Discrete Fourier Transform (DFT):-

(Finite Length sequence)

It is represented by $X(K)$

$$X[K] = \sum_{n=0}^{N-1} x[n] \, e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$N-1 \rightarrow$ Length of sequence

$0 \leq K \leq N-1$

It is a function of Discrete sequence while DTFT is a function of continuous sequence.

Phase Factor or Twiddle Factor

$$W_N = e^{-j(2\pi/N)}$$

$$X(K) = \sum_{n=0}^{N-1} x[n] \, W_N^{kn}$$

(Q) $x[n] = \{1, 1, 0, 1\}$

Length $\Rightarrow N = 4$

$$X[k] = \sum_{n=0}^{3} x[n]\, e^{-j\left(\frac{2\pi}{4}\right)kn}$$

For $K = 0$

$$X[0] = x[0]e^{0} + x[1]e^{0} + x[2]e^{0} + x[3]e^{0}$$

$$= 1 + 1 + 0 + 1 = \boxed{3}$$

$$X[1] = \sum_{n=0}^{3} x[n](-j)^{n}$$

$$= x[0](-j)^{0} + x[1](-j)^{1} + x[2](-j)^{2} + x[3](-j)^{3}$$

$$= 1$$

$X[2] = -1$

$X[3] = 1$

$\because e^{-j(\pi/2)} = -j$

$e^{-j(\pi/2)} = \cos(\pi/2)^{0} - j\sin(\pi/2)^{1} = -j$

$$x[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)kn} \longrightarrow \text{For DFT}$$

$$C_K = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)kn} \longrightarrow \text{For DFS}$$

There is another method (Efficient method) to solve DFT:-

$$X = D_N \, x$$

$$x = \{x[0], x[1], \dots\}^T$$

$$X = \{X[0], X[1], \dots\}^T$$

D_N = $N \times N$ DFT Matrix

There is Inverse to get back the original sequence.

$$x = D_N^{-1} X$$

where

$$D_N^{-1} = \frac{1}{N} D_N^{*}$$

Length.

Example.

$k \backslash n$	0	1	2	3
0	$W_N^{kn} = W_4^0$	W_4^0	W_4^0	W_4^0
1	W_4^0	W_4^1	W_4^2	W_4^3
2	W_4^0	W_4^2	W_4^4	W_4^6
3	W_4^0	W_4^3	W_4^6	W_4^9

W_N^{kn}

* Symmetry Property.

(i) $W_N^0 = 1$

(2) $W_N^{N/2} = -1$

$k \backslash n$	0	1	2	3
0	1	1	1	1
1	1	$-j$	-1	j
2	1	-1	1	-1
3	1	j	-1	$-j$

$\therefore W_n^{kn} = e^{-j\left(\frac{2\pi}{4}\right)4}$

$= \cos 2\pi - j\sin 2\pi$

$= 1$

This table is made from the basic definition.

* :

$$x[n] = \{1, 1, 0, 1\}$$

$$X = D_N\, x$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+0+1 \\ 1-j+0+j \\ 1-1+0-1 \\ 1+j+0-j \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

— x — x — x —

Properties of DFT:-

① Linearity

$$x_1[n] \xleftrightarrow{DTFT} X_1[k]$$

$$x_2[n] \xleftrightarrow{\;''\;} X_2[k]$$

$$a\,x_1[n] + b\,x_2[n] \longleftrightarrow aX_1[k] + bX_2[k]$$

② Periodicity:-

$$x[n+N] = x[n]$$

then $$X[K+N] = X[K]$$

If an interval is repeated Then Fourier transform is also repeated.

③ Multiplication of DFT's or

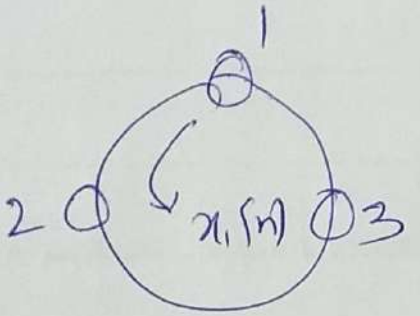
Circular / Periodic / Cyclic Convolution:-

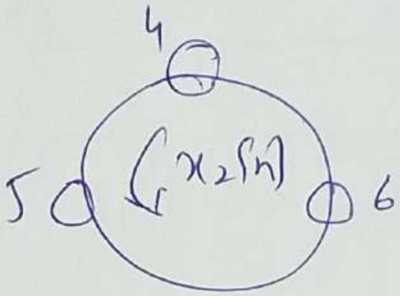
DYS.

(*) Find $y[n]$ by using Zero Padding / Cyclic / Periodic Convolution.

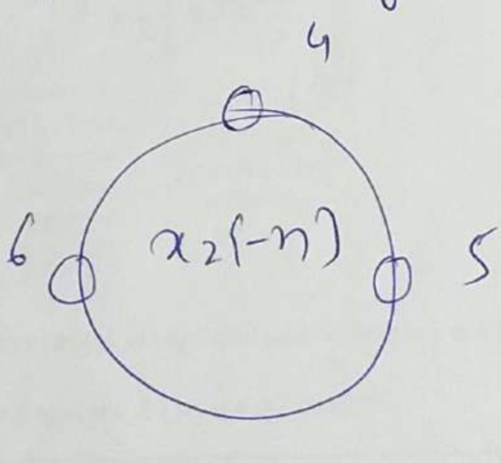
$$x_1[n] = \{\underset{\uparrow}{1}, 2, 3\}$$

