LECTURE # 3

In this lecture you will learn about:

- Fundamental Laws of Mechanics
- Moment
- Couple

Course Name:

“Applied Mechanics”

Course Code: CT-144

Credit Hours: 3

Semester: Summer 2020

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FUNDAMENTAL LAWS OF MECHANICS

- Newton’s First Law.
- Newton’s Second Law.
- Newton’s Third Law.
- Newton’s Law of gravitation.
- Law of transmissibility of Force.
- Parallelogram law of Forces.

NEWTON’S FIRST LAW

- It states that every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by external agency acting on it.

- Newton’s laws of motion of rotation which state that, “Every body continues in its state of rest or of uniform motion of rotation about an axis unless it is acted upon by some external torque”

NEWTON'S SECOND LAW

It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it.

Force α rate of change of momentum

Momentum = Mass x velocity

As mass do not change,

Force α Mass x rate of change velocity

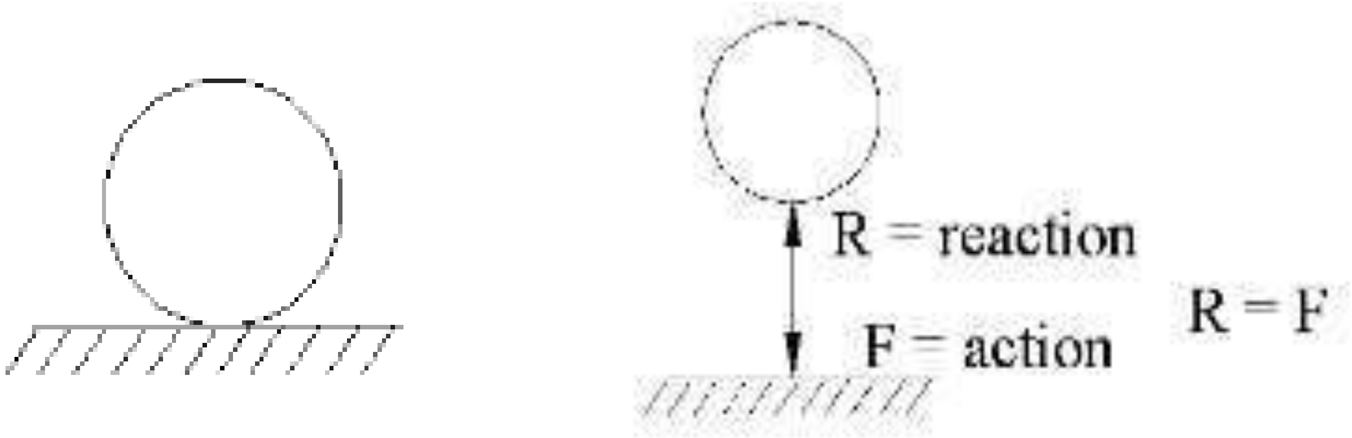
Force α Mass x acceleration

F α ma

F = ma

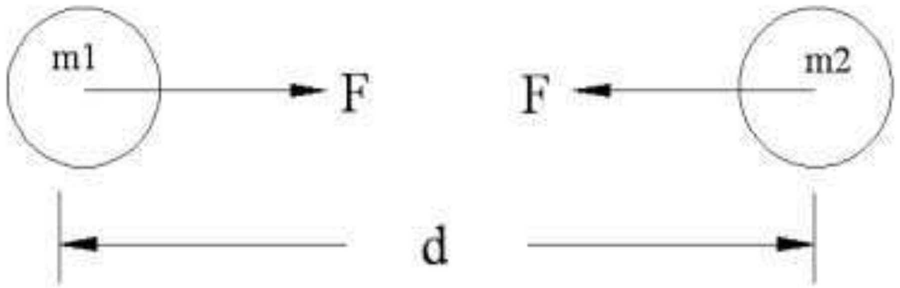
NEWTON’S THIRD LAW

It states that for every action there is an equal and opposite reaction.



NEWTON’S LAW OF GRAVITATION

Everybody attracts the other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them.



