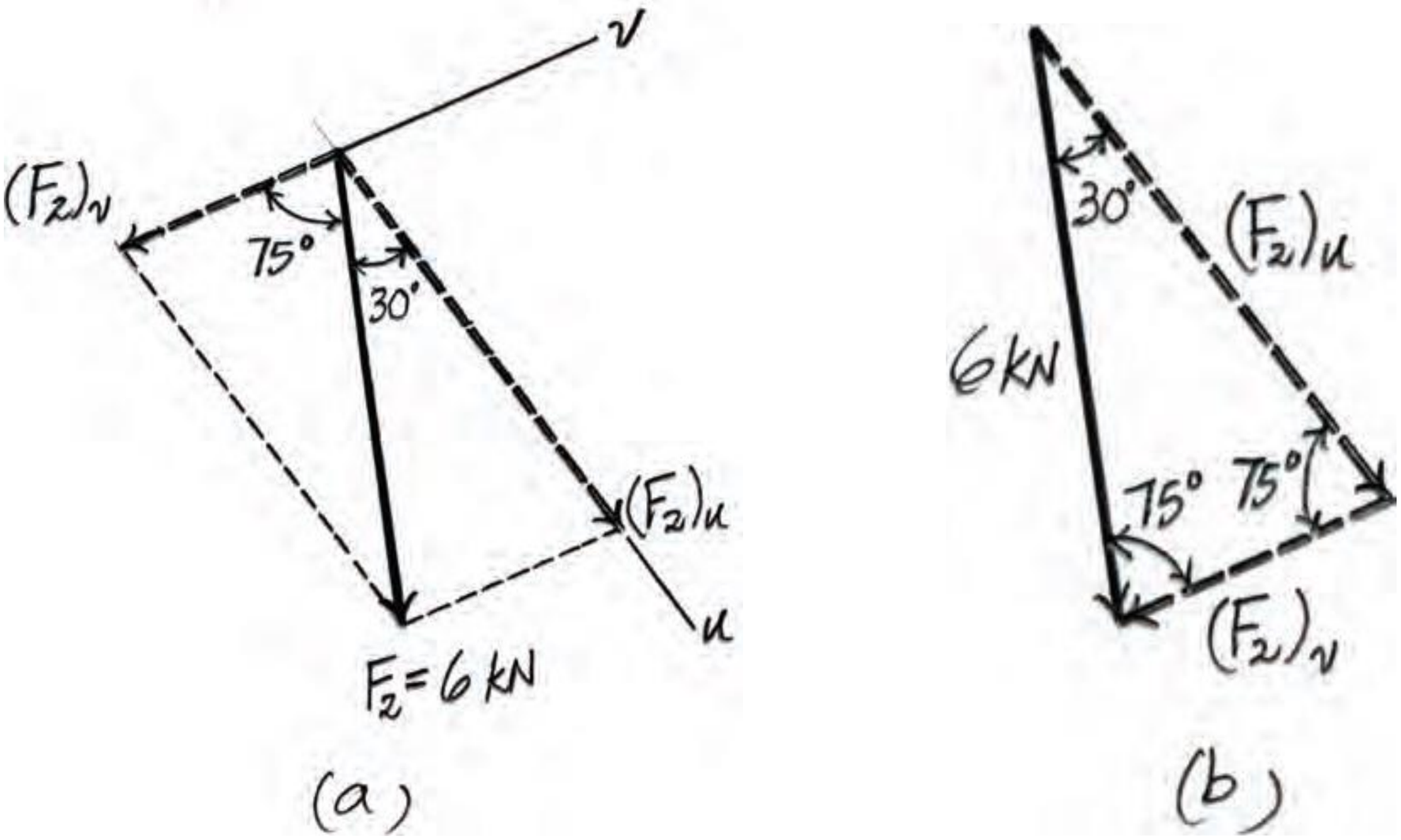
SOLUTION



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SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,

Trigonometry. Applying the sines law of referring to Fig. *b*,

$$\frac{(F_2)_u}{\sin 75^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_u = 6.00\ \text{kN}$$

$$\frac{(F_2)_v}{\sin 30^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_v = 3.106\ \text{kN} = 3.11\ \text{kN}$$

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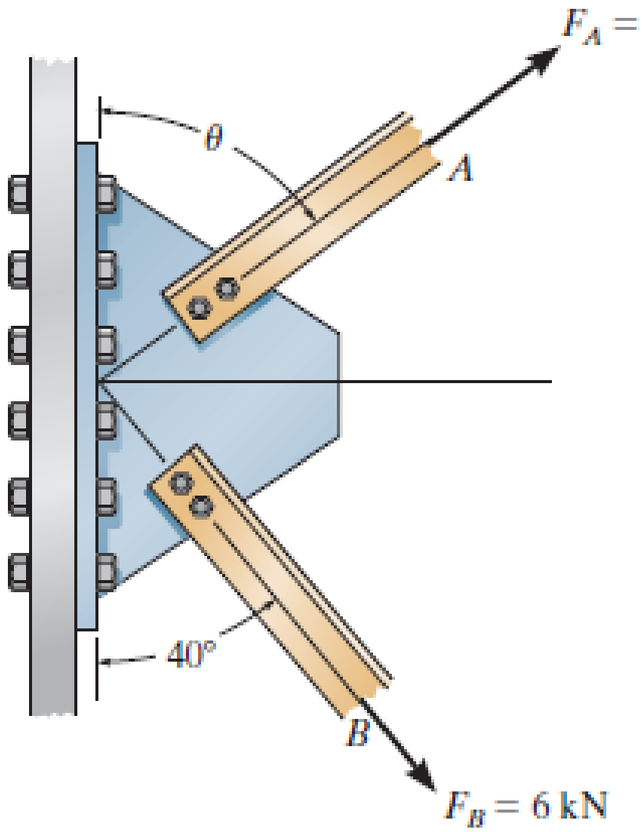
PROBLEM # 5

