Lecture 12:

Ampere Circuit Law, Application of Ampere Circuit Law

- Related YouTube Video Link:
  1) https://www.youtube.com/watch?v=TFpq4PpUHk4
  2) https://www.youtube.com/watch?v=K0zds682kEk
  3) https://www.youtube.com/watch?v=cxNiquRMOfs
  4) https://www.youtube.com/watch?v=WcQMKpq0nVo

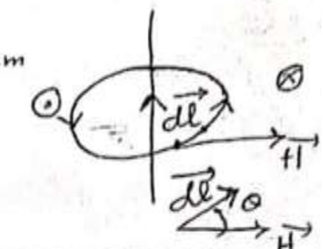
- **Read 8.2 of the Given Book**

- **Lecture Notes on Next Page**

~~A~~mphere's Circuit LAW - Maxwell's Equation

"Ampere's circuit law, states that the line integral of $\vec{H}$ around a closed path is the same as the net current I_{enc} ~~and~~ enclosed by the path".

$$\boxed{\oint_L \vec{H}\cdot\vec{dl} = I_{enc}} \text{ ---- Integral form}$$



"The circulation of $\vec{H}$ equals I_{enc};

Using stoke's Theorem:

$$\int_s (\nabla\times\vec{H})\cdot\vec{ds} = I_{enc} \qquad \oint_L \approx \int_s$$

Also, $I_{en} = \int_s \vec{J}\cdot\vec{ds}$

$$\boxed{\nabla\times\vec{H} = \vec{J}} \longrightarrow \text{Third Maxwell's equation.}$$

$\longrightarrow$ Ampere's Law in differential (or point) form.

Note: since, $\nabla\times\vec{H} \neq 0$; a magnetostatic field is not conservative

Applications of Amperes Law:

1) Used to determine $\vec{H}$ for symmetrical current distribution such as

- Infinite line current
- Infinite sheet of current
- Infinite long coaxial transmission line

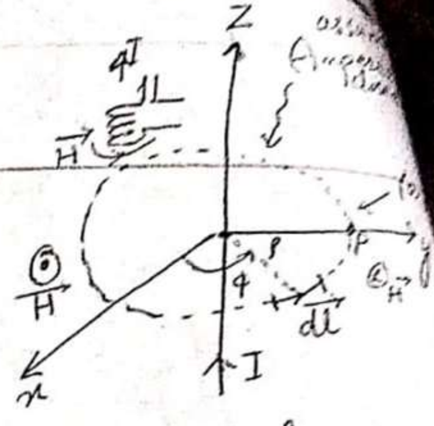
} Apply $\oint_L \vec{H}\cdot\vec{dl} = I_{enc}$

2) For symmetrical current distribution, $\vec{H}$ is either parallel or perpendicular to $\vec{dl}$.

Infinite Line Current:

To determine $\vec{H}$,
Ampere's Circuit law is applied.

Consider closed path ≈ Amperian Path which shows that $\vec{H}$ is constant as ρ is constant.



Cylindrical Coordinate
(ρ, ϕ, z)

Note:
Amperian Path is analogous to Gaussian Surface

$$\oint_L \vec{H} \cdot \vec{dl} = I$$

$$\oint_L H_\phi \vec{a_\phi} \cdot \rho d\phi \vec{a_\phi} = I$$

$$H_\phi \rho \oint_0^{2\pi} d\phi = I \quad (0 \le \phi \le 2\pi)$$

$$H_\phi \rho (2\pi) = I.$$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}} \text{ — magnitude.}$$

$$\rightarrow \boxed{\vec{H} = \frac{I}{2\pi\rho} \vec{a_\phi}} \text{ — vector form}$$

$$\vec{H} = H_\rho \vec{a_\rho} + H_\phi \vec{a_\phi} + H_z \vec{a_z}$$

$$\vec{dl} = d\rho \vec{a_\rho} + \rho d\phi \vec{a_\phi} + dz \vec{a_z}$$

Also, $\rho =$ constant

$z = 0$ for point P.

$\therefore d\rho = dz = 0$

So, $\vec{H} \cdot \vec{dl} = H_\phi \vec{a_\phi} \cdot \rho d\phi \vec{a_\phi}$

Numerical

Q: An infinite filament on the z-axis carries 20π mA in the $\vec{a_z}$ direction. Three uniform cylindrical current sheets are also present: 400 mA/m at $\rho = 1$ cm, -250 mA/m at $\rho = 2$ cm, and -300 mA/m at $\rho = 3$ cm.

Calculate H_ϕ at $\rho = 0.5, 1.5, 2.5$ and 3.5 cm.