Where, G is the constant of proportionality, it is known as constant of gravitation. Experimentally, it is proved that the value of

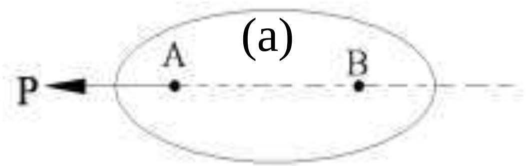
G = 6.673 x 10-11 Nm^2/Kg^2 $F = G\,\frac{m_1\,m_2}{d^2}$

LAW OF TRANSMISSIBILITY OF FORCE

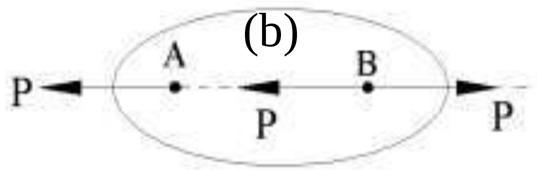
STATEMENT

“The point of application of force may be transmitted along its line of action without changing its effect on the rigid body to which the force is applied”.

(a)



(b)



EXPLANATION

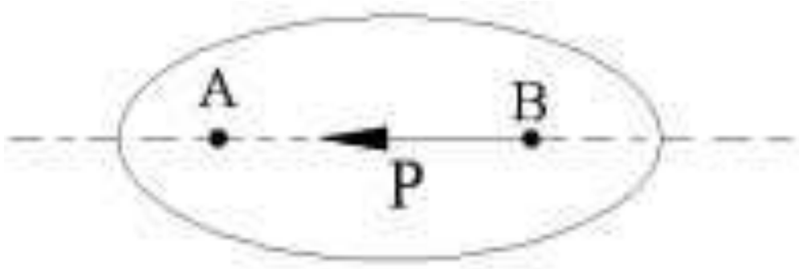
A force is acting at point A along line of action AB on rigid body as shown in Fig. (a).

Two equal and opposite forces of magnitude ‘P’ are added at point ‘B’ along line of action

AB according to the law of superposition as shown in Fig (b).

LAW OF TRANSMISSIBILITY OF FORCE

Two equal and opposite forces of the magnitude ‘P’ at point A and B can be subtracted without changing action of original force P according to the law of superposition as shown in Fig (c).



(c)

Thus the point of application of force P is transmitted along its line of action from A to B.

VARIGNON'S THEOREM OF MOMENTS/ PRINCIPLE OF MOMENTS

STATEMENT

"The algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant about the same point".

$$\text{i.e. } \Sigma M = \Sigma \text{ (Moments of forces)} = \text{Moment of } R$$

PROOF

In above Figure AB and AC represents forces P and Q resp. and ‘O’ is the point about which moment is taken. ABCD represents a parallelogram.

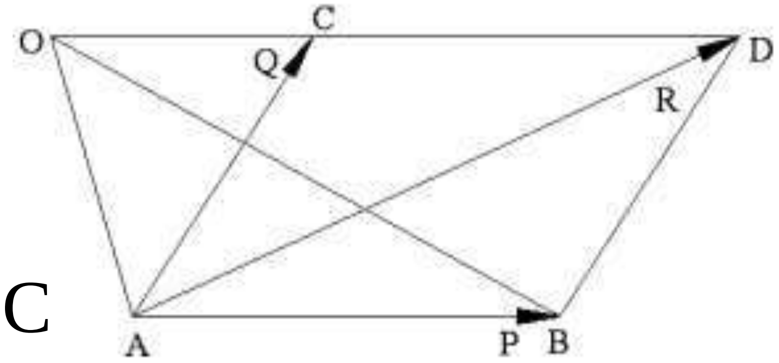
A diagonal AD represents resultant of forces P and Q. Now extend CD up to the point ‘O’ which is the line of CD. Join OA and OB.