The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



Problem

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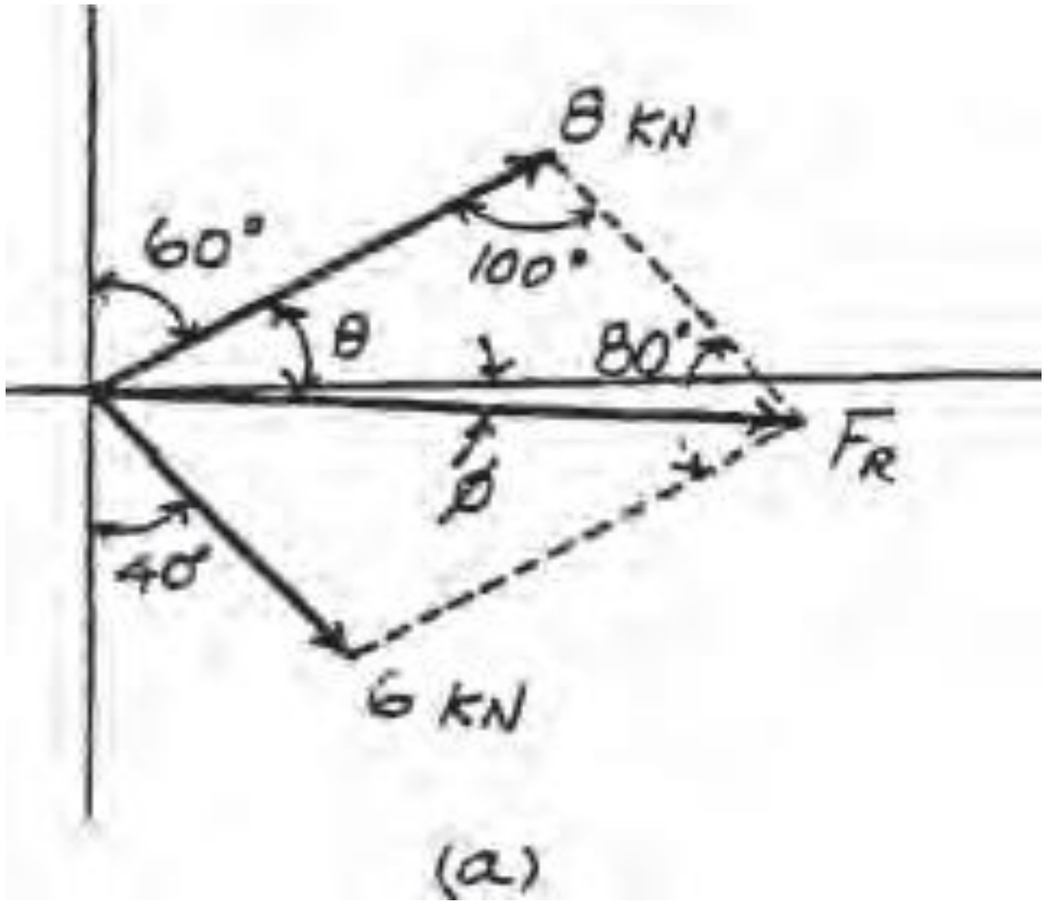
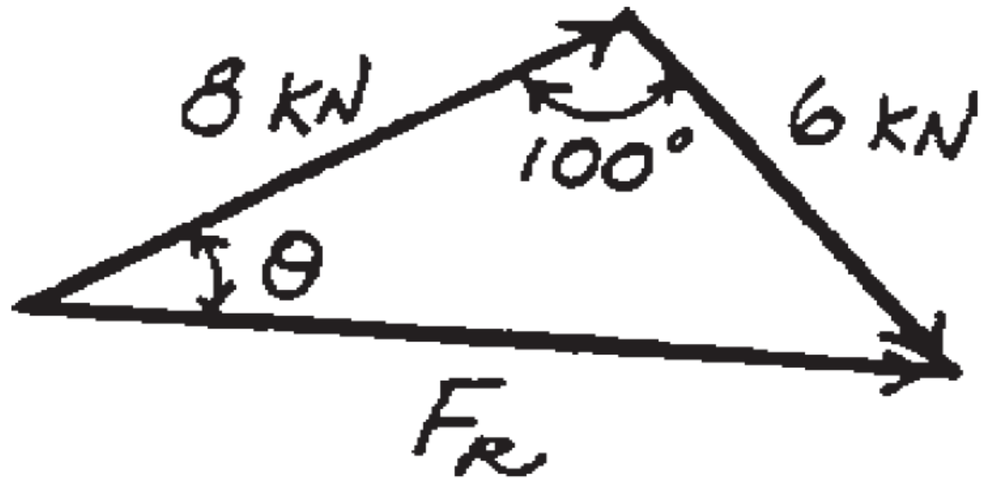
SOLUTION



(a)



(b)

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SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle θ can be determined using law of sines (Fig. *b*).

