Basic Properties Of Systems:-

There are a number of properties of continuous-time and discrete-time systems.

1) Systems With & Without Memory:-

Memoryless System:-

Def:-

"A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent only on the input at that same time."

Examples Of Memoryless Systems:-

1) $y[n] = (2x[n] - x^2[n])^2$

The above system is a memoryless system as the value of $y[n]$ at any particular time n_0 depends only on the value of $x[n]$ at that time.

2) A Resistor Is A Memoryless System:-

Let,

$x(t)$ = Input taken as the current.

$y(t)$ = Voltage taken as output.

Then input-output relationship of a resistor is:

$$y(t) = R\,x(t).$$

Where, R is the resistance.

3) Identity System:

An identity system is a simple memoryless system. Its output is identical to its input. The input-output relationship for an identity system is given by:

For Continuous-time Identity System:

$$y(t) = x(t)$$

For Discrete-time Identity System:

$$y[n] = x[n].$$

System With Memory:

Def:

"Memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time."

Examples Of Systems With Memory:

1) Accumulator or Summer:

Accumulator or summer is a discrete time system with memory.

$$y[n] = \sum_{K=-\infty}^{n} x[K]$$

An accumulator must "remember" or store information about past inputs. The accumulator computes the running sum of all inputs upto the current time, and thus, at each instant of time, the accumulator must add the current input value

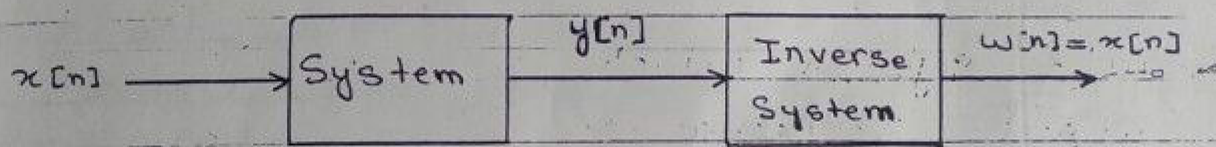
2) Invertibility & Inverse Systems:

Def:-

"A system is said to be invertible if distinct inputs lead to distinct outputs."

Explanation:-

For a discrete-time case, if a system is invertible, then inverse system exists that, when cascaded with the original system, yeilds an output $w[n]$ equal to the input $x[n]$ to the first system.



The above series interconnection has an overall input-output relationship which is the same as that for the identity system.

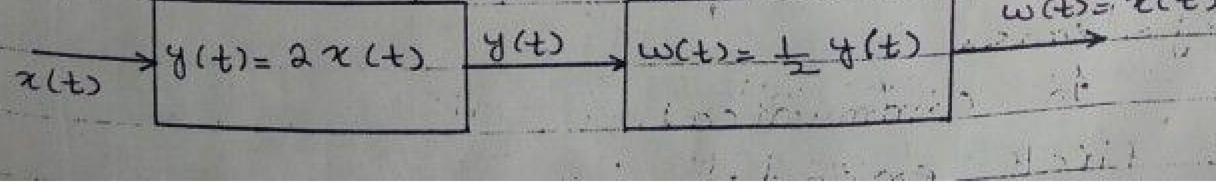
Examples Of Invertible Systems:-

1) An example of an invertible continuous-time system is:

$$y(t) = 2x(t).$$

Because the inverse system also exists: i.e

$$w(t) = \frac{1}{2} y(t).$$



2) Accumulator or Summer:

Accumulator is another example of invertible systems. For this system the difference between two successive values of the output is precisely the last input value. Therefore, in this case, the inverse system is:

$$x[n] \longrightarrow \boxed{y[n] = \sum_{k=-\infty}^{n} x[k]} \xrightarrow{y[n]} \boxed{w[n] = y[n] - y[n-1]} \longrightarrow w[n] = x[n]$$

Examples Of Non-Invertible Systems:

1) The system which produces the zero output sequence for any input sequence is a non invertible system i.e

$$y[n] = 0$$

2) The system in which, we cannot determine the sign of the input from the knowlege of the output is a non-invertible system. i.e.

$$y(t) = x^2(t)$$

The inverse of this system can't be ~~[illegible]~~ judged because we are not sure whether $\sqrt{x^2(t)} = +x(t)$ or $-x(t)$

3) Causality:

Def:

"A system is said to be causal (if the output at any time depends only on values of the input) at the present time and in the past."

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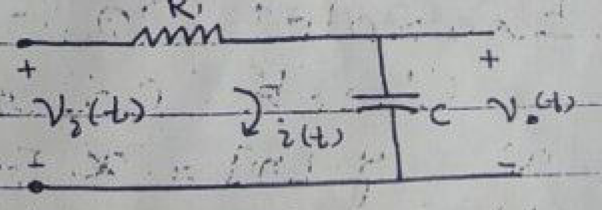
Explanation:

The causal systems are also refered to as non-anticipative systems because the system output does not anticipate future values of the input. Consequently, if two inputs to a causal system are identical upto some point in time t_0 or n_0, the corresponding outputs must also be equal up to this same time.

