✱ Frequency - Domain Representation of DT - signals:

① DTFT (Discrete time Fourier Transform)

② DFT (Discrete Fourier Transform)

③ Z - Transform.

We deccelerate information and then see which one is required.

DTFT :-

If we have a sequence $x[n]$ then

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

DTFT is a complex value fn. I is continuous and complex.

If $x[n]$ for Causal.

$$X(e^{jw}) = \sum_{n=0}^{\infty} x[n]\, e^{-jwn}$$

DTFT is the representation of Discrete time aperiodic sequence by a continuous periodic fn.

(8) $$X(e^{jw}) = X_R(e^{jw}) + jX_I(e^{jw})$$

Rectangular form.

and $$X(e^{jw}) = |X(e^{jw})|\, e^{j\theta(w)}$$

$$\theta(w) = \arg[X(e^{jw})]$$

Polar form.

Convergence Conditions:-

- Unique

$\Rightarrow$ Convergence main features.

① Absolute summable

$$\left|\sum x[n]\right| < \infty$$

② Square summable

$$\left|\sum |x[n]|\right|^2 < \infty$$

Some Series:-

① $$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

② $$\sum_{n=1}^{\infty} x^n = \frac{x}{1-x}$$

③ $$\sum_{n=0}^{N-1} A^n = \frac{1-A^N}{1-A}$$

Exple:- $x[n] = \alpha^n u[n]$ $\quad |\alpha|<1$

Sol:- $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}$

$$= \sum_{n=0}^{\infty} (\alpha)^n (e^{-jw})^n$$

$\therefore$ because of $u[n]$

$$= \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$

From the series

$$\boxed{X(e^{jw}) = \frac{1}{1-(\alpha e^{-jw})}}$$

$\Downarrow$

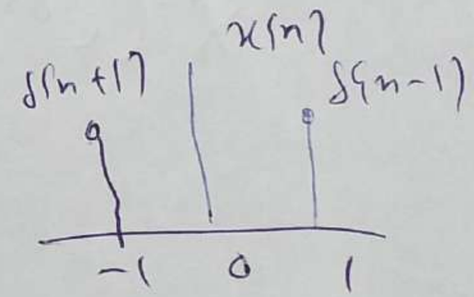
DYS:- Draw spectrum in Matlab.

Ex:- $x[n] = \delta[n-1] + \delta[n+1]$

Sol:- $X(e^{jw}) = \sum_{-\infty}^{\infty} x[n]\, e^{-jwn}$

$$= \sum \delta[n-1]\, e^{-jwn} + \sum \delta[n+1]\, e^{+jwn}$$

$\downarrow$ at $n=1$ $\qquad$ $\downarrow$ at $n=-1$



$$= (1)e^{-jw} + (1)e^{jw}$$ xply & ÷de by 2.

$$= \frac{2\left(e^{jw} + e^{-jw}\right)}{2}$$

$$= \boxed{2\cos w}$$ Ans.

⊛ Properties of DTFT:-

① Periodicity of DTFT:-

$$x[n] \longleftrightarrow X(e^{j\omega})$$

$$X(e^{j(\omega+2\pi)}) = X(e^{j\omega}) \quad \text{Proof DYS}$$

② Linearity:

$$x_1[n] \longleftrightarrow X_1(e^{j\omega})$$

$$x_2[n] \longleftrightarrow X_2(e^{j\omega})$$

If we xply both sequence with scalars and add up.

$$a x_1[n] + b x_2[n] \longleftrightarrow a X_1(e^{j\omega}) + b X_2(e^{j\omega})$$

Proof DYS.