$$\frac{\sin\theta}{6} = \frac{\sin 100^\circ}{10.80}$$

$$\sin\theta = 0.5470$$

$$\theta = 33.16^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ$$

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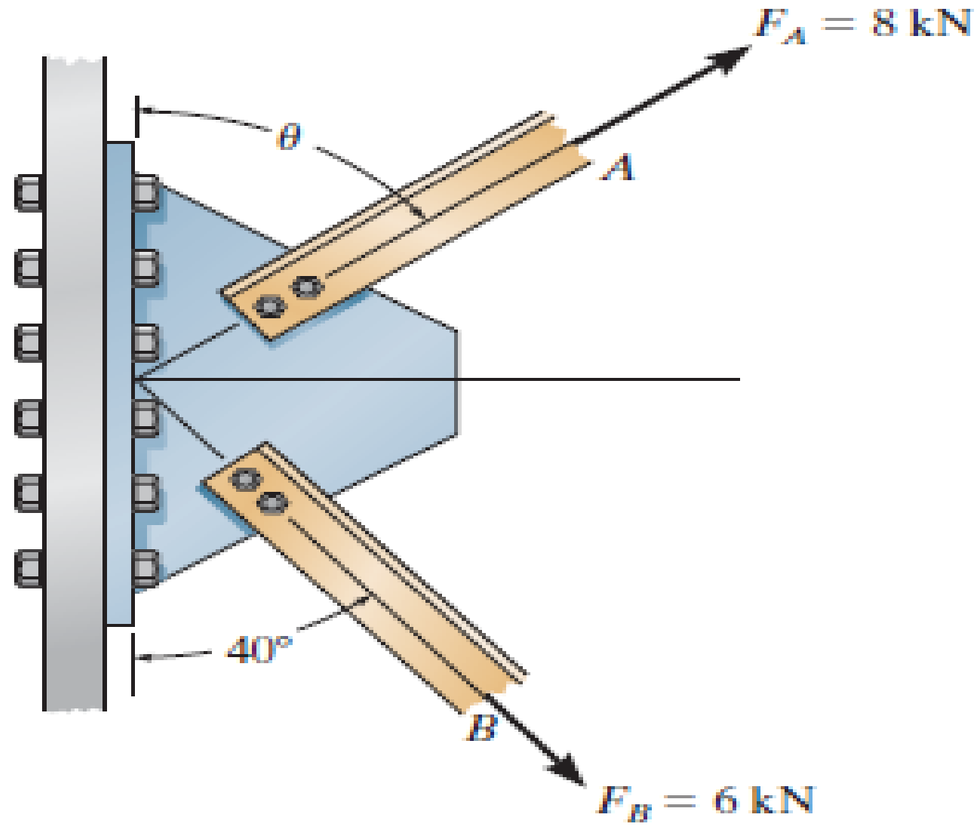
PROBLEM # 6

Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



Problem

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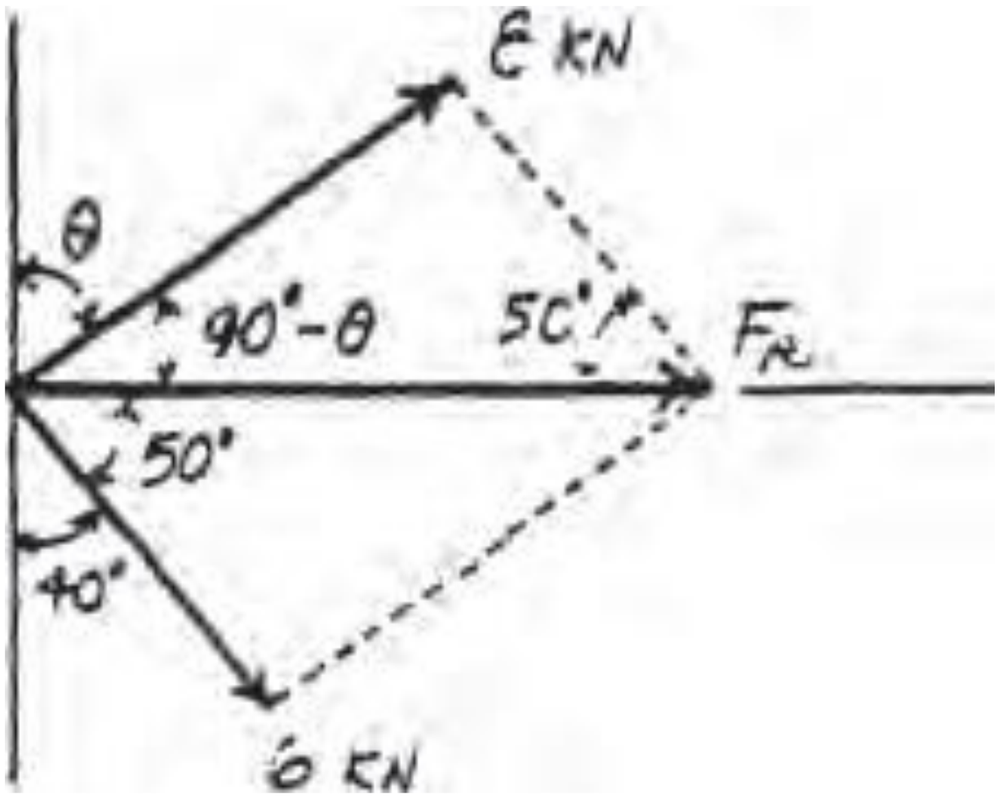
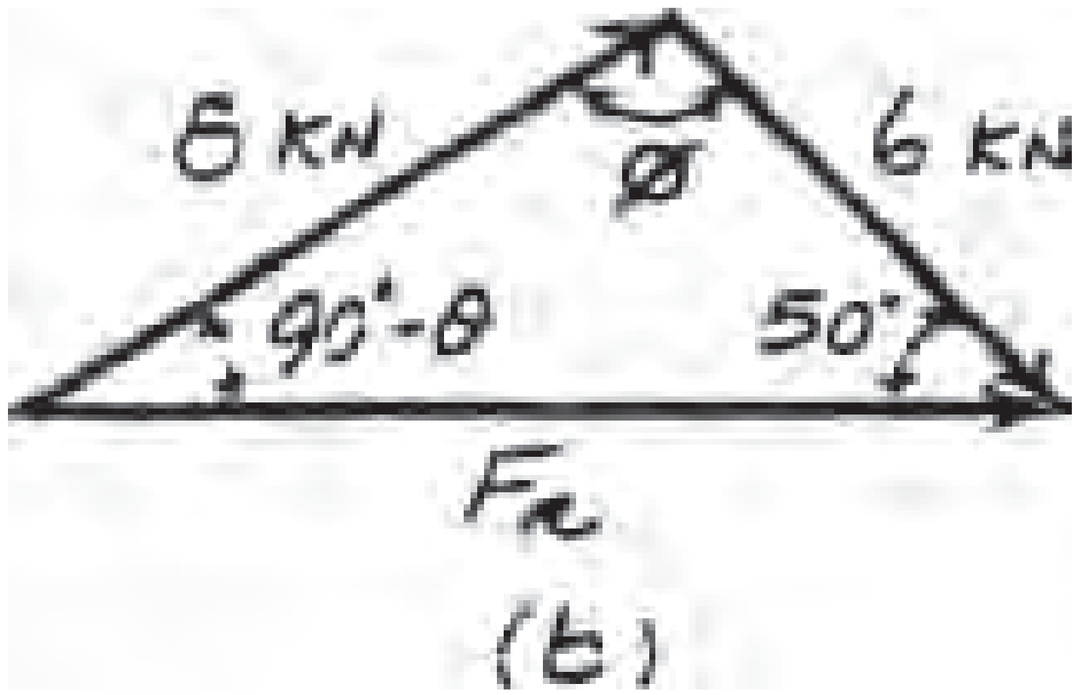
SOLUTION