Now, we know that,

Moment of force = 2(Area of triangle)

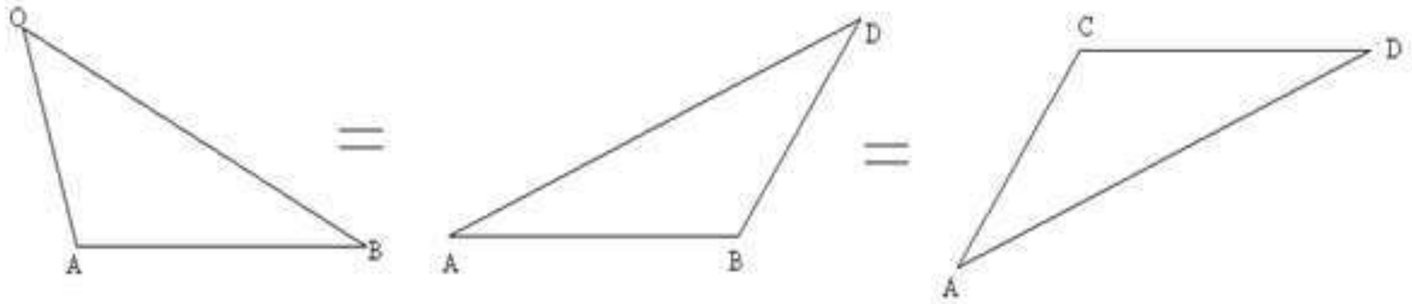
Moment of force P = 2 x Area of Triangle AOB

And Moment of force Q = 2 x Area of Triangle AOC



PROOF

Algebraic sum of Moments of forces P and Q = Σ M = 2 x (Area of ΔADB + Area of ΔAOC)



Now,

Area of Δ AOB = Area of ΔADB = Area of ΔACD

Since, AB = CD (base is same) and height is same

Σ M = 2 x (Area of Δ ACD + Area of Δ AOD) = 2 x (Area of Δ AOD)

Σ M = Moment of Resultant Force 'R'

APPLICATION

1) It is generally used to locate the **point of application** of resultant.

2) In case of coplanar non-concurrent system of forces this concept is used to locate the **line of action** of the resultant.

PARALLELOGRAM LAW OF FORCES

STATEMENT

"If two forces acting simultaneously on a body at a point are presented in magnitude and direction by the two adjacent sides of parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces".

PARALLELOGRAM LAW OF FORCES

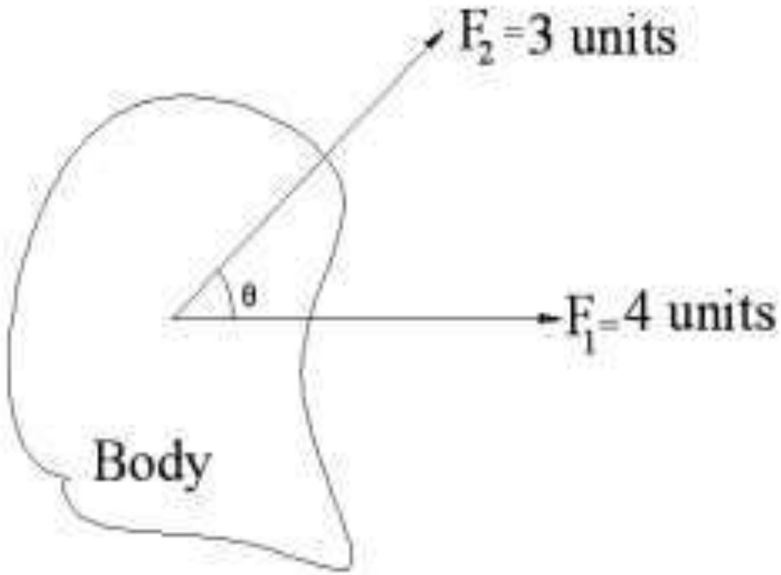


Fig. (a)

