

⊕ Solution of Linear Constant Difference Equation:-

There are two methods used for solutions of Linear Const DE.

- ① Direct Method
- ② Indirect Method (By using Z-Transform)

① Direct Method:-

This method is called Direct Method because it is almost same as

Differential method actually not

been diff eqn is used for

continuous but some how.

Total solution is the sum of

the following two cases:-

- (a) Homogeneous and Particular Solution
- (b) Zero input and Zero state Solution.

(a) Homogeneous and Particular Solution:-

$$y[n] = y_h[n] + y_p[n]$$

Total Solution = Sum of (Homog + Particular Sol)

Steps → (i) Characteristic Equation
(ii) Roots

Exple:-

Find Homogeneous Solution of

$$y[n] + a_1 y[n-1] = x[n]$$

Characteristic Eqn:

$$\lambda^n + a_1 \lambda^{n-1} = 0$$

∴ Make the input = zero

$$\lambda^{n-1} (\lambda + a_1) = 0$$

$$\lambda + a_1 = 0$$

$$\boxed{\lambda = -a_1} \text{ root}$$

∴ Roots depends on order of the system.

Root is real and non-repeated

So, we have

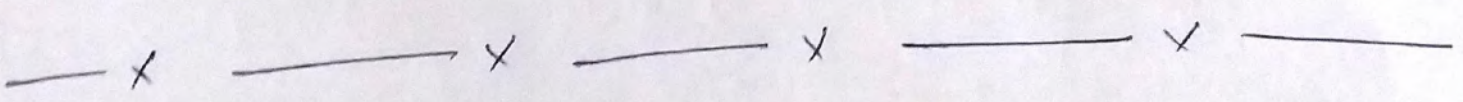
$$y_n[n] = C_1 \lambda^n + C_2 \lambda^{n_1} + \dots$$

but here only one root

So,

$$y_n[n] = C_1 \lambda^n$$

$$\boxed{y_n[n] = C_1 (-a)^n}$$



∴ If roots are repeated and real.

$$y[n] = C_1 \lambda^n + C_2 n \lambda^n + C_3 n^2 \lambda^n + \dots$$

Exp:- $y[n] + a_1 y[n-1] = u[n]$

Find the particular solution?

Sol:-

Suppose $y_p[n] = K u[n]$

$$K u[n] + a_1 K u[n-1] = u[n]$$

$$K + a_1 K = 1$$

$$K(1 + a_1) = 1$$

$$K = \frac{1}{1 + a_1}$$

$$y_p[n] = \frac{1}{1 + a_1} u[n]$$

_____ x _____ x _____

Ex: $y[n] + y[n-1] - 6y[n-2] = x[n]$

For unit step; $x[n] = 8u[n]$

with $y[-1] = 1$

$y[-2] = -1$

Sol:

Homogeneous Sol:

$$\lambda^n + \lambda^{n-1} - 6\lambda^{n-2} = 0$$

$$\lambda^{n-2} (\lambda^2 + \lambda - 6) = 0$$

$$\lambda^2 + \lambda - 6 = 0 ; \lambda^{n-2} = 0$$

$$\lambda^2 + 3\lambda - 2\lambda - 6 = 0$$

$$\lambda(\lambda + 3) - 2(\lambda + 3) = 0$$

$$(\lambda + 3) = 0 ; (\lambda - 2) = 0$$

$$\boxed{\lambda = -3 ; 2}$$

$$\begin{array}{r} \lambda^n \\ \lambda^{n-2} \\ \hline \lambda^{n-1} \\ \lambda^{n-2} \\ \hline \lambda^{n-1} \\ \lambda^{n-2} \\ \hline \lambda^{n-1-n+2} \\ \lambda \end{array}$$

Roots are real & non repeated.

$$\begin{aligned}
 Y_n[n] &= C_1 \lambda_1^n + C_2 \lambda_2^n \\
 &= C_1 (-3)^n + C_2 (2)^n
 \end{aligned}$$

④ Particular Solution:-

$$Y_p[n] = 8Ku[n]$$

$$\begin{aligned}
 8Ku[n] + 8Ku[n-1] - 6(8)Ku[n-2] \\
 = 8u[n]
 \end{aligned}$$

$$8Ku[n] + 8Ku[n-1] - 48Ku[n-2] = 8u[n]$$

$$8K + 8K - 48K = 8$$

$$16K - 48K = 8$$

$$-32K = 8$$

$$K = -1/4$$

$$y_p[n] = 2.8 \left(-\frac{1}{4}\right) u[n]$$

$$y_p[n] = -2 u[n]$$

$$y_p[n] = -2$$

$$\text{Total Solution } y[n] = C_1 (-3)^n + C_2 (2)^n - 2$$

$$\therefore \text{Total Solution} = \text{Sum of Homog} + \text{Sum of Part Sol.}$$

Now applying initial conditions:-

$$y[-1] = C_1 (-3)^{-1} + C_2 (2)^{-1} - 2 = 1$$

$$y[-2] = C_1 (-3)^{-2} + C_2 (2)^{-2} - 2 = -1$$

$$C_1 = -1.8$$

$$C_2 = 4.8$$

DYS
=

Putting this in Total solution.

$$y[n] = (-1.8)(-3)^n + (4.8)(2)^n - 2$$

Now for total response
at $n=2$.

$$y[2] = (-1.8)(-3)^2 + (4.8)(2)^2 - 2$$

$$y[2] = 22.6$$

\therefore We also plot the amplitude
by putting different values
up to 4. So we have

$$y[0], y[1], y[2], y[3]$$

_____ x _____ x _____