

⊕ Zero Input, Zero State Responses:-

⇒ Zero input is also called Free

response / natural response. Zero input

depends on initial condition.

System is free and did not

give input as the system depends

on initial condition.

⇒ Zero state response is also called

as Forced response.

We apply specific input and system

does not depend on initial conditions.

$$y[-1] = 0$$

means memory is zero  
i.e. Relax system.

Exps:- Find the Zero input response  
and Zero State response of the  
following DE.

$$y[n] - 3y[n-1] = x[n]$$

$$y[-1] = 1$$

$$x[n] = 4u[n]$$

Sol:-

$$y[n] - 3y[n-1] = x[n] \rightarrow \textcircled{1}$$

Zero input Response:-

$$\lambda^n - 3\lambda^{n-1} = 0$$

$$\lambda^{n-1} (\lambda - 3) = 0$$

$$\lambda - 3 = 0$$

$$\boxed{\lambda = 3}$$

$$y_n[n] = C(3)^n \rightarrow \textcircled{2}$$

$$y[-1] = C(3)^{-1} = 1$$

$$\frac{C}{3} = 1$$

$$\boxed{C = 3}$$

$$y_n[n] = 3(3)^n$$

$$\boxed{y_{z_i}[n] = 3(3)^n}$$

Zero state Response:-

$$y_p[n] = 4K u[n]$$

$$4K u[n] - 3(4)K u[n-1] = 4 u[n]$$

$$4K - 12K = 4$$

$$K(4-12) = 4$$

$$K \frac{(-8)}{-8} = \frac{4}{-8}$$

$$K = -1/2$$

$$y_p[n] = 4^2 \left(-\frac{1}{2}\right) u[n]$$

$$y_p[n] = -2$$

Particular solution.

Total solution

$$y[n] = C(3)^n - 2 \rightarrow \textcircled{2}$$

Applying condition Zs.

$$\text{i.e. } y[-1] = 0$$

$$y_{Zs}[-1] = C(3)^{-1} - 2 = 0$$

$$\frac{C}{3} - 2 = 0$$

$$C = (2)(3)$$

$$\boxed{C = 6}$$

Another Method:- $n=0$  in equation ①

$$y[0] - 3y[-1] = x[0] \rightarrow \textcircled{a}$$

Now

 $n=0$  in eqn (2)

$$y[0] = C(3)^0 - 2$$

$$y[0] = C - 2 \rightarrow \textcircled{b}$$

$$\textcircled{a} \Rightarrow y[0] - 3y[-1] = x[0]$$

$$y[0] = 4 \rightarrow \textcircled{c}$$

$$\therefore x[0] = 4n[0]$$

$$x[0] = 4$$

comparing  $\textcircled{b}$  &  $\textcircled{c}$ 

$$C - 2 = 4$$

$$\boxed{C = 6}$$