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SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

$$\sin(90^\circ - \theta) = 0.5745$$

$$\theta = 54.93^\circ = 54.9^\circ \qquad \textbf{Ans.}$$

From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93^\circ}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$

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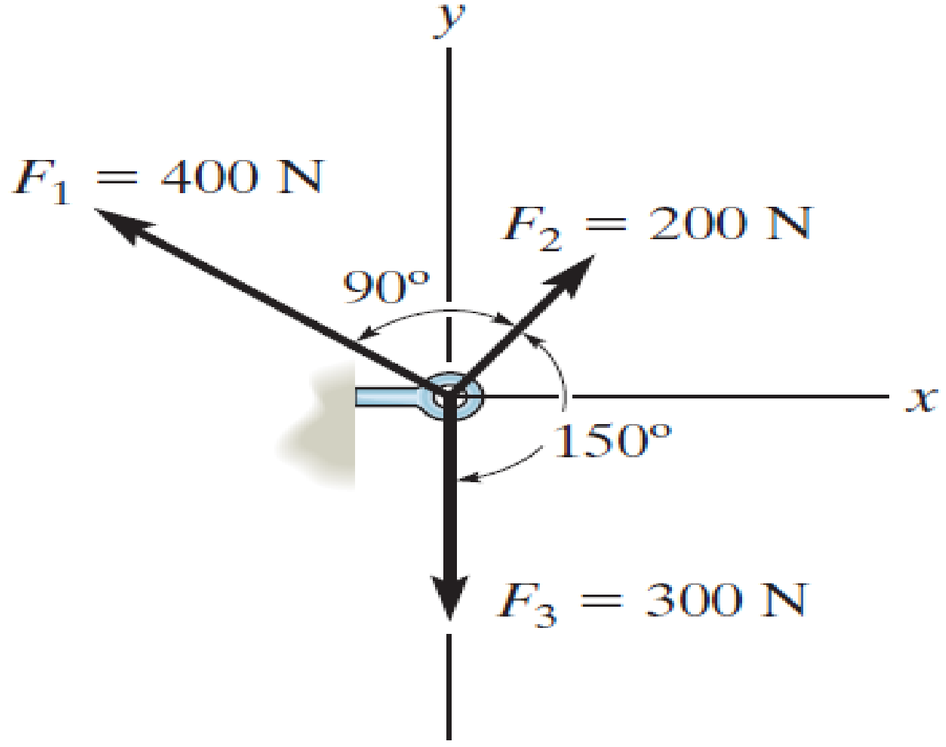
PROBLEM # 7

Determine the magnitude and direction of the resultant force, $\mathbf{F}_R$ measured counterclockwise from the positive x axis. Solve the problem by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



