Decomposition of a Sequence into Impulses:-

Suppose we have given a sequence. we can decompose it into scaled and shifted impulse.

$$\boxed{x[n] = \sum_{K=-\infty}^{\infty} x[K]\,\delta[n-K]}$$

$\Downarrow$

Scalled & Shifted

e.g $\quad x[n] = \{2, \underset{\uparrow}{4}, 1, 3\}$

$K = -1$ to 2.

$$= x[-1]\,\delta[n+1] + x[0]\,\delta[n-0] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2]$$

$$\delta[n] = 2\,\delta[n+1] + 4\,\delta[n] + \delta[n-1] + 3\,\delta[n-2]$$

Magnitude   Location

We decompose a sequence into Impulse Response.

⇒ If we know the impulse response of a system we can easily determine its output without physically performed it.

#

$$\delta[n] \longrightarrow \boxed{\text{System}} \longrightarrow h[n]$$

Time-Invariant

$$\delta[n-k] \longrightarrow \boxed{} \longrightarrow h[n-k]$$

Linear (Homoginity / Scaling)

$$a\delta[n] \longrightarrow \boxed{} \longrightarrow a h[n]$$

$$b\delta[n-1] \qquad b h[n-1]$$

$$c\delta[n-1] \qquad c h[n-1]$$

Linear (Additive)

$$a\delta[n] + b\delta[n-1] + c\delta[n-1] \longrightarrow \boxed{} \longrightarrow a h[n] + b h[n-1] + c h[n-2]$$

This is called Linear Time Invarient System (LTI System).

(#) Linear Convolution of DT signal:-

Convolution is just a mathematical operator like subtraction, addition & Multiplication. Denoted by " * ".

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[n] h[n-k]$$

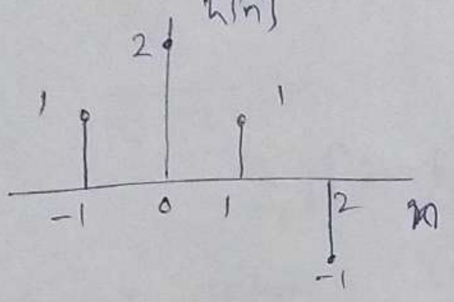
(#) Geometrically convolutions means Flip & Drag until unless its multiplication is zero.

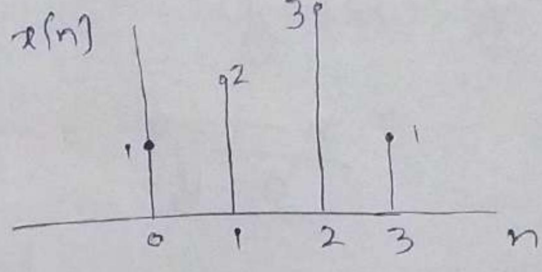
(#) Steps to denote it Graphically:-

1. Folding
2. Shifting
3. Product sequence
4. Sum.

Graphical Method:-

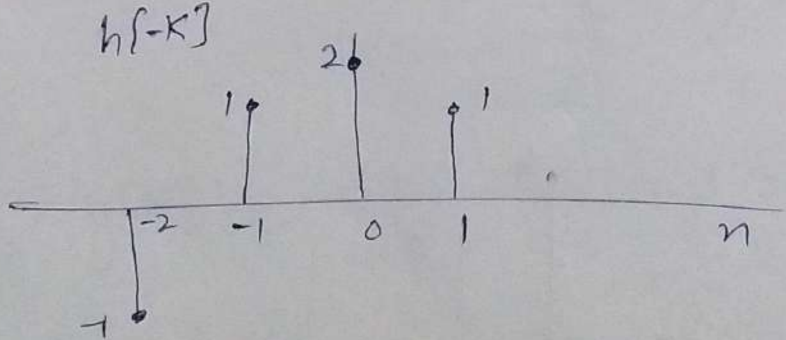
$h[n] = \{1, \underset{\uparrow}{2}, 1, -1\}$ $x[n] = \{1, 2, 3, 1\}$





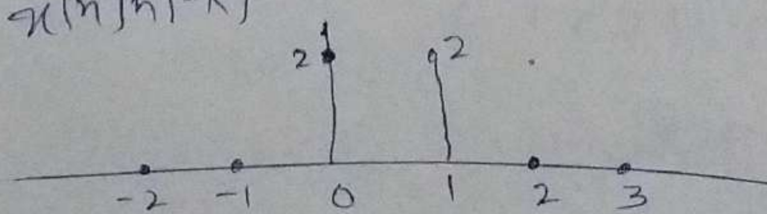
Length of output $= L = 4+4-1 = \boxed{7}$

Fold any one but we usually fold impulse response.