③ Time Shifting Property:-

$$x[n] \longleftrightarrow X(e^{j\omega})$$

$$x[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$$

Proof:-

$$x[n] = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$F\{x[n-n_0]\} = \sum_{n=-\infty}^{\infty} x[n-n_0] e^{-j\omega n}$$

Put, $n - n_0 = m$

$$= \sum_{m=-\infty}^{\infty} x[m] e^{-j\omega(m+n_0)}$$

$$= \sum \{x[m] e^{-j\omega m}\} e^{-j\omega n_0}$$

$$= X(e^{j\omega}) e^{-j\omega n_0}$$

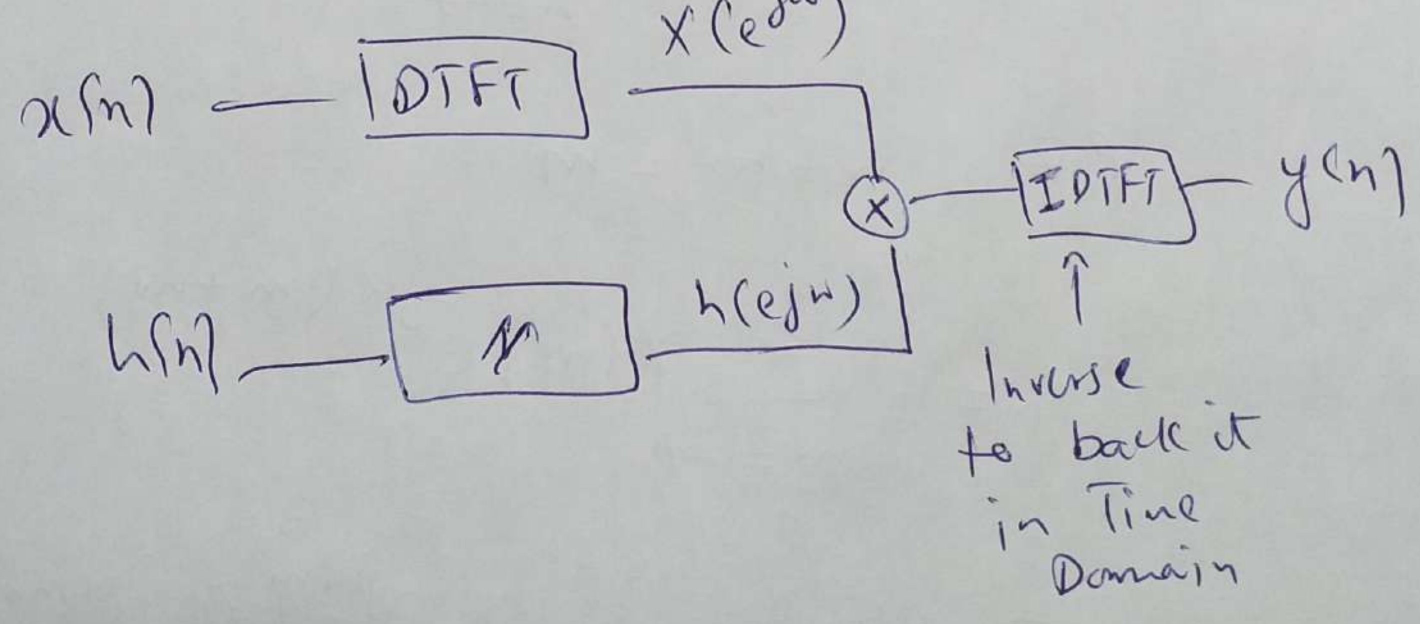
(4) Convolution Property:-

$$x[n] = h[n]$$

$$y[n] = x[n] * h[n] \Rightarrow \text{Time Domain}$$

$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$$

But if we implement it in Freq domain.



$\Rightarrow$ Convolution in Time domain is equal to xplication in Freq Domain.

(8) Time Reversal Property:-

$$x(n) \longleftrightarrow X(e^{j\omega})$$

$$x[-n] \longleftrightarrow X(e^{-j\omega})$$

(#) Differentiation in freq:-

$$n\,x(n) \longleftrightarrow j\frac{d}{d\omega}X(e^{j\omega})$$

(#) Inverse discrete time Fourier Transform (IDTFT):-

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega n}\,d\omega$$

For Inverse we adopt Partial fraction method because the above tool is very complicated.

Application of DTFT:-

① Impulse Response (IR)

② Frequency " (FR)

③ Digital Filter Designing (DFD)

(DYS)