Problem

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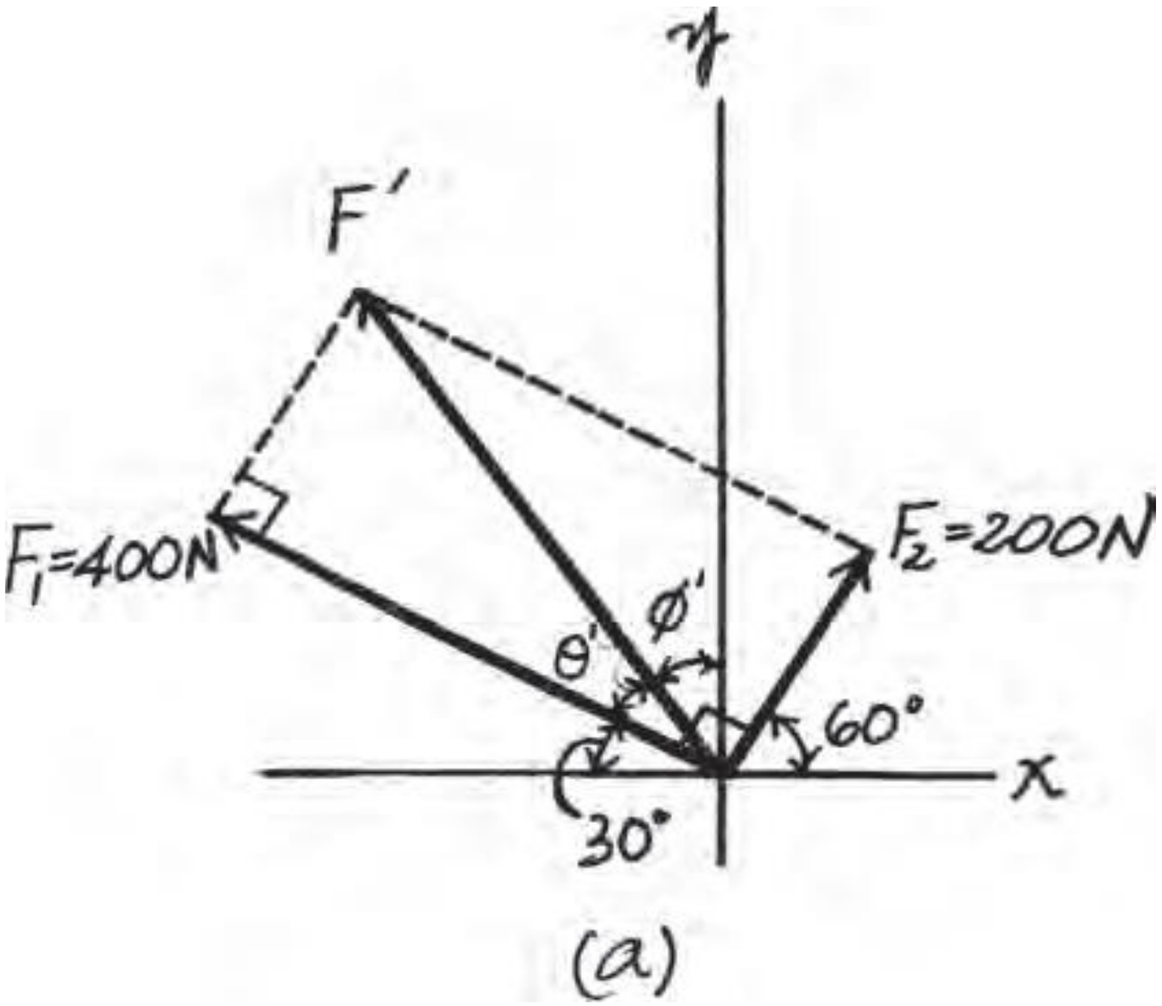
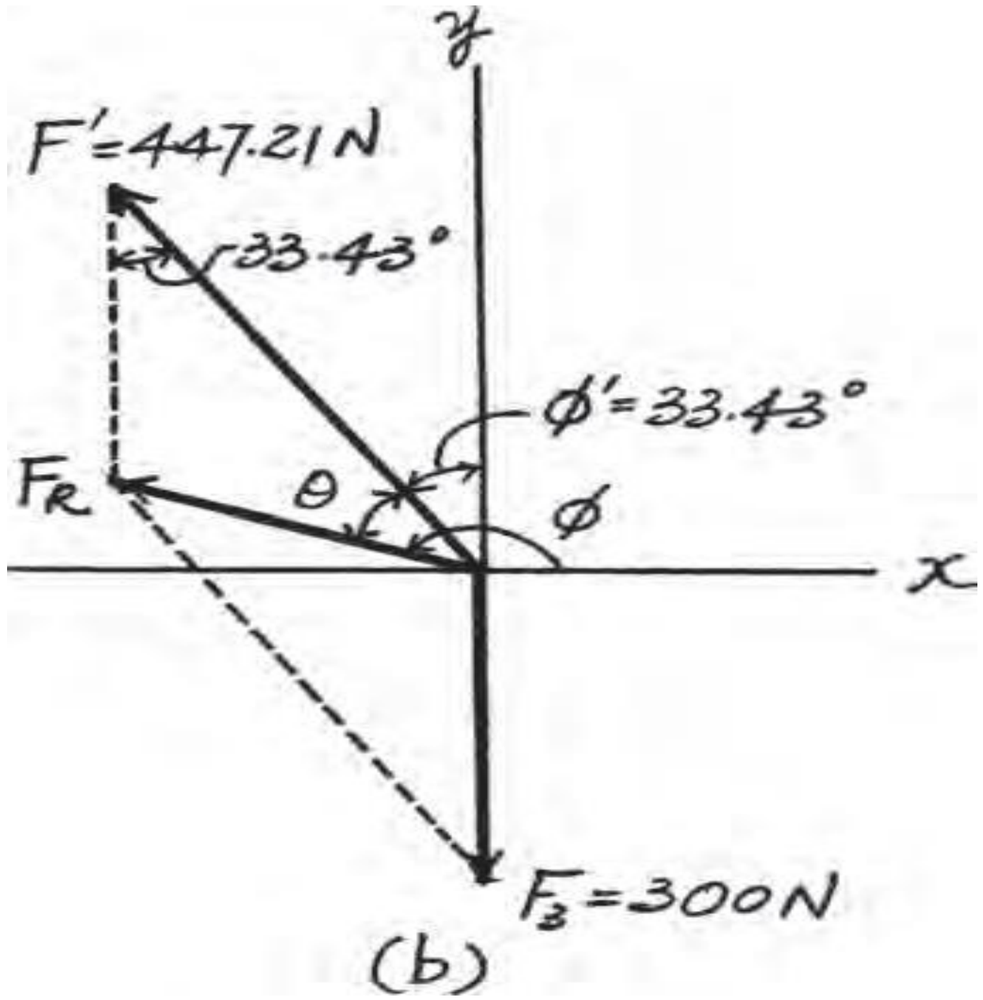
SOLUTION