Now for product sequence

$x[n] * h[n]$



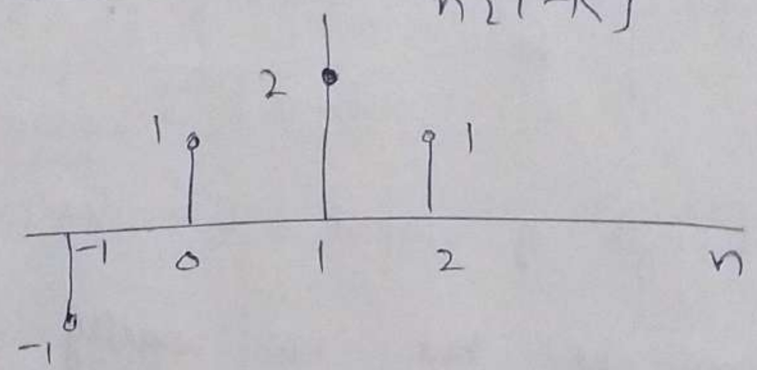
Sum:

$$y[0] = 0 + 0 + 2 + 2 + 0 + 0$$
$$= 4$$

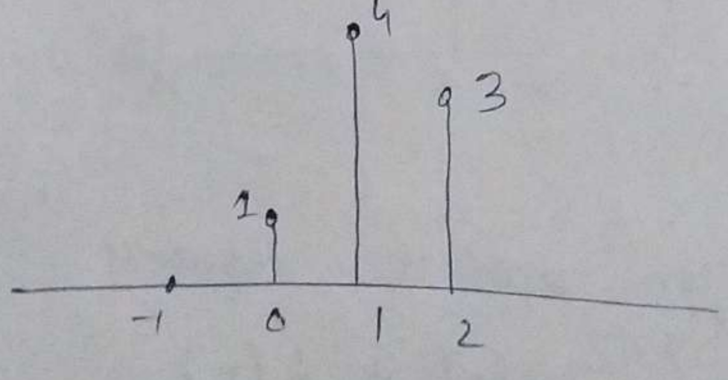
* Shifting

$n - 1 = 0$

$\boxed{n = 1}$



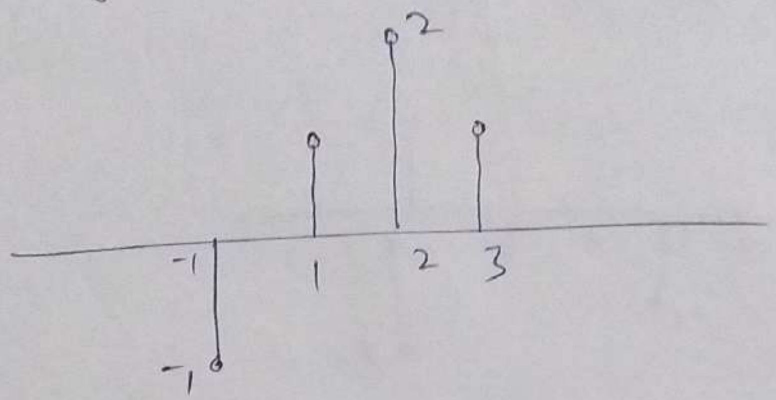
$x[n]\, h[1-k]$



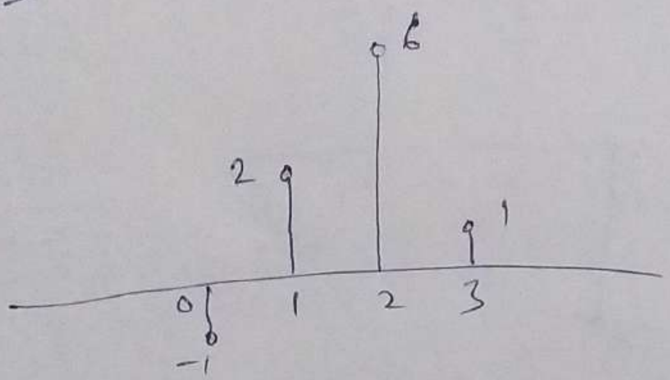
$$y[1] = 1 + 4 + 3$$
$$= 8$$

$\boxed{n=2}$

$h[2-k]$



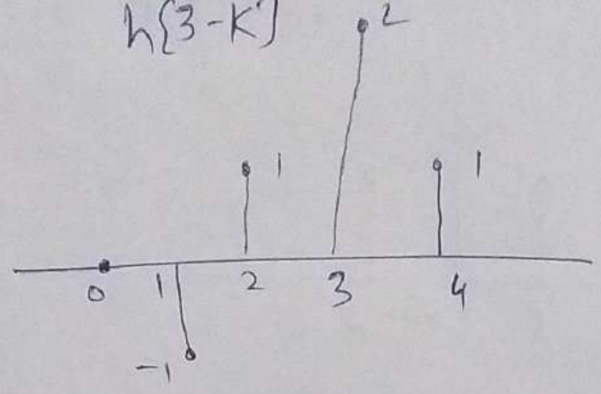
Product sequence:

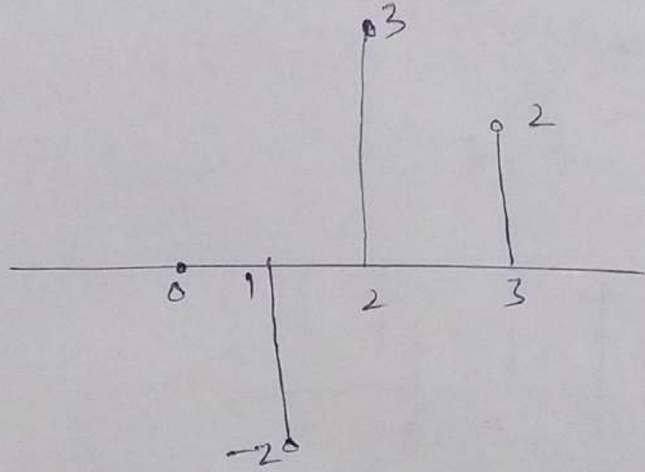


$$y[2] = 6+2-1+1 = 8$$

$\boxed{n=3}$

$h[3-K]$

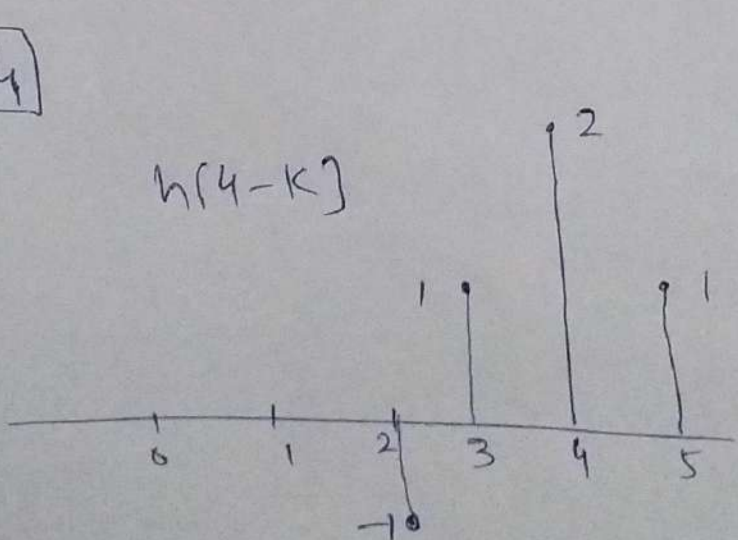


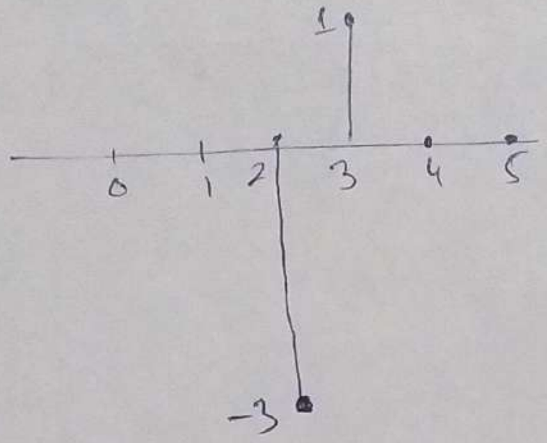


$$y[3] = -2+3+2 = \boxed{3}$$

$\boxed{n=4}$

$h[4-K]$

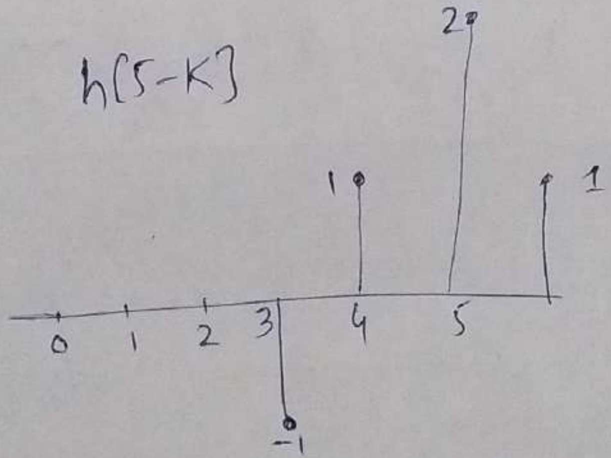




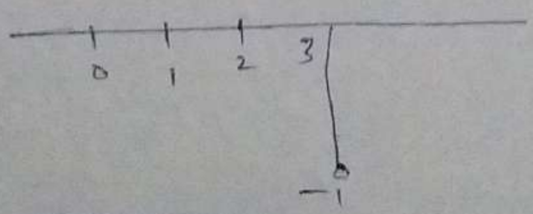
$y[4] = -3+1$