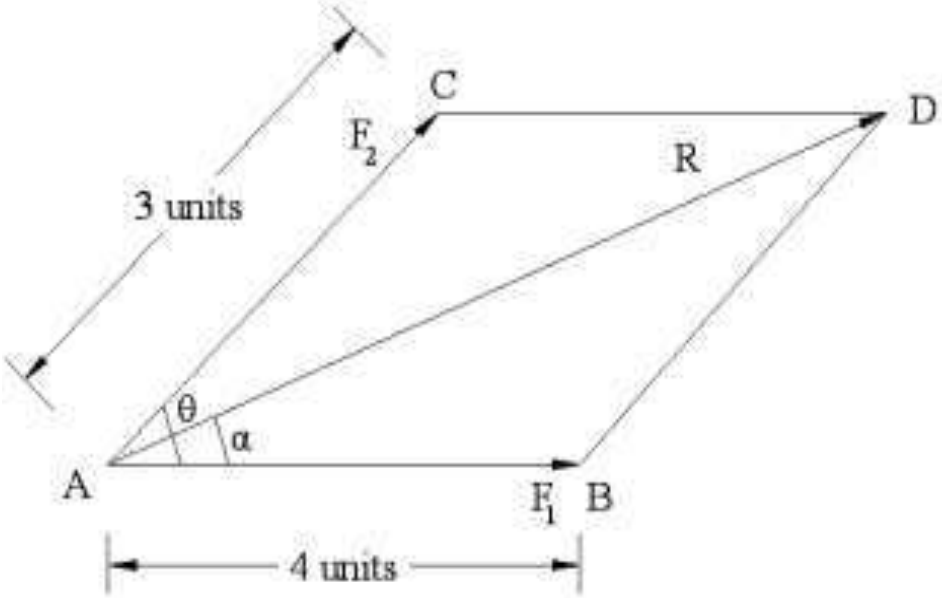


Fig. (b)

The length of diagonal in Fig. (b) will indicate the magnitude of resultant of 'R'.

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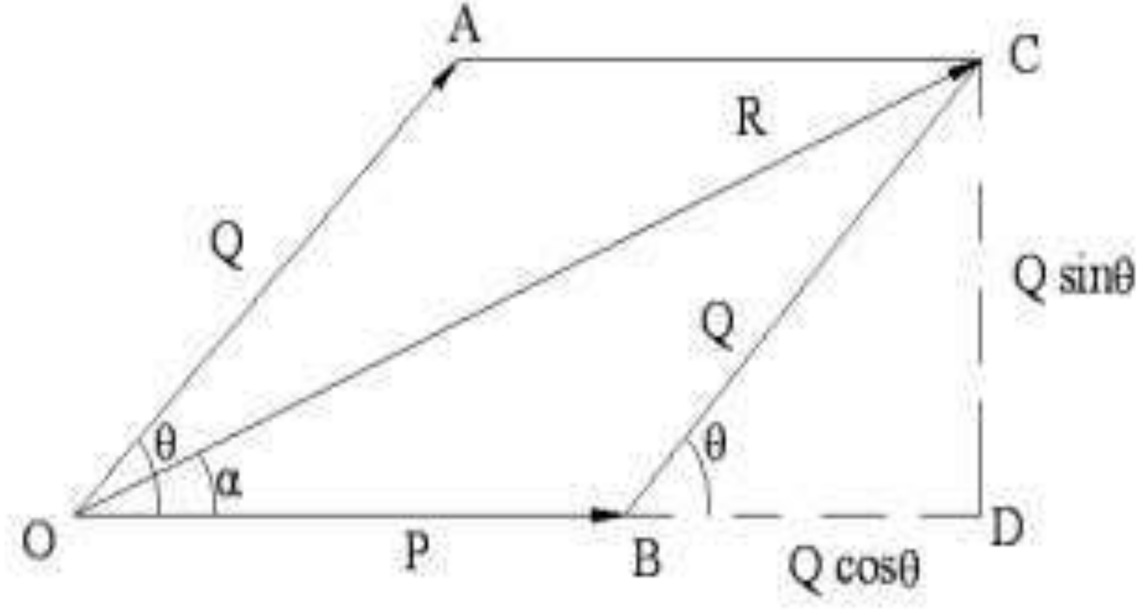
DERIVATION