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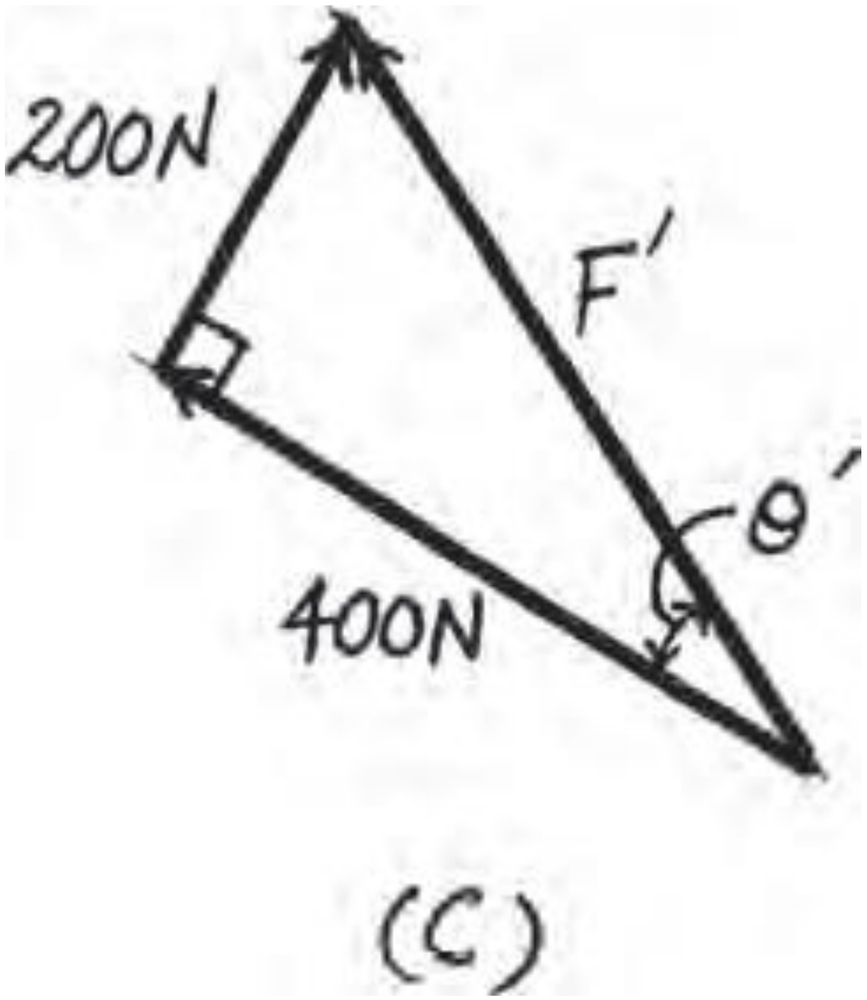
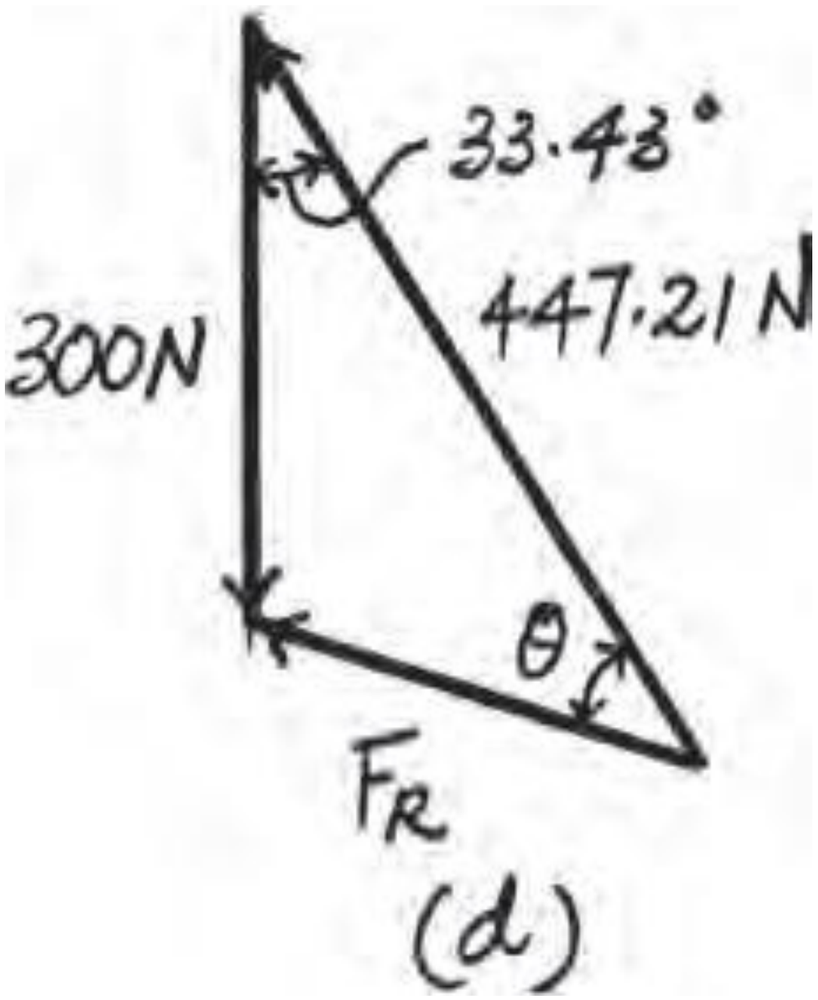
SOLUTION





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SOLUTION

Parallelogram Law. The parallelogram law of addition for $\mathbf{F}_1$ and $\mathbf{F}_2$ and then their resultant $\mathbf{F}'$ and $\mathbf{F}_3$ are shown in Figs. *a* and *b*, respectively.
Trigonometry. Referring to Fig. *c*,

$$F' = \sqrt{200^2 + 400^2} = 447.21 \text{ N} \qquad \theta' = \tan^{-1}\left(\frac{200}{400}\right) = 26.57°$$

Thus $\phi' = 90° - 30° - 26.57° = 33.43°$

Using these results to apply the law of cosines by referring to Fig. *d*,

$$F_R = \sqrt{300^2 + 447.21^2 - 2(300)(447.21)\cos 33.43°} = 257.05 \text{ N} = 257 \text{ kN} \quad \textbf{Ans.}$$

Then, apply the law of sines,

$$\frac{\sin\theta}{300} = \frac{\sin 33.43°}{257.05}; \qquad \theta = 40.02°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\phi = 90° + 33.43° + 40.02° = 163.45° = 163° \quad \textbf{Ans.}$$

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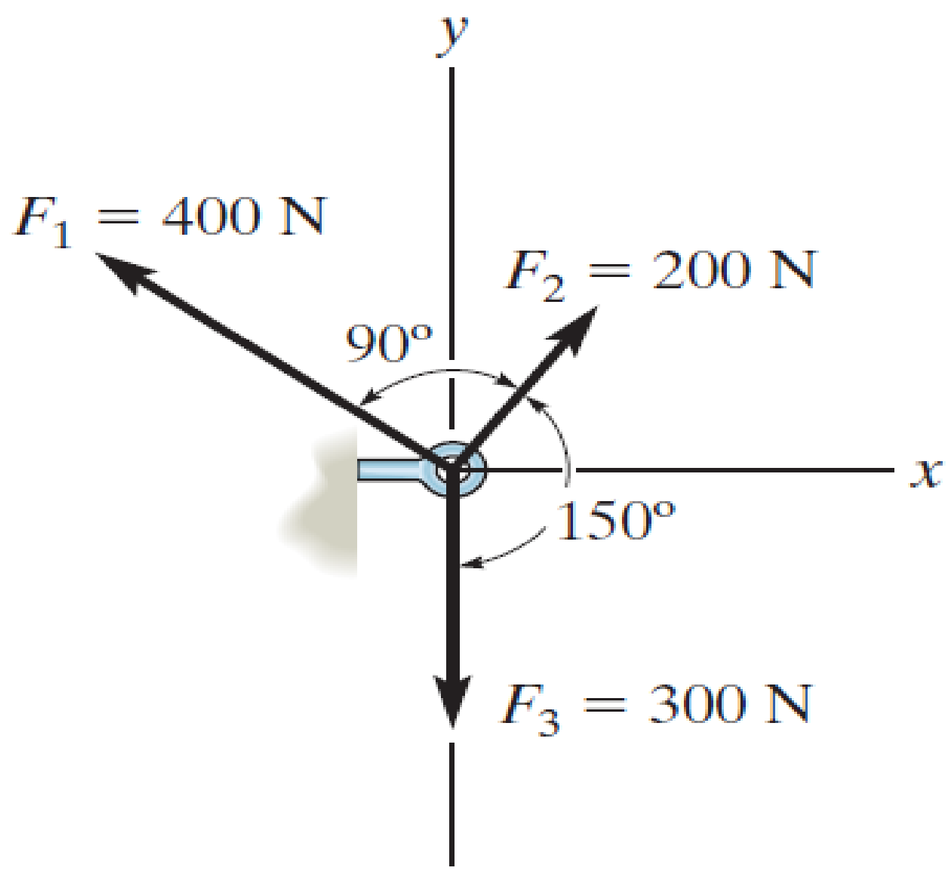
PROBLEM # 8

