Lecture 08

Design of Reinforced Concrete Column

By: Prof Dr. Qaisar Ali

Civil Engineering Department

UET Peshawar

drqaisarali@uetpeshawar.edu.pk

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Organization of the Presentation

- The PowerPoint presentation of this lecture has been organized in such a manner that a "General Introduction" to the topic has been given in the beginning and the remaining portion has been divided into to parts. In part-I the design of concentrically loaded columns and in part-II design of eccentrically loaded columns have been discussed.

Contents

- General Introduction
- Design of RC Columns
  - Part-I
    - Concentrically loaded Columns
      - Mechanics
      - ACI Code Recommendations
      - Examples

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Contents

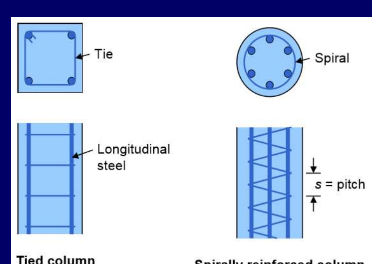
- Part-II
  - Eccentrically loaded Columns
    - Mechanics
    - Interaction Diagram
    - Use of Design Aids
    - Examples
    - Development of Interaction Diagram
    - Examples

Introduction

- Columns are defined as members that carry loads chiefly in compression.
- However, columns would generally carry bending moments as well, about one or both axes of the cross section.
- The bending action may produce tensile forces over a part of the cross section.
- Columns are generally referred to as compression members, because the compression forces dominate their behavior

Introduction

- **Types of RC columns according to the lateral (shear) reinforcement**
  1. Tied: columns reinforced with longitudinal bars (at least 4) and lateral ties. (stirrups in beams are called ties in columns)
  2. Spiral: columns reinforced with longitudinal bars (At least 6) and continuous spirals.

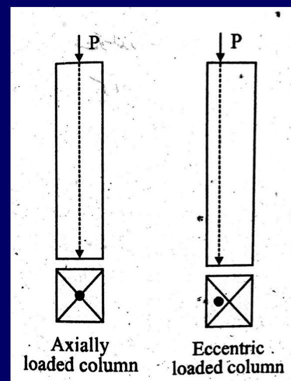


Introduction

- **Types of RC columns according to Loading**
  1. Concentrically/Axially loaded

     No eccentricity. Centre of gravity and centroid matches
  2. Eccentrically loaded

     eccentric load is applied.
     a. Uniaxially eccentric
     b. Biaxially eccentric



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Introduction

- **Types of RC columns according to slenderness**
- **Short Columns**
  - The failure in such columns occurs due to the failure of materials.
  - Most of the concrete columns falls in this category
- **Slender columns**
  - Failure in such columns occurs due to lateral deflections (buckling).

Introduction

- **Longitudinal/Main reinforcement**
  - The main reinforcement in columns is longitudinal, parallel to the direction of the load.
  - They are provided to resist bending moment and to take the compression

Introduction

- **Lateral ties**
  - The lateral ties are bars arranged in a square, rectangular, or circular pattern.
  - They are provided to resist buckling, to hold the main bars and to resist shear.
  - The continuous spiral contain/retain concrete, thus increasing the load taking capacity.

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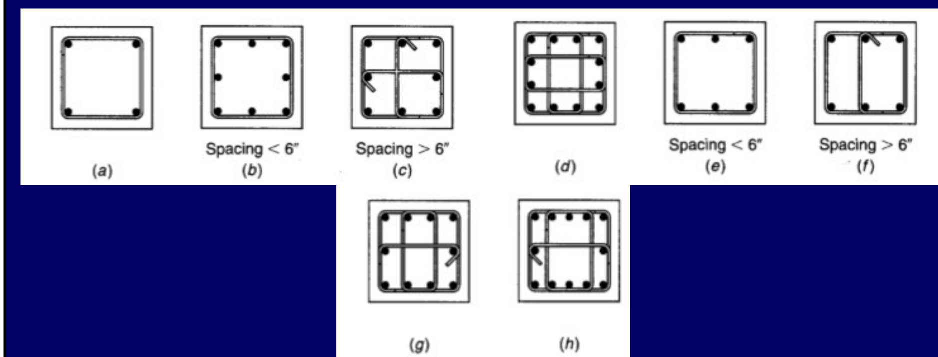
Introduction

- **Arrangement of lateral ties in square column**



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Design of RC Columns

Part-I

Concentrically Loaded Columns

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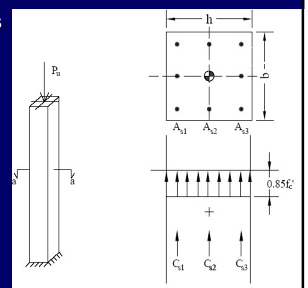
Mechanics

- Consider a Rectangular Section subjected to axial load P_u.
  - To avoid failure; $\Phi P_n \geq P_u$
  - Nominal Axial Capacity P_n can be calculated as follows;
    - $C_{s1} + C_{s2} + C_{s3} + C_c = P_n$
      - $C_{s1} = A_{s1} * f_{s1}$
      - $C_{s2} = A_{s2} * f_{s2}$
      - $C_{s3} = A_{s3} * f_{s3}$
      - $C_c = A_c * f_c$
    - $A_{s1} * f_{s1} + A_{s2} * f_{s2} + A_{s3} * f_{s3} + A_c * f_c = P_n$