From right angle triangle BCD
BD = Q sinθ
CD = Q cosθ
Using Pythagoras theorem to the ΔOCD
OC2 = CD2 + OD2



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DERIVATION

$OC^2 = CD^2 + (OB + BD)^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$R = \sqrt{P2 + Q2 + 2PQ\cos\theta}$ ---------------------------------- (1)

Angle α of resultant R with force P is given by,

$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P+Q\cos\theta}\right]$ -- (2)

PARTICULAR CASES

- When $\theta = 90^0$ R = $\sqrt{P^2 + Q^2}$
- When $\theta = 0^0$ R = P + Q (acting along Same Direction)
- When $\theta = 180^0$ R = P – Q (acting in Opposite Direction)

MOMENT

The **turning effect** caused by a force on the body is called as a moment of force.

DEFINITION

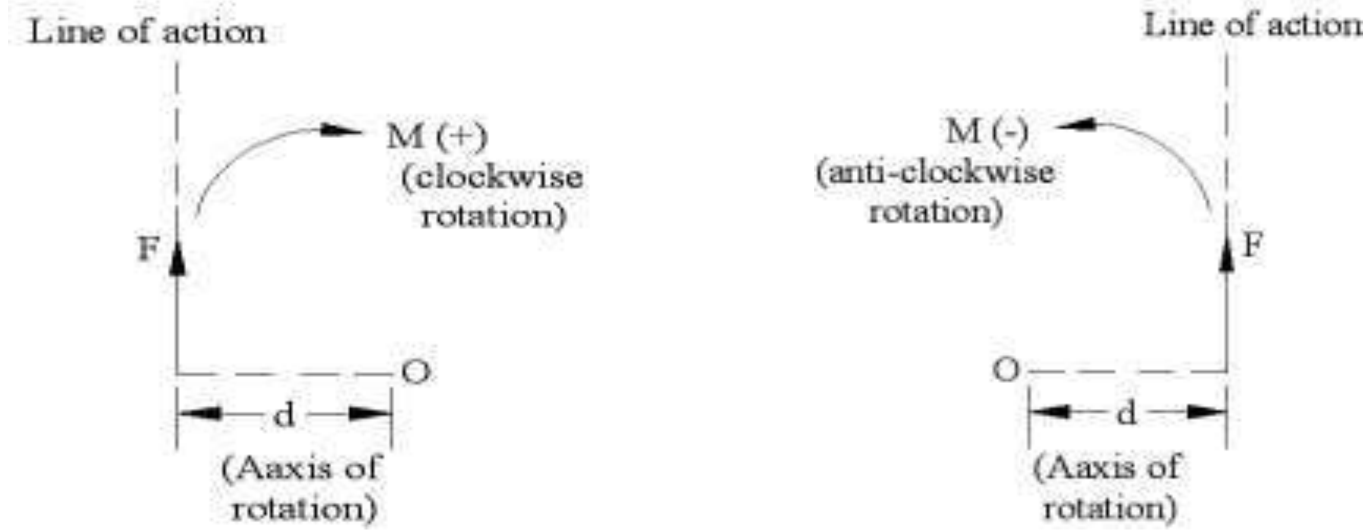
The moment of a force (M) is equal to the magnitude of the force (F) multiplied by the perpendicular distance (d) between the line of action of the force and the axis of rotation.

Moment = Force x Perpendicular Distance

$$M = F \times d$$

SIGN CONVENTION

- If the moment of the force producing clockwise rotation is the clockwise moment and it is taken as positive as shown in Fig. (a).
- If the moment of the force producing anticlockwise rotation is the anticlockwise moment and it is taken as negative as shown in Fig. (b).