Determine the magnitude and direction of the resultant force, measured counterclockwise from the positive x axis. Solve l by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



Problem

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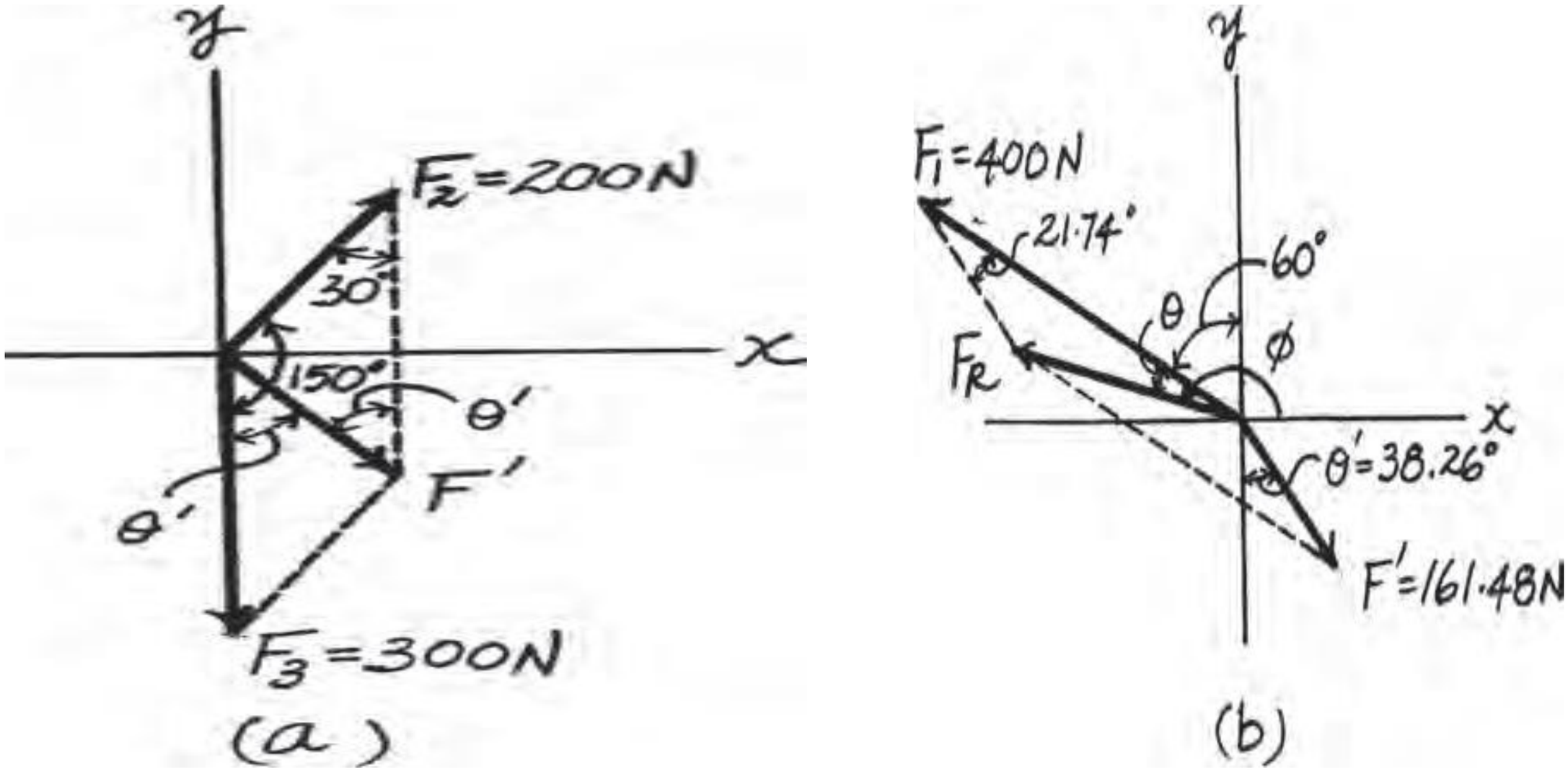
SOLUTION