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Mechanics

- The section will reach its axial capacity when strain in concrete reaches a value of 0.003.
- The yield strain values of steel for grade 40 and 60 are 0.00138 and 0.00207 respectively.
- Therefore steel would have already yielded at 0.003 strain. Hence $f_{s1} = f_{s2} = f_{s3} = f_{s4} = f_y$ and $f_c = 0.85 f_c'$
- Let $A_{s1} + A_{s2} + A_{s3} = A_{st}$ and $A_c = A_g - A_{st}$, where A_g = gross area of column section, A_{st} = total steel area
- **$A_{st} f_y + 0.85 f_c'(A_g - A_{st}) = P_n$** ------------(A)

Mechanics

- $P_n = A_{st} f_y + 0.85 f_c'(A_g - A_{st})$
- $\Phi P_n = P_u$
- As per ACI code (22.4.2.1), Φ = 0.65 for tied column and Φ = 0.75 for spiral column
- Additional reduction factors 'α' are used to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than f_c' under sustained high loads. **R22.4.2.1**

Mechanics

- **Axial capacity for tied columns**
  - $\alpha \Phi P_n = P_u$; α = 0.80, Φ = 0.65
  - $0.80 \times 0.65 [0.85f_c'(A_g - A_{st}) + f_y A_{st}] = P_u$
- **Axial capacity for spiral columns**
  - $\alpha \Phi P_n = P_u$; α = 0.85, Φ = 0.75
  - $0.85 \times 0.75 [0.85f_c'(A_g - A_{st}) + f_y A_{st}] = P_u$

ACI Code Recommendations

- **Ratio of longitudinal reinforcement**
  - According to ACI Code 10.6.1.1, for columns, area of longitudinal reinforcement shall be at least $0.01A_g$ but shall not exceed $0.08A_g$.
  - Most columns are designed with ratios below 0.04
  - Lower limit → To prevent failure mode of plain concrete
  - Upper limit → To maintain proper clearance between bars

$$0.01 \le \left[\rho_g = \frac{A_{st}}{A_g} \right] \le 0.08$$

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ACI Code Recommendations

- **Minimum number of bars**
  - According to ACI Code 10.7.3.1, the minimum number of longitudinal bars shall be;
    - A minimum of four longitudinal bars must be used when the bars are enclosed by spaced rectangular or circular ties
    - A minimum of six bars must be used when the longitudinal bars are enclosed by a continuous spiral.

ACI Code Recommendations

- **Spacing between bars**
  - According to ACI Code 25.2.3, For longitudinal reinforcement in columns, clear spacing between bars shall be at least the greatest of;
    - i. 1.5 in.
    - ii. $1.5d_b$
- **Clear cover to bars**
  - Cover shall be 1.5 in. minimum over primary reinforcement, ties or spirals

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ACI Code Recommendations

- **Maximum spacing (minimum reinforcement) of lateral ties**
  - According to ACI 25.7.2.1;

    Center-to-center spacing shall not exceed the least of;
    - i. $16d_b$ of longitudinal bar
    - ii. $48d_b$ of tie bar
    - iii. Smallest dimension of member

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ACI Code Recommendations

- **Minimum diameter of lateral ties**
  - According to ACI 25.7.2.2; Diameter of tie bar shall be at least (a) or (b):
    a. No. 3 enclosing No. 10 or smaller longitudinal bars
    b. No. 4 enclosing No. 11 or larger longitudinal bars or bundled longitudinal bars

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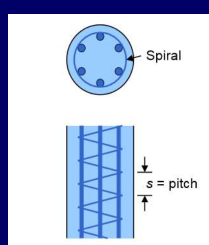
ACI Code Recommendations

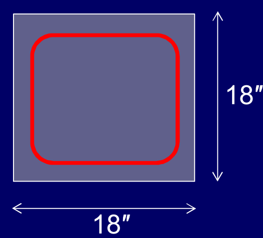
- **Spacing and diameter of spiral reinforcement**
  - According to ACI 25.7.3.1, Spacing/pitch of spiral must not be less than 1 in. and greater than 3 in.
  - According to ACI 25.7.3.2 the minimum spiral size is 3/8 in.



Spirally reinforced column

Examples

- Design a 18″ × 18″ tied column for a factored axial compressive load of 300 kips. The material strengths are f_c' = 3 ksi and f_y = 40 ksi.



Examples

- **Solution**
  - Nominal strength ($\alpha\Phi P_n$) of axially loaded column is:
    - $\alpha\Phi P_n = 0.80\Phi\{0.85f_c'(A_g - A_{st}) + A_{st} f_y\}$
      - $A_g = 18 \times 18 = 324$ in^2
      - Let A_{st} = 1% of $A_g = 0.01 \times 324 = 3.24$
      - $\alpha\Phi P_n = 0.80 \times 0.65 \times \{0.85 \times 3 \times (324 - 3.24) + 3.24 \times 40\}$

        = 492 kips > (P_u = 300 kips), O.K.