$= \boxed{-2}$

$\boxed{n=5}$ $h[5-K]$



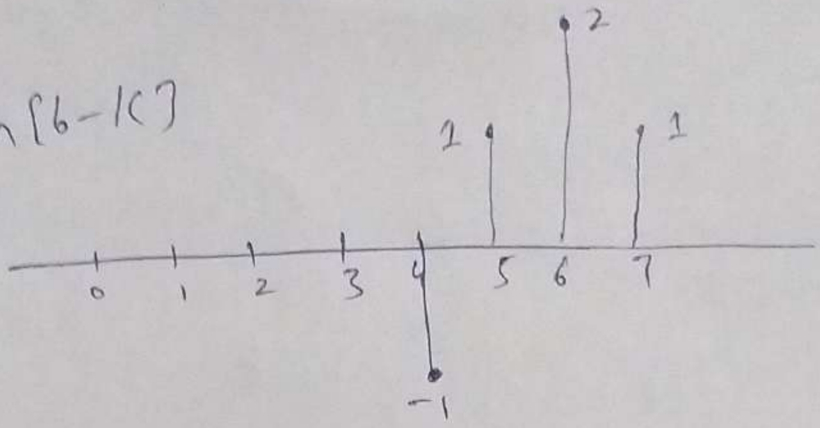
$x[n]\,h[5-K]$



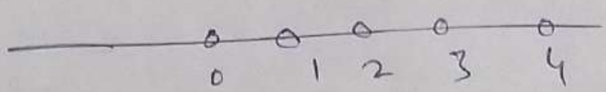
$\boxed{y[5] = -1}$

$\boxed{n=6}$

$h[6-k]$



$x[n]\,h[6-k]$



$\boxed{y=0}$

$$\boxed{y[n] = 1,\ \underset{\uparrow}{4},\ 8,\ 8,\ 3,\ -2,\ -1}$$

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