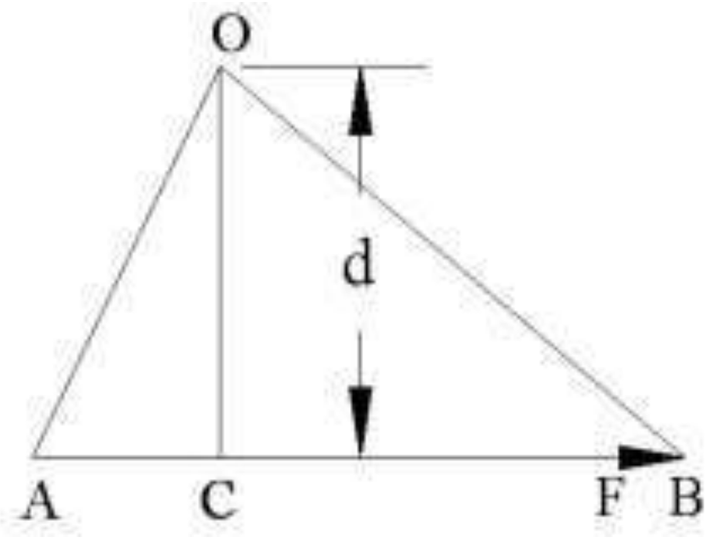
UNIT

If the force is measured in Newton and the distance in meter, the SI unit of the moment is Newton meter (Nm).

GEOMETRICAL REPRESENTATION OF MOMENT

- As shown in Fig. below, AB represents force F and O is the point about which the moment of
- force M is taken. Let OC be the perpendicular distance 'd'. Moment of Force F is given by,

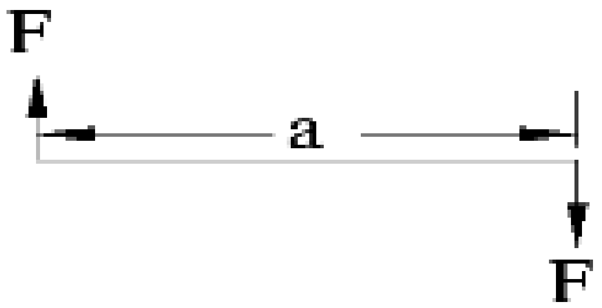
$$M = F \times d$$
$$M = AB \times OC$$
$$M = 2 \times (½\ AB \times OC)$$
$$M = 2\ (\text{Area of triangle OAB})$$



Thus Moment of Force about any point is geometrically equal to twice the area of the triangle having base representing a point about which moment is taken.

COUPLE

Two equal, opposite and parallel (non-collinear) forces are said to form a couple as shown in Fig. below.



Arm of couple:

The distance ‘a’ between the lines of action of the two forces of a couple is known as ‘arm of couple’.

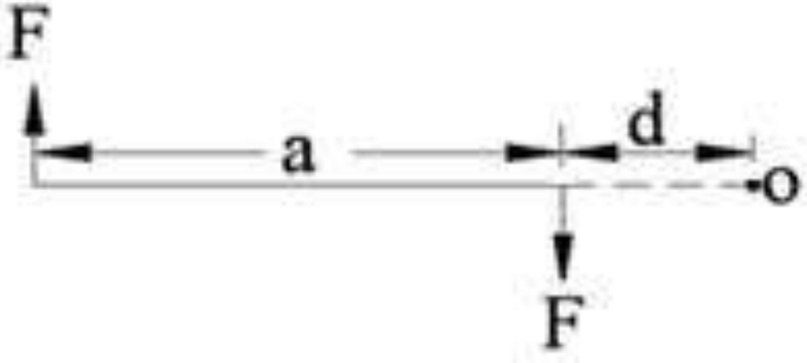
PROPERTIES

a) Couple cannot be replaced by a **single resultant force.**

b) Couple cannot produce **rotation or moment** but it cannot produce straight line motion.

MOMENT OF COUPLE

From above Fig. moment of couple about any point 'O' (moment of centre) is given by



F (a +d) – (Fd) = Fa + Fd – Fd = Fa **Nm**

Moment of Couple = Force x Arm

Thus the moment of the couple has a constant value irrespective of the point about which moment is taken.

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Thank You