      Therefore, $A_{st} = 0.01 \times 324 = 3.24$ in^2

Examples

- **Solution**
  - Main Bars:
    - Using #6 bar, with bar area A_b = 0.44 in^2
    - No. of bars = A_s/A_b = 3.24 / 0.44 = 7.36 ≈ 8 bars
    - Use 8 #6 bars

Examples

- **Solution**
  - Tie Bars:
    - Using #3 bar, with bar area A_b = 0.11 in^2

      Center-to-center spacing shall not exceed the least of;

      i. 16d_b of longitudinal bar = 16 x 0.75 = 12″

      ii. 48d_b of tie bar = 48 x 3/8 = 18″

      iii. smallest dimension of member = 18″

      Therefore use #3 ties @ 12″ c/c

Examples

- Design a 24 inch circular spiral column to support an axial service dead load of 500 kips and an axial service live load of 230 kips. The material strengths are f_c' = 4.0 ksi and f_y = 60 ksi.

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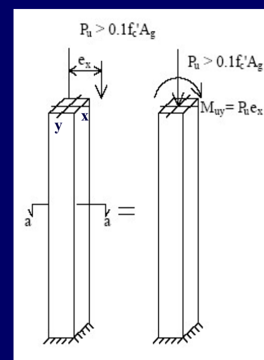
Design of RC Columns

Part-II

Eccentrically Loaded Columns

Mechanics

- Consider a rectangular column subjected to eccentric load P_u. The load will also produce bending moment M_u, therefore the column is subjected to an axial load P_u and bending moment $M_u = P_u\, e$
- To avoid failure; $\Phi P_n \geq P_u$



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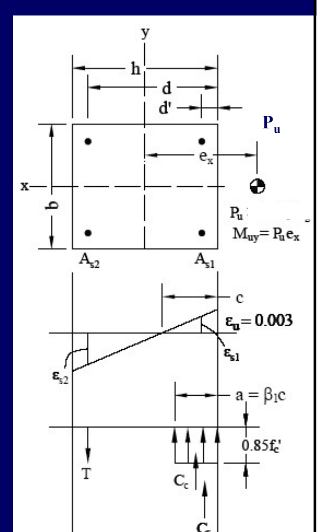
Mechanics

- **Nominal axial capacity P_n can be calculated as follows;**
- $P_n = C_c + C_s - T$

Where; $C_c = 0.85f_c'ab$, $C_s = A_{s1}f_{s1}$ and $T = A_{s2}\,f_{s2}$

- **$P_n = 0.85f_c'\,ba + A_{s1}\,f_{s1} - A_{s2}\,f_{s2}$ ------(1)**



Mechanics

- $\Phi M_n = M_u$

M_n will be calculated by taking moment about centroid

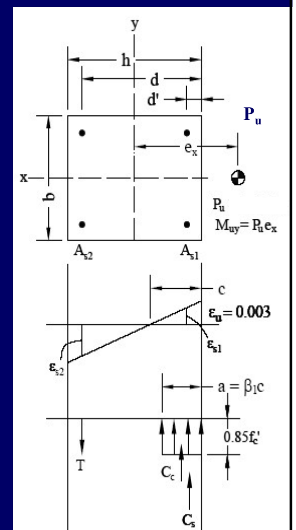
$M_n = C_c (h/2 – a/2) + C_s (h/2 - d') + T (h/2 - d')$

Where;

$C_c = 0.85f_c'ab$

$C_s = A_{s1}f_{s1}$ and

$T = A_{s2} f_{s2}$



Mechanics

- $M_u = \Phi M_n$
- $M_u = \Phi\, C_c (h/2 – a/2) + C_s (h/2 - d') + T (h/2 - d')$ -------(2a)

$A_{s1} = A_{s2} = A_s$

The equation (2a) becomes (2b) as follows:

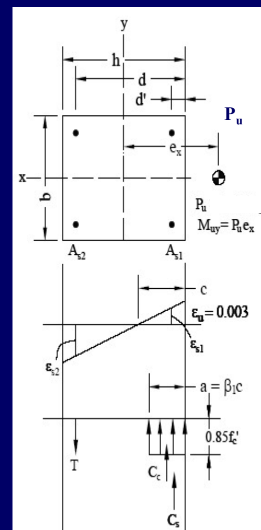
- **$M_u = \Phi\, [\, 0.425\, f_c'\, ab\, (h – a) + A_s \{h / 2 – d' \}\, (f_{s1} + f_{s2})]$------(2b)**

Mechanics

It is important to note that equation (1) & (2b) are valid for 2 layers of reinforcements only

- $P_u = \Phi\{0.85f_c'ab + A_{s1}f_{s1} - A_{s2}f_{s2}\}$ --------(1)
- $M_u = \Phi\ [\ 0.425\ f_c'\ ab\ (h - a) + A_s\{h / 2 - d'\}$ $(f_{s1} + f_{s2})]$------(2b)

For intermediate layers of reinforcement, the corresponding terms with "A_s" shall be added in the equations.



