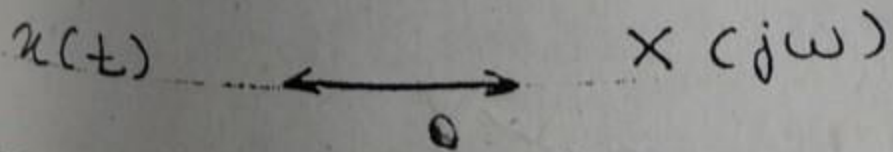


## Properties Of CTFT:-

1) Linearity:-

Property of linearity holds good in the case of continuous-time Fourier transform.

Now, consider two CT signals & their Fourier transforms i.e.



$$ax(t) + by(t) \xrightarrow{F} aX(j\omega) + bY(j\omega)$$

Thus, linearity property is easily extended to a linear combination of an arbitrary number of signals.

## 2) Time Shifting:

Def.:

"If a signal  $x(t)$  is given a certain time shift  $t_0$ , then the continuous-time Fourier transform will also be shifted by the same amount."

Explanation:

Consider a signal  $x(t)$  & its CTFT.

$$x(t) \xrightarrow{F} X(j\omega)$$

Applying a time shift of " $t_0$ "

$$x(t-t_0) \xrightarrow{F} X(j\omega) e^{-j\omega t_0}$$

Proof:

The inverse Fourier transform is given by:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$\begin{aligned}
 x(t-t_0) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega(t-t_0)} d\omega \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ e^{-j\omega t_0} X(j\omega) \right\} e^{j\omega t} d\omega
 \end{aligned}$$

$(\mathcal{F}^{-1}) \{ X(j\omega) \}$   
 $(\mathcal{F}^{-1}) \{ X(j\omega) e^{-j\omega t_0} \}$

Recognizing this as the synthesis equation for  $x(t-t_0)$ , we conclude that:

$$\mathcal{F}\{x(t-t_0)\} = e^{-j\omega t_0} X(j\omega)$$

Hence, proved.

different

$(\mathcal{F}^{-1}) \{ X(j\omega) \}$   
 $(\mathcal{F}^{-1}) \{ X(j\omega) e^{-j\omega t_0} \}$



## Conjugate Property Of CTF.T:

As we know that a conjugate of a vector is a vector whose real part is the same ~~and~~ and the imaginary is also same but with negative sign.

Consider a continuous-time signal  $x(t)$  such that  $x(t)$  is real. Then CFT of  $x(t)$  is given by,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad \longrightarrow (1)$$

Now taking the conjugate on both the sides,

$$w) = \int_{-\infty}^{\infty} \left\{ x(t) e^{-j\omega t} dt \right\}$$

$$w) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$$

w put,

$$\omega = -\omega$$

The above equation becomes,

$$w) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \longrightarrow (2)$$

Comparing eq (1) & (2)

$$X(j\omega) = X^*(-j\omega) \quad \text{if } x(t) \text{ is real}$$

or put  $\omega = -\omega$ .

then:

$$X(-j\omega) = X^*(j\omega)$$



# Fourier Transform Of Differentiation & Integration Of Continuous-Time Signals:-

Let  $x(t)$  be a continuous-time signal with a fourier transform of  $X(j\omega)$ .

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

Differentiating both sides w.r.t "t"

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \frac{d}{dt} \{ e^{j\omega t} \} d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \{ e^{j\omega t} \cdot j\omega \} d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{ j\omega X(j\omega) \} e^{j\omega t} d\omega$$

=>

$$F\left\{\frac{dx(t)}{dt}\right\} = j\omega X(j\omega)$$

Result:-

We conclude that if a function is differentiated in time domain, it is multiplied by  $j\omega$  in frequency domain.

From the above result we also conclude that if a function is differentiated in time domain, it is multiplied with " $j\omega$ " in frequency domain. Similarly, if a function is integrated in time domain, then it is divided by " $j\omega$ " in frequency domain.



We know that differentiation in the time domain corresponds to multiplication by  $j\omega$  in frequency domain. From the duality property, we might suspect that multiplication by  $jt$  in the time domain corresponds roughly to differentiation in frequency domain.

As we know that:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Differentiating both sides w.r.t.  $\omega$

$$\frac{dX(j\omega)}{d\omega} = \int_{-\infty}^{\infty} -jt x(t) e^{-j\omega t} dt$$

$$\frac{dX(j\omega)}{d\omega} = -jt \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\frac{dX(j\omega)}{d\omega} = -jt \mathcal{F}\{x(t)\}$$