Examples Of Causal Systems:-



1) The RC circuit shown is a causal system, since the capacitor voltage responds only to the present and past values of the source voltage.

3) A summer or an accumulator is a causal system and is given by

$$y[n] = \sum_{K=-\infty}^{n} x[K]$$

4) A delay system is also causal & given by:

$$y[n] = x[n-1].$$

Note:-

All Memoryless systems are caus since the output responds only the current value of input.

Non Causal Systems:- (Examples)

The following two systems a not causal because they also an ipate the future values of the in

) $y[n] = x[n] - x[n+1] \Rightarrow$ Not Caus

) $y(t) = x(t+1) \Rightarrow$ Not Causal.

5) Time Invariance:

Def:

"A system is said to be time invariant if a time shift in the input signal results in an identical time shift in the output signal."

Explanation:

Let

$y[n]$ = Output of the discret time invariant system.

Check time invar...

$$y[n] = n\,x[n]$$

Solution:

Let $x_1[n]$ be the arbitrary input to the system.

Therefore,

$$y_1[n] = n\,x_1[n] \longrightarrow (1)$$

Now, consider another input $x_2[n]$ to the system such that $x_2[n]$ is the delayed version of $x_1[n]$, i.e.

$$x_2[n] = x_1[n-n_0].$$

$\therefore$ For this input, the output of the system is given by:

$$y_2[n] = n\,x_2[n]$$

$$y_2[n] = n\,x_2[n-n_0] \longrightarrow (2)$$

Now, giving a time shift of "n_0" to eq$^{(1)}$.

$$y_1[n-n_0] = (n-n_0)\,x_1[n-n_0] \longrightarrow (3)$$

Comparing eq (2) & (3)

$$\boxed{y_2[n] \neq y_1[n-n_0]}$$

6) Linearity:-

Def:- "A linear system, in continuo[us] or discrete-time, is a system that [has] the important property of superpositio[n]

"OR"

Def # 2:-

"A system is said to be lin[ear] if the following two properties [are] satisfied i.e

1) Additive Property.
2) Scaling or homogeneity property"

Explanation:-

Let $y_1(t)$ be the response of [a] continuous time system to an [input] $x_1(t)$, and let $y_2(t)$ be the outpu[t] corresponding to the input $x_2(t)$, th[e] system is linear if additive property is satisfied. i.e

$x_1(t) + x_2(t)$ gives response $y_1(t) + y_2(t)$

2) Homogeneity or Scaling Property: is satisfied i.e

The response to $ax_1(t)$ is $ay_1(t)$ where "a" is any complex constant.

The two properties defining a linear system can be combined into a single statement.

Case of Continuous-time Case:

$$\boxed{ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)}$$

Case Of Discrete-time Case:

$$\boxed{ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]}$$

Here, "a" & "b" are complex constants

Superposition Property:

The superposition property hol if the linear combination of inputs

$$x[n] = \sum_k a_k x_k[n] = a_1 x_1[n] + a_2 x_2[n] + \text{------}$$

give the response

$$y[n] = \sum_k a_k y_k[n] = a_1 y_1[n] + a_2 y_2[n] + \text{----}$$

Note:

It should be noted that a system can be linear without being time invariant and it can be time invariant without being

Example 1.17)

Check linearity of the system described by the equation:

$$y(t) = t\,x(t)$$

Solution:

Let an input $x_1(t)$ is given to the system. i.e.

$$y_1(t) = t\,x_1(t) \longrightarrow (1)$$

Again an input $x_2(t)$ is given to the system such that:

$$y_2(t) = t\,x_2(t) \longrightarrow (2)$$

Now, we apply input $x_3(t)$ to the system to get response $y_3(t)$ such that:

$$x_3(t) = a\,x_1(t) + b\,x_2(t)$$

$$y_3(t) = t\,x_3(t)$$
$$= t\,[a\,x_1(t) + b\,x_2(t)]$$
$$y_3(t) = a\,t\,x_1(t) + b\,t\,x_2(t) \longrightarrow (3)$$

Using the values from eq (1) & (2) in eq (3)

$$\boxed{y_3(t) = a\,y_1(t) + b\,y_2(t)}$$

As superposition property is satisfied therefore, the given system is linear.

Check linearity of the system.
$y(t) = x^2(t)$.

Solution:

Let an input $x_1(t)$ is given to the system to get $y_1(t)$ such that:

$$y_1(t) = x_1^2(t) \longrightarrow (1)$$

Again, another input $x_2(t)$ is given to the system to get $y_2(t)$ such that:

$$y_2(t) = x_2^2(t) \longrightarrow (2)$$

Now, we apply input $x_3(t)$ to get the response $y_3(t)$ such that:

$$x_3(t) = a x_1(t) + b x_2(t)$$

$\therefore$

$$y_3(t) = x_3^2(t)$$
$$= [a x_1(t) + b x_2(t)]^2$$
$$= a^2 x_1^2(t) + b^2 x_2^2(t) + 2ab\, x_1(t) x_2(t)$$

$$y_3(t) = a^2 y_1(t) + b^2 y_2(t) + 2ab\, x_1(t) x_2(t)$$

It is obvious that the superposition does not apply here.

Hence the given system is non linear.