Mechanics

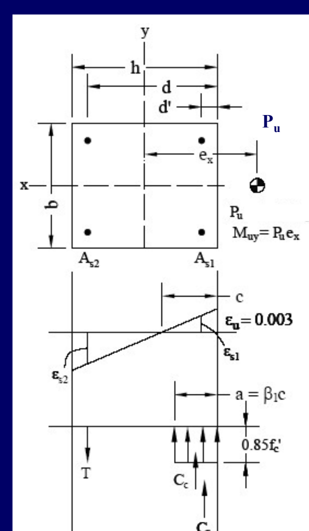
- The stress in compression (f_{s1}) and stress tensile (f_{s2}) in steel is calculated as follows;
- <u>Stress in Compression Steel:</u>

  $\varepsilon_{s1} / (c - d') = \varepsilon_u / c$

  $\varepsilon_{s1} = \varepsilon_u (c - d') / c$

  $f_{s1} = E_s\ \varepsilon_{s1}$

  $f_{s1} = E_s\ \{\varepsilon_u (c - d') / c\}$



Mechanics

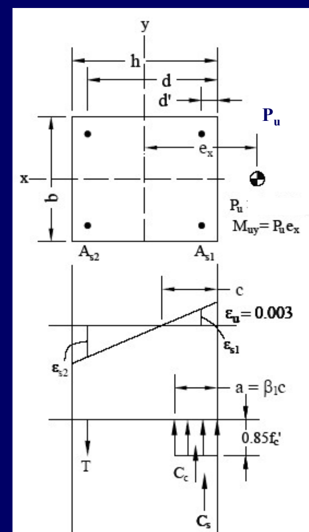
- Stress in Tensile Steel:

  $\varepsilon_{s2}/(d - c) = \varepsilon_u/c$

  $\varepsilon_{s2} = \varepsilon_u (d-c) / c$

  $f_{s2} = E_s\, \varepsilon_{s2}$

  $f_{s2} = E_s\, \{\varepsilon_u (d - c) / c\}$



Mechanics

- **Design by trial and success method**
  - As discussed in previous lectures, the singly reinforced flexural member can be designed by trial and success method using following formulae:
    - $A_s = M_u / \{\Phi f_y (d - a/2)\}$ & $a = A_s f_y / 0.85 f_c' b$
  - In the same way, equations (1) and (2b) may be used for design of RC member subjected to compressive load with uniaxial bending
    - $P_u = \Phi\{0.85 f_c' ab + A_{s1} f_{s1} - A_{s2} f_{s2}\}$ ....(1)
    - $M_u = \Phi[0.425 f_c' ab(h - a) + A_s\{(h/2) - d'\}(f_{s1} + f_{s2})]$ ....(2b)

Mechanics

- **Design by trial and success method**
  - However unlike equations for beam where $f_s = f_y$, here we don't know values of f_{s1} and f_{s2}. But we do know that steel stress shall be taken equal to or less than yield strength. Therefore
    - $f_{s1} = E\varepsilon_{s1} = 0.003E\ (c - d')/c \le f_y$
    - $f_{s2} = E\varepsilon_{s2} = 0.003E\ (d - c)/c \le f_y$
  - Equation (1) can be now written in the following form
  - $P_u = \Phi\ \{0.85f_c'\beta_1 cb + A_sE \times 0.003(c - d')/c - A_sE \times 0.003(d - c)/c\}$---(1)

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Mechanics

- **Design by trial and success method**
  - Equation (1) can be transformed into a quadratic equation to obtain the value of "c" for a particular demand P_u and assumed A_s:
    - $\Phi 0.85f_c'\ \beta_1 bc^2 + (\Phi 174A_s - P_u)c - \Phi 87A_s\ (d - d') = 0$
  - However such approach will not be convenient because the check that stresses in reinforcement layers f_{s1} and f_{s2} shall not exceed f_y can not be applied in the above equation.

Mechanics

- **Design by trial and success method**
  - As an example, with M_u = 40 ft-kip, P_u = 145 kips, A_s = 0.88 in^2, f_c' = 3 ksi, b = h = 12″, d = 9.5″ and d′ = 2.5″, c comes out to be 6.08″ from quadratic equation.
  - For c = 6.08″, now f_{s1} and f_{s2} shall be ≤ f_y
  - $f_{s1} = E\varepsilon_{s1} = 0.003E\,(c - d')/c$ = 51 ksi ; greater than 40 ksi
  - $f_{s2} = E\varepsilon_{s2} = 0.003E\,(d - c)/c$ = 49 ksi ; greater than 40 ksi

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Mechanics

- **Design by trial and success method**
  - It means that every time when we obtain value of c, we have to check stresses in steel and only that value of c will be used when f_{s1} and f_{s2} are ≤ f_y .
  - Therefore this method of trial and success will not work in members subjected to axial load and flexure together. We now look at another approach.

Mechanics

- **Alternative approach**
  - Instead of calculating c, we assume c and calculate ΦP_n and ΦM_n for a given set of data such as follows:
  - $\Phi P_n = \Phi\{0.85f_c'ab + A_sE \times 0.003(c - d')/c - A_sE \times 0.003(d - c)/c\}$
  - $\Phi M_n = \Phi\ [0.425f_c'\beta_1 c\ b\ (h - a) + A_s\ \{(h/2) - d'\}\ (f_{s1} + f_{s2})]$
  - For $A_s = 0.88$ in^2, $f_c' = 3$ ksi, b = h = 12″, d = 9.5″ and d′ = 2.5″ , all values in the above equations are known except "c".