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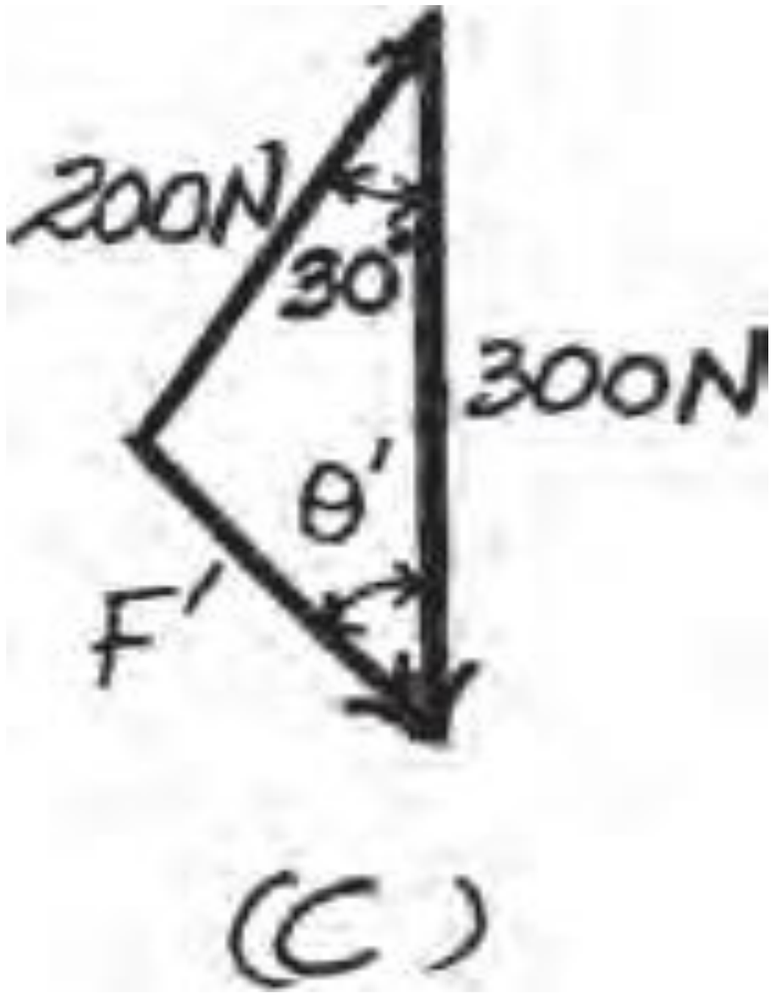
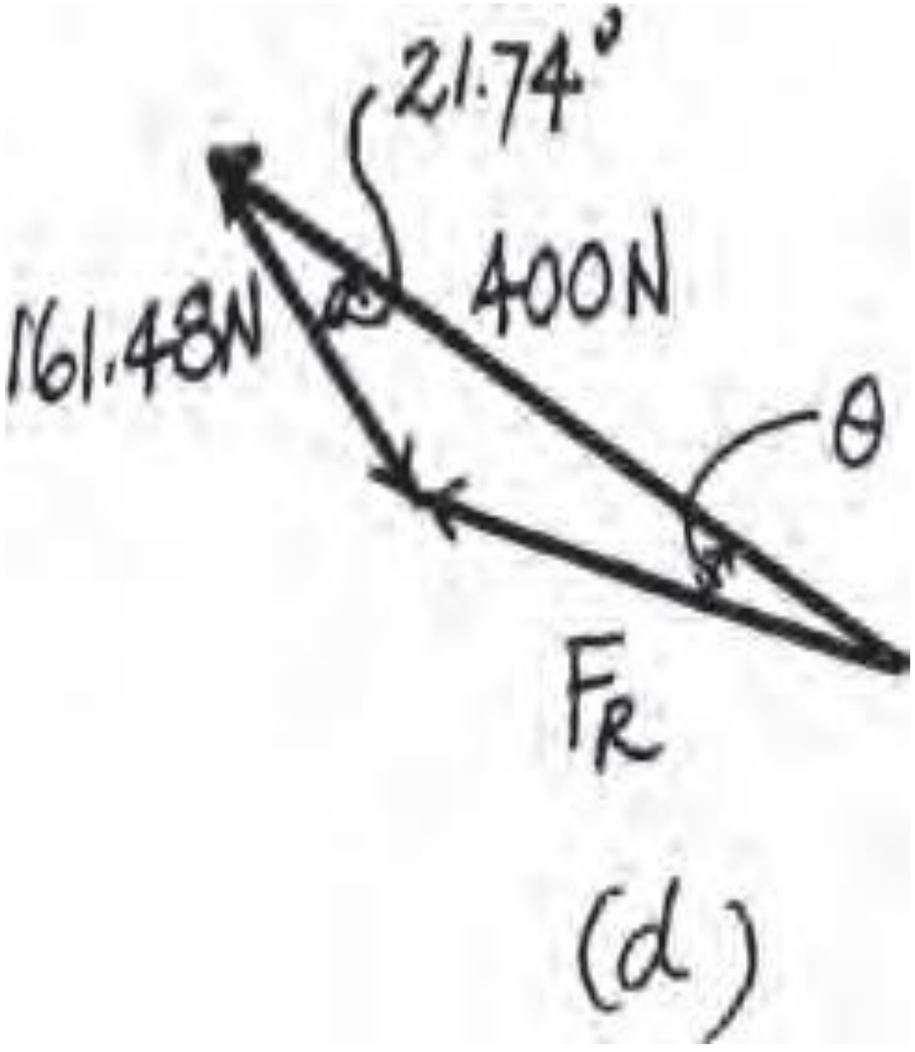
SOLUTION





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SOLUTION

Parallelogram Law. The parallelogram law of addition for $\mathbf{F}_2$ and $\mathbf{F}_3$ and then their resultant $\mathbf{F}'$ and $\mathbf{F}_1$ are shown in Figs. *a* and *b*, respectively.

Trigonometry. Applying the law of cosines by referring to Fig. *c*,

$$F' = \sqrt{200^2 + 300^2 - 2(200)(300)\cos 30^\circ} = 161.48\ \text{N} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *c*,

$$\frac{\sin\theta'}{200} = \frac{\sin 30^\circ}{161.48}; \qquad \theta' = 38.26^\circ$$

Using the results of $\mathbf{F}'$ and θ' to apply the law of cosines by referring to Fig. *d*,

$$F_R = \sqrt{161.48^2 + 400^2 - 2(161.48)(400)\cos 21.74^\circ} = 257.05\ \text{N} = 257\ \text{N} \quad \textbf{Ans.}$$

Then, apply the sines law,

$$\frac{\sin\theta}{161.48} = \frac{\sin 21.74^\circ}{257.05}; \qquad \theta = 13.45^\circ$$

Thank You