$$x_2[n] = \{\underset{\uparrow}{4}, 5, 6\}$$

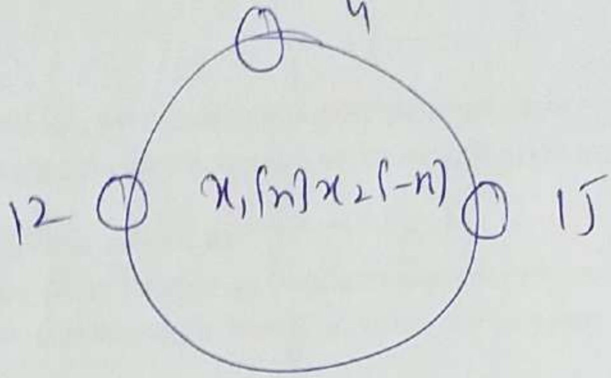




① Folding :- In this we take clockwise mirror image of one sequence

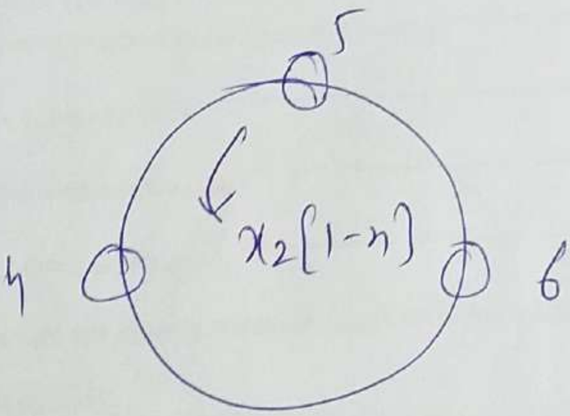


② Multiplication:-

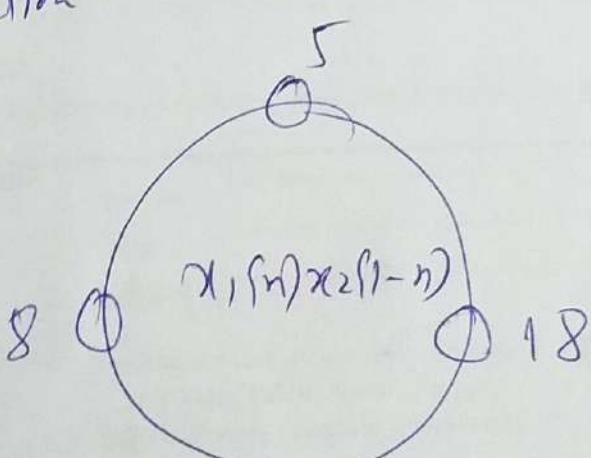


③ Sum $y(0) = 31$

Shift the folded seq (Anticlock wise)



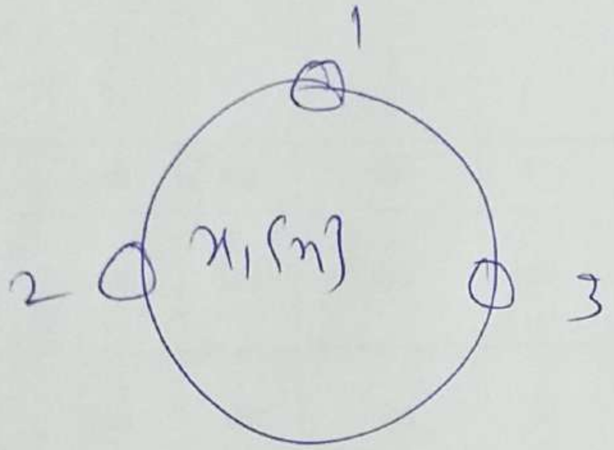
Multiplication

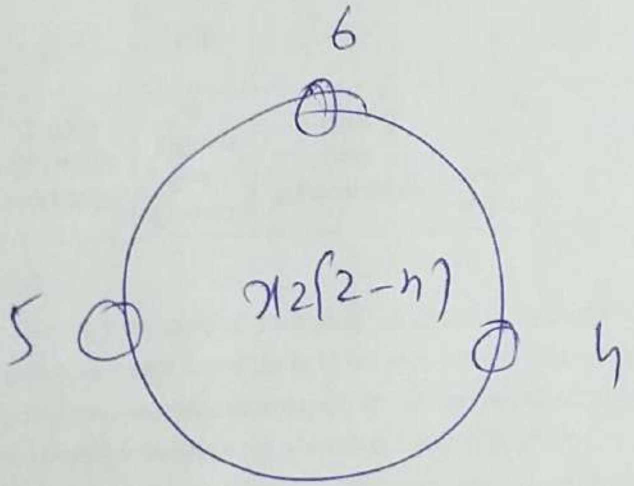


Sum

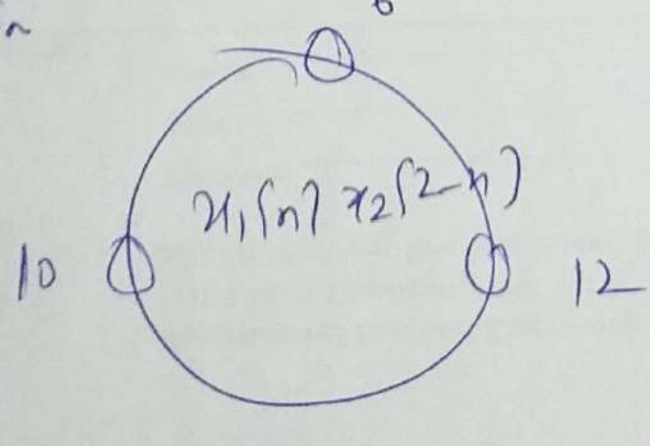
$$\boxed{y[1] = 31}$$

Second shift:-





Multiplication



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Sum = $\boxed{y[2] = 28}$

$\underline{So}$ $y[n] = \{ \underset{\uparrow}{31}, 31, 28 \}$

———— x ———— x ————