Mechanics

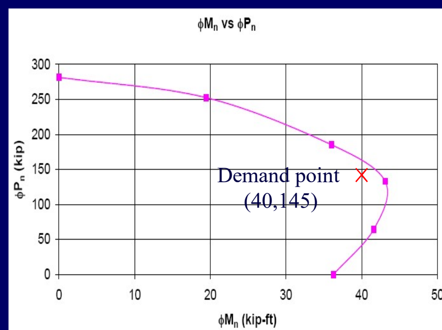
- **Alternative approach**
  - ΦP_n and ΦM_n are calculated for various values of "c" from 0 to h, with the check that during calculations f_{s1} and f_{s2} do not exceed f_y for both equations.

Table 4

c (in) $0 \le c \le (h = 12)$	ΦP_n (kips)	ΦM_n (kip-ft)
3.69	0	36.25
5	64.6	41.59
7	133	43.09
9	185.3	36
12	252.64	19.44
Axial capacity	281	0

Mechanics

- **Alternative approach**
  - Plot the values and check the capacity of the column for the demand equal to M_u = 40 ft-kip and P_u = 145 kips



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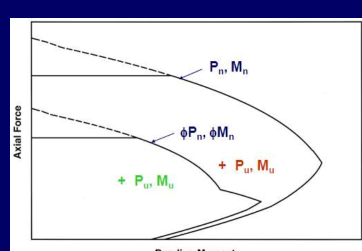
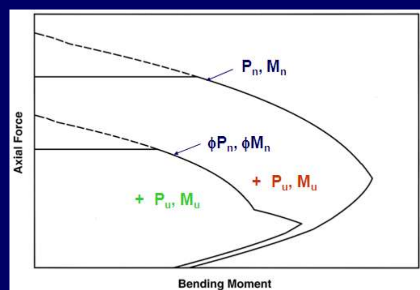
Interaction Diagram

- **General:**
  - For a column of known dimensions and reinforcement, several pairs of P and M from various values of "c" using equations 1 and 2b can be obtained and plotted as shown. Such a graph is known as capacity curve or interaction diagram. Nominal and Design diagram are given in the figure.



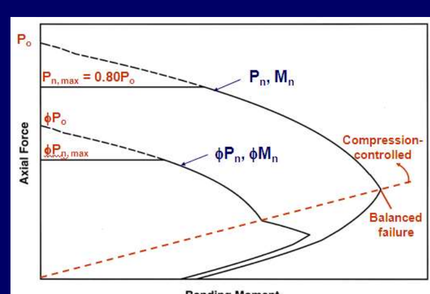
Interaction Diagram

- **General:**
  - If the factored demand in the form of P_u and M_u lies inside the design interaction diagram, the given column will be safe against that demand.



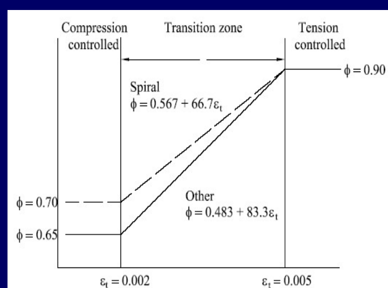
Interaction Diagram

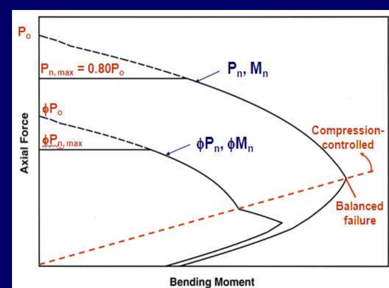
- **Important features of Interaction diagram:**
  - Horizontal Cutoff: The horizontal cutoff at upper end of the curve at a value of $\alpha\Phi P_{nmax}$ represents the maximum design load specified in the ACI 10.3.5 for small eccentricities i.e., large axial loads.



Interaction Diagram

- **Important features of Interaction diagram:**
  - Linear Transition of Φ from 0.65 to 0.90 is applicable for $\varepsilon_t \le f_y/E_s$ to $\varepsilon_t = 0.005$ respectively.

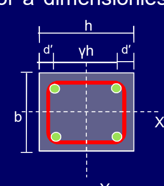




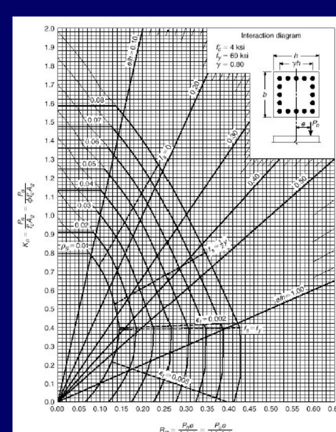
Use of Design Aids

- Use of Design Aids:
  - The uniaxial columns can be designed using design aids e.g, normalized interaction diagrams such as given in graph A5-A16 (Nilson). These diagrams require the calculation of a dimensionless constant γ.
    - $h = \gamma h + 2d'$

      $\gamma = (h - 2d')/h$



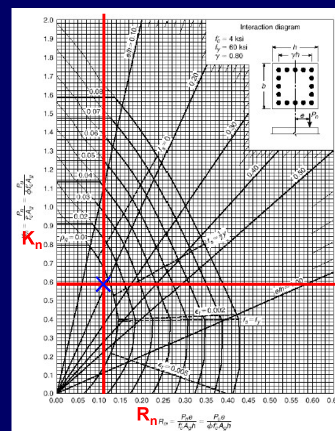
  - Once γ is calculated, the interaction diagram corresponding to the value of γ is selected & then column can be designed using steps given on the next slides.



