"The Continuous-Time Fourier Transform"

The continuous-time fourier transform of a function (signal) $x(t)$ is given by:

$$x(t) \longleftrightarrow X(j\omega)$$

i.e

$X(j\omega)$ is the Continuous-time fourier transform of $x(t)$ & is defined as.

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

The inverse Fourier transform of a Fourier transform of a signal is the signal itself and is given by:

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega}$$

Example # 4.1.

$x(t) = e^{-at}\, u(t)$; $a > 0$

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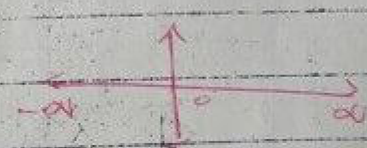


$X(j\omega) = ?$

Solution:-

CTFT (Continuous-Time Fourier Transform) of a signal $x(t)$ is given by:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

As the given signal is multiplied by a step function, hence limits $0 \leq t \leq \infty$.

$$X(j\omega) = \int_{0}^{\infty} e^{-at}(1)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= \frac{-1}{a+j\omega}\left[e^{-\infty} - e^{0} \right]$$

$e^{0} = 1$

$e^{-\infty} = 0$

$$= \frac{-1}{a+j\omega}\left[0 - 1 \right]$$

$$\boxed{X(j\omega) = \frac{1}{a+j\omega}}$$

Clear

Plot:

Since, this Fourier transform is a complex valued, so to plot it as a function of "ω", we express $X(j\omega)$ in terms of its magnitude and phase.

i.e

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \Rightarrow \text{Magnitude.}$$

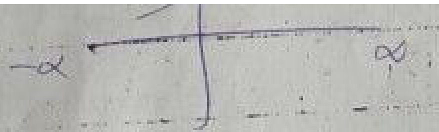
$$\measuredangle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

Note:

We have taken -ve sign with the phase because:

"Whenever the whole vector is in denominator, the angle is negative because we always express angle in numerator.

e.g: $\dfrac{1\angle 0^\circ}{26\angle 36^\circ} = \dfrac{1}{26}\angle -36^\circ$



Example 4.2 ::

$$x(t) = e^{-a|t|} \qquad a > 0$$

$$X(j\omega) = ?$$

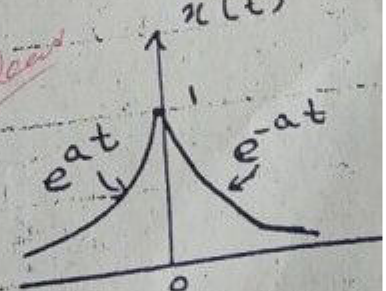
Solution::

The Fourier transform of the given function $x(t)$ is given by:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt.$$

Clear



Note::

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$\therefore$

$$X(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\,dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\,dt$$

$$= \left.\frac{e^{(a-j\omega)t}}{a-j\omega}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0}-e^{-\infty}\right] - \frac{1}{(a+j\omega)}\left[e^{-\infty}-e^{0}\right]$$

$$= \frac{1}{(a-j\omega)}\left[1-0\right] - \frac{1}{(a+j\omega)}\left[0-1\right]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2-(j\omega)^2}$$

$$\boxed{(j\omega) = \frac{2a}{a^2+\omega^2}}$$

t: