Properties Of CTFT :-

1) Linearity ::

Property of linearity holds good in the case of continuous-time Fourier transform.

Now, consider two CT signals & their Fourier transforms i.e.

$$x(t) \longleftrightarrow X(j\omega)$$

&

$$\boxed{a x(t) + b y(t) \xleftrightarrow{F} a X(j\omega) + b Y(j\omega)}$$

Thus, linearity property is easily extended to a linear combination of an arbitrary number of signals.

2) Time Shifting:

Def:

"If a signal $x(t)$ is given a certain time shift t_0, then the continuous-time Fourier transform will also be shifted by the same amount

Explanation:

Consider a signal $x(t)$ & its CTFT.

$$x(t) \xleftrightarrow{F} X(j\omega).$$

Applying a time shift of "t_0"

$$\boxed{x(t-t_0) \xleftrightarrow{F} X(j\omega)\, e^{-j\omega t_0}}$$

Proof:

The inverse Fourier transform is given by:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$x(t-t_0) = \frac{1}{2\pi}\int_{-\infty} X(j\omega)\, e^{j\omega(t-t_0)}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{e^{-j\omega t_0} X(j\omega)\right\} e^{j\omega t}\, d\omega$$

Recognizing this as the synthesis equation for $x(t-t_0)$, we conclude that:

$$\boxed{\mathcal{F}\{x(t-t_0)\} = e^{-j\omega t_0} X(j\omega)}$$

different

Hence, proved.

Conjugate Property Of CTFT:

As we know that a conjugate of a vector is a vector whose real part is the same and the imaginary is also same but with negative sign.

Consider a continuous-time signal $x(t)$ such that $x(t)$ is real. Then CTFT of $x(t)$ is given by.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \longrightarrow (1)$$

Now taking the conjugate on both the sides.

$$\omega) = \int_{-\infty}^{\infty} \left\{ x(t) \, e^{-j\omega t} \, dt \right\}$$

$$\omega) = \int_{-\infty}^{\infty} x(t) \, e^{j\omega t} \, dt$$

w put,

$$\omega = -\omega$$

The above equation becomes,

$$\omega) = \int_{-\infty}^{\infty} x(t) \, e^{-j\omega t} \, dt \longrightarrow (2)$$

omparing eq (1) & (2)

$$\boxed{X(j\omega) = X^{*}(-j\omega)}$$ if $x(t)$ is re

or put $\omega = -\omega$.

en:

$$\boxed{X(-j\omega) = X^{*}(j\omega)}$$

Fourier Transform Of Differentiation & Integration Of Continuous-Time Signals:-

Let $x(t)$ be a continuous-time signal with a fourier transform of $X(j\omega)$.

i.e

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$

Differentiating both sides w.r.t "t".

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, \frac{d}{dt}\{e^{j\omega t}\}\, d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, \{e^{j\omega t}\cdot j\omega\}\, d\omega.$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{j\omega\, X(j\omega)\}\, e^{j\omega t}\, d\omega$$

$\Rightarrow$

$$\boxed{\mathcal{F}\left\{\frac{d}{dt}x(t)\right\} = j\omega\, X(j\omega)}$$

Result:

We conclude that if a function is differentiated in time domain, it is multiplied by $j\omega$ in frequency domain.

From the above result we also conclude that if a function is differentiated in time domain, it is multiplied with "$j\omega$" in frequency domain. Similarly, if a function is integrated in time domain, then it is divided by "$j\omega$" in frequency domain.

We know that differentiation in time domain corresponds to multiplication by $j\omega$ in frequency domain. From the duality property, we might suspect that multiplication by jt in the time domain corresponds roughly to differentiation in frequency domain.

As we know that:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Differentiating both sides w.r.t. "ω"

$$\frac{dX(j\omega)}{d\omega} = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-j\omega t}\, dt$$

$$\frac{dX(j\omega)}{d\omega} = -jt \int_{-\infty}^{-\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\frac{dX(j\omega)}{d\omega} = -jt\, \mathcal{F}\{x(t)\}$$

$\Rightarrow$

$$\boxed{-jt\, x(t) \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{d}{dt} X(j\omega)}$$