Reference: Design of Concrete Structures 13th Ed. by Nilson, Darwin and Dolan.

Use of Design Aids

- Use of Design Aids: Graph A.5 to A.16 (Nilson)
  - Calculate $\gamma = (h - 2 \times d') / h$, select the relevant interaction diagram.
  - Given P_u, e, A_g, f_y, and f_c'
  - Calculate $K_n = P_u/(\Phi f_c' A_g)$
  - Calculate $R_n = P_u e/(\Phi f_c' A_g h)$
  - From the values of K_n & R_n, find ρ from the graph as shown.
  - $A_{st} = \rho A_g$



Reference: Design of Concrete Structures 13th Ed. by Nilson, Darwin and Dolan.

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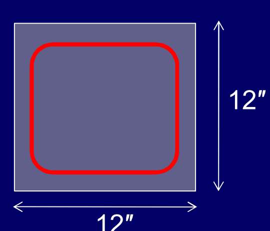
Examples

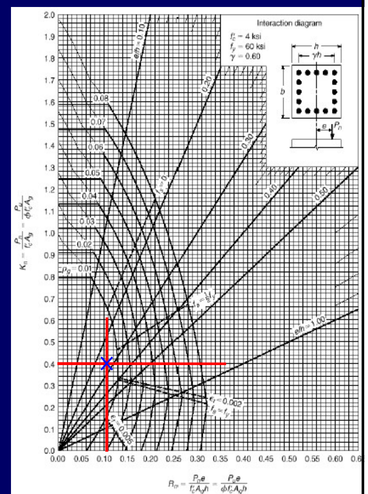
- Using design aids, design a 12″ square column to support factored load of 145 kip and a factored moment of 40 kip-ft. The material strengths are f_c' = 4 ksi and f_y = 60 ksi.



Examples

- Solution: Design Aids (using f_c' = 4 ksi and f_y = 60 ksi)
  - With d′ = 2.5 in, γ = (12 − 2 × 2.5)/12 = 0.60.
  - $K_n = P_u/(\Phi f_c' A_g)$ = 145/(0.65 × 4 × 144) = 0.40
  - $R_n = P_u e/(\Phi f_c' A_g h)$ = (40 × 12)/ (0.65 × 4 × 144 × 12) = 0.11
  - ρ = 0.007
  - A_{st} = 0.007 × 144 = 1.0 in2. < 1 % of A_g =1.44
  - Using #6 bar,

    No. of bars = A_{st}/A_b = 1.44/0.44 ≈ 4 bars



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Examples

- Using design aids, design a 15″ square column to support factored load of 200 kip and a factored moment of 80 kip-ft. The material strengths are f_c' = 4 ksi and f_y = 60 ksi.
- [Design aids will be provided in the examination]

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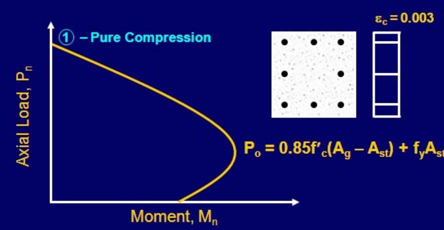
Development of Interaction Diagram

- Interaction diagram can be developed by calculation of certain points as discussed below:
  - Point 01: Point representing capacity of column when concentrically loaded.
  - This represents the point for which $M_n = 0$.



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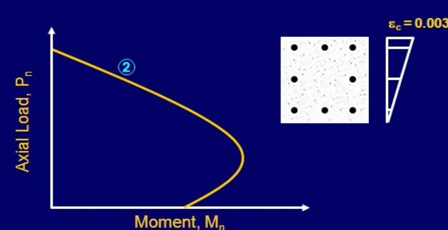
Development of Interaction Diagram

- Point 02: c = h
- Point 2 corresponds to crushing of the concrete at the compression face of the section and zero stress at the other face.



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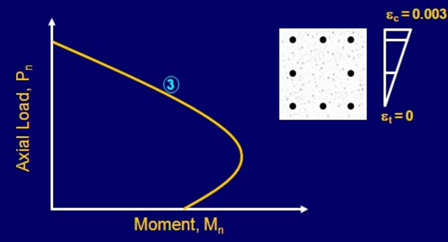
Development of Interaction Diagram

- Point 03: c = (h-d′)
- At Point 3, the strain in the reinforcing bars farthest from the compression face is equal to zero.



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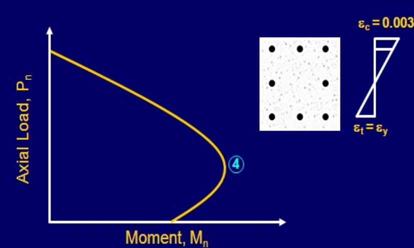
Development of Interaction Diagram

- Point 04: c = 0.68d (Grade 40)
  c = 0.58d (Grade 60)
- Point representing capacity of column for balance failure condition (ε_c = 0.003 and $\varepsilon_t = \varepsilon_y$).



$c = d\,\{\varepsilon_c / (\varepsilon_c + \varepsilon_y)\}$
$\varepsilon_c = 0.003$
$\varepsilon_y = 0.0013$ (Grade 40)
$\varepsilon_y = 0.0021$ (Grade 60)

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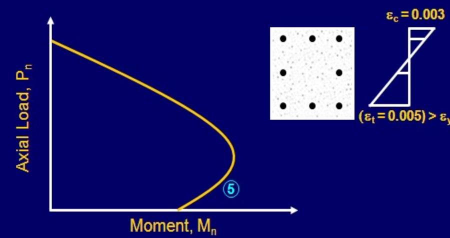
Development of Interaction Diagram

- Point 05: c = 0.375d
- Point in tension controlled region for net tensile strain (ε_t) = 0.005, and Φ = 0.90, (ε_c = 0.003).



$c = d\,\{\varepsilon_c / (\varepsilon_c + \varepsilon_t)\}$
$\varepsilon_c = 0.003$
$\varepsilon_t = 0.005$

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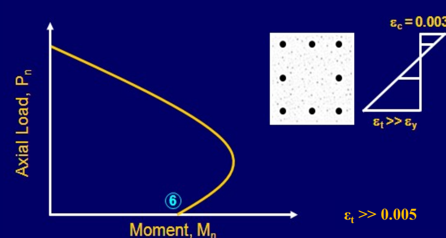
Development of Interaction Diagram

- Point 06: c = 0.23d
- Point on capacity curve for which ε_t >> 0.005 and ε_c = 0.003.



$c = d\,\{\varepsilon_c / (\varepsilon_c + \varepsilon_t)\}$
$\varepsilon_c = 0.003$
$\varepsilon_t >> 0.005$

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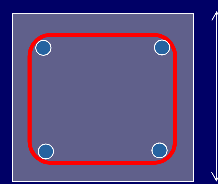
Examples

- Develop interaction diagram for the given column. The material strengths are $f_c' = 3$ ksi and $f_y = 40$ ksi with 4 no. 6 bars.



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Examples

- **Solution:**
  - Design interaction diagram will be developed by plotting (06) points as discussed earlier.
    - Point 1: Point representing capacity of column when concentrically loaded: Therefore
    - $\Phi P_n = \Phi\ [0.85f_c'(A_g - A_{st}) + f_yA_{st}]$

      $= 0.65 \times [0.85\times3\times(144 - 1.76) + 40 \times 1.76] = 281.52$ kip
    - $\Phi M_n = 0$

Examples

- **Solution:**
  - Point 2: c = h
    - c = 12 ″ (c = h); a = β_1c = 0.85 × 12 = 10.2″
    - f_{s1} = 0.003E (c – d′)/c = 0.003×29000(12 – 2.25)/12 = 70.69 ksi > f_y, use fy = 40 ksi.
    - f_{s2} = 0.003E (d – c)/c = 0.003×29000(9.75 – 12)/12 = -16.31 ksi< f_y
    - Therefore, $\Phi P_n = \Phi$ {0.85f_c′ab + A_s f_{s1} – $A_s f_{s2}$}
      = 0.65{0.85×3×10.2×12 +0.88×40+0.88×16.31} = 235.09 kip
    - $\Phi M_n = \Phi$ [0.425f_c′ab (h – a) + A_s {(h/2) – d′} (f_{s1} + f_{s2})]
      = 0.65[0.425×3×10.2×12×(12–10.2)+0.88×{(12/2) – 2.25}(40-16.31)]
      = 233.41 in-kip = 19.45 ft-kip

Examples

- **Solution:**
  - Point 3: c = (h-d ′)
    - c =12-2.25=9.75; a = β_1c = 0.85 × 9.75 = 8.29″
    - f_{s1} = 0.003E (c – d′)/c = 0.003×29000(9.75 – 2.25)/9.75 = 66.92 ksi > f_y, use fy = 40 ksi.
    - f_{s2} = 0.003E (d – c)/c = 0.003×29000(9.75 – 9.75)/9.75 = 0 ksi< f_y
    - Therefore, $\Phi P_n = \Phi$ {0.85f_c′ab + A_s f_{s1} – $A_s f_{s2}$}
    - = 0.65{0.85×3×8.29×12 +0.88×40} = 187.77 kip
    - $\Phi M_n = \Phi$ [0.425f_c′ab (h – a) + A_s {(h/2) – d′} (f_{s1} + f_{s2})]
      = 0.65[0.425×3×8.29×12×(12–8.29)+0.88×{(12/2) – 2.25}(40)]
      = 391.67 in-kip = 32.64 ft-kip

Examples

- **Solution:**
  - Point 4: Point representing balance failure: The neutral axis for the balanced failure condition is easily calculated from $c = d\ \{\varepsilon_u/ (\varepsilon_u + \varepsilon_y)\}$ with ε_u equal to 0.003 and ε_y = 40/29000 = 0.001379, c = 0.68d
    - $c_b = d\ \{\varepsilon_u/ (\varepsilon_u + \varepsilon_y)\}$ = 9.75 × 0.003/ (0.003 + 0.001379) = 0.68d = 6.68″ giving a stress-block depth;

