④ Power Flow Solution by Gauss-Seidel Method:-

This is the first iterative method to find out the Power flow Equations.

For this method we again start with the basics of Network equations

i-e $I_{Bus} = Y_{Bus} \cdot V_{Bus}$

and for any particular bus K.

$$I_K = \sum_{n=1}^{N} Y_{Kn} V_n$$

The complex Power

$$S_K = P_K + jQ_K = V_K I_K^*$$

$$P_K + jQ_K = V_K \left[\sum_{n=1}^{N} Y_{Kn} V_n \right]^*$$

where $K = 1, 2, \ldots N$.

From Complex Power,

$$I_K = \frac{P_K - jQ_K}{V_K^*}$$

Also

$$I_K = \sum_{n=1}^{N} Y_{Kn} V_n \quad \text{or}$$

$$I_K = Y_{K1}V_1 + Y_{K2}V_2 + \dots + Y_{KK}V_K + \dots + Y_{KN} \cdot V_N$$

From the above eq,

$$V_K = \frac{1}{Y_{KK}} \left[I_K - \left(\sum_{n=1}^{K-1} Y_{Kn}V_n + \sum_{n=K+1}^{N} Y_{Kn}V_n \right) \right]$$

or

$$V_K = \frac{1}{Y_{KK}} \left[\frac{P_K - jQ_K}{V_K^*} - \left(\sum_{n=1}^{K-1} Y_{Kn}V_n + \sum_{n=K+1}^{N} Y_{Kn}V_n \right) \right]$$

Where $K = 1, 2, \dots N$.

This is the equation we get for finding out V_K. Where $K = 1, 2, \dots N$ i-e for each or any bus we can used this equation.

⇒ We can write this equation for any buses except the swing bus.

Iterative Procedure:-

① Make an initial guess $|V_i|(0)$ and $\delta_i(0)$. Normally we use a flat start that is

$$|V_i|(0) = 1.0 \text{ P.U} \ \& \ \delta_i(0) = 0.0^\circ$$

because in normal condition the voltage magnitudes at all buses are going to be very near to 1.0 P.U.

② Use this solution in PFE to obtain a better first solution and this solution is called improved estimate of Vk.

③ First solution is used to obtain a better second solution and so on.

i.e We substitute the new value in place of the old values to again get an improved estimate of Vk and continues till the values did not converge i.e the change in values from the previous iteration to the current iteration is not so much.

So, in Mathematical form we can write,

$$V_K^{i+1} = \frac{1}{Y_{KK}}\left[\frac{P_K - jQ_K}{V_K^{i*}} - \left(\sum_{n=1}^{K-1} Y_{Kn}V_n^{i+1} + \sum_{n=K+1}^{N} Y_{Kn}V_n^{i}\right)\right]$$

$K = 1, 2, \dots, N$, i is iteration count.

Algorithm Steps:-

① With P_{gi}, Q_{gi}, P_{Li} and Q_{Li} known. Calculate bus injections P_i, Q_i

② Form Y_{Bus} Matrix

③ Set initial Voltage $V_i^{(0)}$, $\delta_i(0)$

④ Iteratively solve equation.

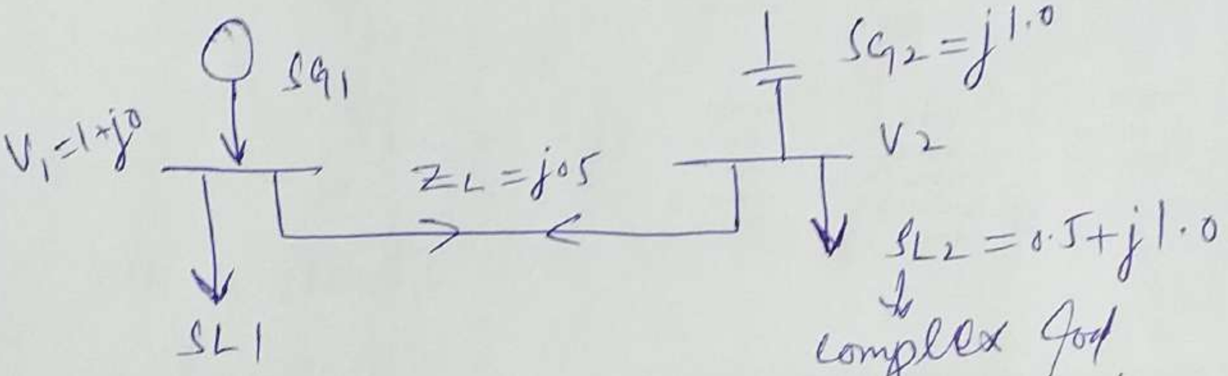
$$V_K^{i+1} = \frac{1}{Y_{KK}}\left[\frac{P_K - jQ_K}{V_K^{*}} - \left(\sum_{n=1}^{K-1} Y_{Kn}V_n^{i+1} + \sum_{n=K+1}^{N} Y_{Kn}V_n^{i}\right)\right]$$

