Solution of Linear Constant Difference Equation:-

There are two methods used for solutions of Linear Const D.E.

(i) Direct Method

(ii) Indirect Method (By using Z-Transform)

(i) Direct Method:-

This method is called Direct Method because it is almost same as Differential method actually not been diff eqtn is used for continuous but some how.

Total solution is the sum of

the following two cases:-

(a) Homogeneous and Particular Solution

(b) Zero input and Zero state solution.

(a) Homogeneous and Particular Solution:-

$$y[n] = y_n[n] + y_p[n]$$

Total Solution = Sum of (Homog + Particular Sol)

Steps → (i) characteristic Equation
→ (ii) Roots

Exple:-

Find Homogenous solution of

$$y[n] + a_1 y[n-1] = x[n]$$

characteristic Eqtn:

$$\lambda^n + a_1 \lambda^{n-1} = 0$$

∴ Make the input = zero

$$\lambda^{n-1}(\lambda + a_1) = 0$$

$$\lambda + a_1 = 0$$

$$\boxed{\lambda = -a_1} \text{ root}$$

∴ Roots depends on order of the system.

Root is real and non-repeated

So, we have

$$y_n[n] = C_1 \lambda^n + C_2 \lambda^{n_1} + \cdots$$

but here only one root

so,

$$y_n[n] = C_1 \lambda^n$$

$$\boxed{y_n[n] = C_1 (-a)^n}$$

∴ If roots are repeated and real

$$y[n] = C_1 \lambda^n + C_2 n \lambda^n + C_3 n^2 \lambda^n + \cdots$$

Exp:- $y[n] + a_1 y[n-1] = u[n]$

Find the particular solution?

Sol:-

Suppose $y_p[n] = K u[n]$

$$K \cancelto{1}{u[n]} + a_1 K \cancelto{1}{u[n-1]} = \cancelto{1}{u[n]}$$

$$K + a_1 K = 1$$

$$K(1 + a_1) = 1$$

$$K = \frac{1}{1+a_1}$$

$$y_p[n] = \frac{1}{1+a_1} u[n]$$

Exp: $y[n] + y[n-1] - 6y[n-2] = x[n]$

For unit step ; $x[n] = 8u[n]$

with $y[-1] = 1$

$y[-2] = -1$

Sol:-

Homogeneous Sol:-

$$\lambda^n + \lambda^{n-1} - 6\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 + \lambda - 6) = 0$$

$$\lambda^2 + \lambda - 6 = 0 \; ; \; \lambda^{n-2} = 0$$

$$\lambda^2 + 3\lambda - 2\lambda - 6 = 0$$

$$\lambda(\lambda+3) - 2(\lambda+3) = 0$$

$$(\lambda+3) = 0 \; ; \; (\lambda-2) = 0$$

$$\boxed{\lambda = -3 \; ; \; 2}$$

$\because \dfrac{\lambda^n}{\lambda^{n-2}}$

$\lambda^{\cancel{n}-\cancel{n}+2}$

λ^2

$\dfrac{\lambda^{n-1}}{\lambda^{n-2}}$

$\lambda^{\cancel{n}-1-\cancel{n}+2}$

λ

Roots are real & non repeated.

$$y_h[n] = C_1 \lambda_1^n + C_2 \lambda_2^n$$

$$= C_1 (-3)^n + C_2 (2)^n$$

(*) Particular Solution:-

$$y_p[n] = 8Ku[n]$$

$$8Ku[n] + 8Ku[n-1] - 6(8)Ku[n-2] = 8u[n]$$

$$8Ku[n] + 8Ku[n-1] - 48Ku[n-2] = 8u[n]$$

$$8K + 8K - 48K = 8$$

$$16K - 48K = 8$$

$$-32K = 8$$

$$\boxed{K = -1/4}$$

$y_p[n] = {}^{2}8(-1/4)\,u[n]$

$y_p[n] = -2\,u[n]$

$$\boxed{y_p[n] = -2}$$

Total Solution $y[n] = C_1(-3)^n + C_2(2)^n - 2$

$\therefore$ Total Solution = Sum of Homog + Sum of Pat Sol.

Now applying initial conditions:-

$y[-1] = C_1(-3)^{-1} + C_2(2)^{-1} - 2 = 1$

$y[-2] = C_1(-3)^{-2} + C_2(2)^{-2} - 2 = -1$

$$\boxed{C_1 = -1.8}$$

DYS

$$\boxed{C_2 = 4.8}$$

Putting this in Total Solution.

$$y[n] = (-1.8)(-3)^n + (4.8)(2)^n - 2$$

Now for total response

at $n = 2$

$$y[2] = (-1.8)(-3)^2 + (4.8)(2)^2 - 2$$

$$\boxed{y[2] = 22.6}$$

∴ We also plot the amplitude by putting different values up to 4. So we have

$y[0]$, $y[1]$, $y[2]$, $y[3]$

——— x ——— x ———