$I = 20\pi$ mA

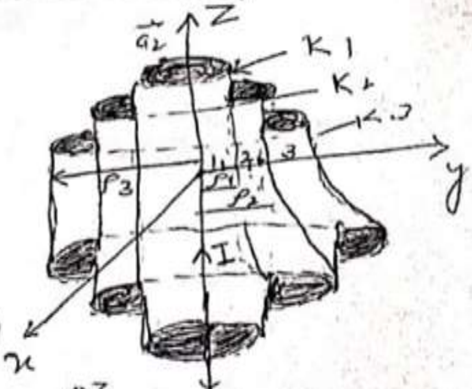
$K_1 = 400$ mA/m, $\rho_1 = 1$ cm.

$K_2 = -250$ mA/m, $\rho_2 = 2$ cm.

$K_3 = -300$ mA/m, $\rho_3 = 3$ cm.

$H_\phi = ?$ at $\rho = 0.5, 1.5, 2.5, 3.5$ cm.

Using Ampere Circuital law.



① At $\rho = 0.5$ cm — $\rho < \rho_1$ (I enclosed only)

$\oint_L \vec{H} \cdot \vec{dl} = I_{enc}$

$H_{\phi_1} \cdot (2\pi \times 0.5 \times 10^{-2}) = I = 20\pi \times 10^{-3}$ A

$H_{\phi_1} = \dfrac{2\pi \times 10^{-3}}{2\pi \times 0.5 \times 10^{-2}} = \dfrac{10 \times 10^{-1}}{0.5} = 2$ A/m Ans.

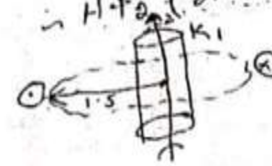
② At $\rho = 1.5$ cm — $\rho_1 < \rho < \rho_2$ — $(I + (K_1 \times 2\pi \times 10^{-2}))$ enclosed

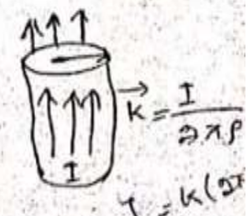
$\therefore H_{\phi_2} (2\pi \times 1.5 \times 10^{-2}) = 20\pi \times 10^{-3} + 400 \times 10^{-3} \times 2\pi \times 10^{-2}$

$= \dfrac{(10+4) \times 10^{-3}}{1.5 \times 10^{-2}} = 9.33 \times 10^{-1}$

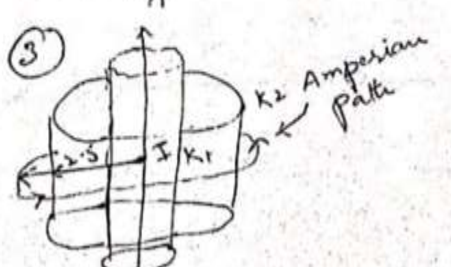
$= 0.933$ A/m

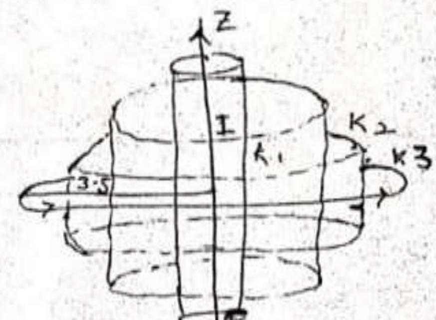
$H_{\phi_2} = 933$ mA/m Ans.





③





③ At $\rho = 2.5\,cm$ ... $\rho_2 < \rho < \rho_3$ ..... $(I + K_1 \times 2\pi \times 10^{-2} + K_2 \times 2\pi \times 2 \times 10^{-2})$

$$H_{\phi 3}\, 2\pi (2.5 \times 10^{-2}) = \overset{I}{\underline{20\pi \times 10^{-3}}} + 400 \times 10^{-3} \times 2\pi \times 10^{-2} - 250 \times 10^{-3} \times 2\pi \times 2 \times 10^{-2}$$

$$H_{\phi 3} = \frac{(10 + 4 - 5)\, 10^{-3}}{2.5 \times 10^{-2}} = \frac{9 \times 10^{-1}}{2.5} = 3.6 \times 10^{-1} = 0.36\ A/m$$

$$= 360\ mA/m.$$

---

④ At $\rho = 3.5$ .... $\rho > \rho_3$ ... $(I + K_1\, 2\pi \times 10^{-2} + K_2 \times 2\pi \times 2 \times 10^{-2} + K_3 \times 2\pi \times 3 \times 10^{-2})$

$$H_{4\phi} (2\pi \times 3.5 \times 10^{-2}) = 20\pi \times 10^{-3} + 400 \times 10^{-3} \times 2\pi \times 10^{-2} - 250 \times 10^{-3} \times 2\pi \times 2 \times 10^{-2} - 300 \times 10^{-3} \times 2\pi \times 3 \times 10^{-2}$$

$$H_{4\phi} = \frac{(10 + 4 - 5 - 9)\, 10^{-3}}{3.5 \times 10^{-2}} = 0\ A/m.$$