      $a_b = \beta_1 c_b$ = 0.85 × 6.68 = 5.67″

Examples

- **Solution:**
  - Point 4: Balance failure: For the balanced failure condition, $f_s = f_y$.
    - $f_{s1} = 0.003E\ (c - d')/c$ = 0.003×29000(6.68–2.25)/6.68= 57.69 ksi > f_y,
    - $f_{s2} = 0.003E\ (d - c)/c$ = 0.003×29000(9.75 – 6.68)/6.68 = 40 ksi = f_y
    - Therefore, $\Phi P_b = \Phi\ \{0.85f_c'ab + A_s f_{s1} - A_s f_{s2}\}$

      = 0.65{0.85×3×5.67×12 +0.88×40–0.88×40} = 112.77 kip
    - $\Phi M_b = \Phi\ [0.425f_c'ab\ (h - a) + A_s\ \{(h/2) - d'\}\ (f_{s1} + f_{s2})]$

      = 0.65[0.425×3×5.67×12×(12–5.67)+0.88×{(12/2) – 2.25}(40 + 40)]

      = 528.54 in-kip = 44.05 ft-kip

Examples

- **Solution:**
  - Point 5: This point is in tension controlled region for which ε_t = 0.005, Φ = 0.90:
    - For ε_t = 0.005; c = d {ε_u/ (ε_u + ε_t)} = 9.75× {0.003/ (0.003 + 0.005)} = 0.375d = 3.66″
    - a = β_1c = 0.85 × 3.66 = 3.11″
    - f_{s1} = 0.003E (c – d′)/c = 0.003×29000(3.66 – 2.25)/3.66 = 33.51 ksi < f_y
    - f_{s2} = 0.003E (d – c)/c = 0.003×29000(9.75 – 3.66)/3.66 = 144.76 ksi > f_y, use f_y = 40 ksi.
    - Therefore, ΦP_n = Φ{0.85f_c′ab + A_sf_{s1} – A_sf_{s2}}

      = 0.90{0.85×3×3.11×12 +0.88×33.51–0.88×40}= 80.50 kip
    - ΦM_n = Φ [0.425f_c′ab (h – a) + A_s {(h/2) – d′} (f_{s1} + f_{s2})]
    - = 0.90[0.425×3×3.11×12×(12–3.11)+0.88×{(12/2) – 2.25}(33.51+40)]

      = 599 in-kip = 49.91 ft-kip

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Examples

- **Solution:**
  - Point 6: Point on capacity curve for which εt >> 0.005:
    - Let ε_t = 2 × 0.005 = 0.01; c = d {ε_u/ (ε_u + ε_t)} = 9.75× {0.003/ (0.003 + 0.01)} = 0.23d = 2.25″
    - a = β1c = 0.85 × 2.25 = 1.91″
    - fs1 = 0.003E (c – d′)/c = 0.003×29000(2.25 – 2.25)/2.25 = 0 < fy
    - fs2 = 0.003E (d – c)/c = 0.003×29000(9.75 – 2.25)/2.25 = 290 ksi > fy, use fy = 40 ksi.
    - Therefore, ΦPn = Φ{0.85fc′ab + A_sf_{s1} – A_sf_{s2}

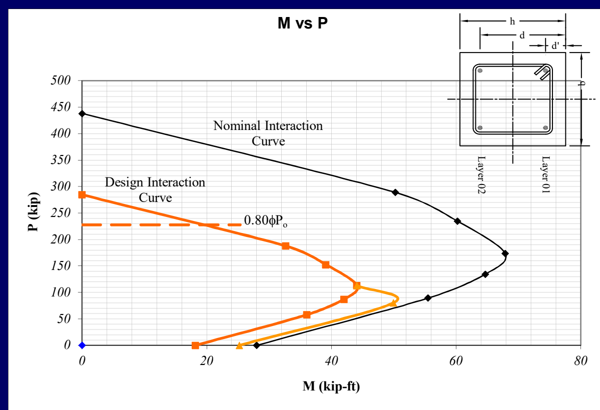
      = 0.90{0.85×3×1.91×12 +0.88×0 – 0.88×40} = 20.90 kip
    - ΦMn = Φ [0.425fc′ab (h – a) + As {(h/2) – d′} (f_{s1} + f_{s2})]

      = 0.90[0.425×3×1.91×12×(12–1.91)+0.88×{(12/2) – 2.25}(0 +40)

      = 384.16 in-kip = 32.01 ft-kip

Examples

- **Solution:**



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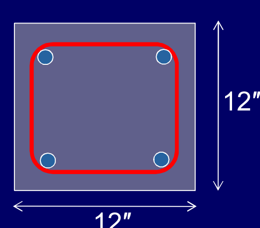
Examples

- Develop interaction diagram for the given column. The material strengths are f_c' = 3 ksi and f_y = 60 ksi with 4 no. 8 bars.



References

- Design of Concrete Structures 14^{th} / 15^{th} edition by Nilson, Darwin and Dolan.
- ACI 318-14