To obtain new values of bus Voltages.

(Q):- For the system shown,

$Z_L = j0.5$, $V_1 = 1\angle 0^\circ$, $S_{G2} = j1.0$

and $S_{L2} = 0.5 + j1.0$. Find V_2 using Gauss-Seidal Iteration Technique?



Sol:- Bus 1 can be taken as a slack bus because the voltage angle and magnitude are known and also we don't know the generation because we assigned all the losses to this bus 1.

First, we calculate the elements of the Y_{Bus}.

For $Z_L = j0.5$, we have

$$Y_{11} = \frac{1}{Z_{11}} = \frac{1}{j0.5} = -j2$$

So $Y_{11} = -j2$

$Y_{12} = j2$

$Y_{21} = j2$

$Y_{22} = -j2$

We iterate on V_2 using the equation below:

$$V_2^{q+1} = \frac{1}{Y_{22}}\left[\frac{S_2^*}{(V_2^n)^*} - Y_{21}.V_1\right] \longrightarrow ①$$

Given $V_1 = 1\angle 0^\circ$

$S_2 = S_{G2} - S_{L2}$

$= (0+j1.0) - (0.5+j1.0)$

$= -0.5$

Putting the values of V_1, S_2, Y_{22} and Y_{21} in eq ① we get:-

$$V_2^{n+1} = \frac{Y_{21}}{Y_{22}}\left[\left[\frac{0.5}{Y_{21}(V_2^n)^*}\right] + V_1\right]$$

$$= \frac{j2}{-j2}\left(\left[\frac{-0.5}{-j2(V_2^n)^*}\right] + 1.0\right]$$

$$= -1\left\{\left[\frac{-0.5}{-j2\ (V_2^n)^*}\right] + 1.0\right]$$

$$= -1\left\{\frac{0.25j}{(V_2^n)^*}\right] + 1.0$$

$$= -j\left[\frac{0.25}{(V_2^n)^*}\right] + 1.0 \quad (2)$$

We start with the guess, taking $V_2^{(0)} = 1\angle 0^\circ$ and iterate eq ②.

We have $V_2^0 = 1 + j0$

Putting in eq (2) and iteration for V_2, we get.

$$V_2^1 = -j\left[0.25/(1+j0)^*\right] + 1.0$$

$$= 1.0 - j0.25$$

$$= 1.3077 \angle -14.036^\circ$$

$$V_2^2 = -j\left[0.25/(1.0-j0.25)^*\right] + 1.0$$

$$= \frac{(1.0 - j0.25)}{(1.0 + j0.25)}$$

$$= 0.970 \angle -14.036^\circ$$

Similarly we can iterate it further. The results of the iterations are as:-

Iteration Number	V_2
0	$1\angle 0^\circ$
1	$1.030776 \angle -14.036^\circ$
2	$0.970143 \angle -14.036^\circ$
3	$0.970261 \angle -14.9310^\circ$
4	$0.96623 \angle -14.931416^\circ$
5	$0.966236 \angle -14.995078^\circ$
6	$0.96548 \angle -14.995072^\circ$

Since, the difference in the values for the voltage does not change much b/w 5th and 6th iteration, So we can stop after 6th iteration.

Hence, we can see that starting with the value $V_2^\circ = 1\angle 0^\circ$, Convergence is reached in sixth step.