Lecture 06

Design of Reinforced Concrete Beam for Shear

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Topics Addressed

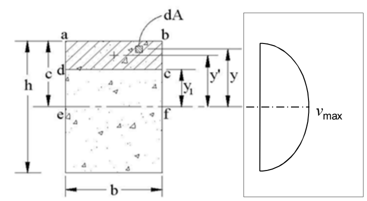
- Shear Stresses in Rectangular Beams
- Diagonal Tension in RC Beams subjected to Flexure and Shear Loading
- Types of Cracks in Reinforced Concrete Beam
- Shear Strength of Concrete
- Web Reinforcement Requirement in Reinforced Concrete Beams
- ACI Code Provisions for Shear Design
- Example

Shear Stresses in Rectangular Beams

- **Homogeneous Elastic Rectangular Beams**

  The shear stress (v) at any point in the cross section is given by

$$v = \frac{VQ}{Ib}$$



Where; V = total shear at section
I = moment of inertia of cross section about neutral axis
b = width of beam at a given point
Q = statical moment about neutral axis of that portion of cross section lying between a line through point in question parallel to neutral axis and nearest face (upper or lower) of beam

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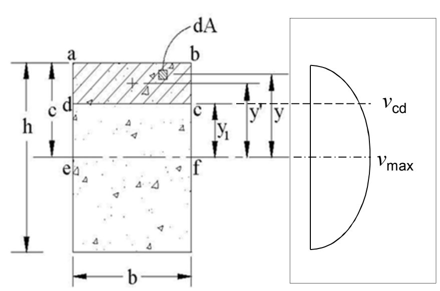
Shear Stresses in Rectangular Beams

- **Homogeneous Elastic Rectangular Beams**

  For the calculation of shear stress at level cd in the given figure, Q will be equal to A_{abcd} y′ , where A_{abcd} is area abcd and y′ is the moment arm.



Shear Stresses in Rectangular Beams

- **Homogeneous Elastic Rectangular Beams**

  For shear at neutral axis ef:

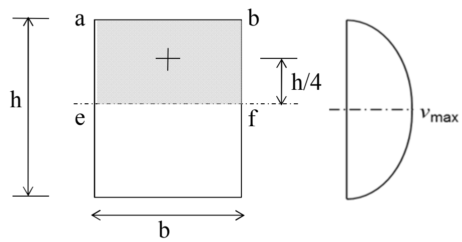
  $Q = A_{abef} \times h/4 = bh^2/8$

  $I = bh^3/12$

  Therefore,

  $v = VQ/Ib = V(bh^2/8)/\{(bh^3/12)\times b\}$

  $v_{max} = (3/2)V/bh$



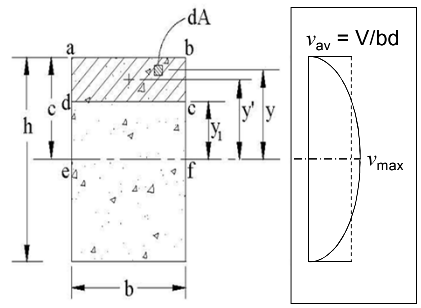
Shear Stresses in Rectangular Beams

- **RC Beams in Non-Linear Inelastic Range**
  - When load on the beam is such that stresses are no longer proportional to strain, then equation $v = VQ/Ib$ for shear stress calculation does not govern.
  - The exact distribution of shear stresses over the depth of reinforced concrete member in such a case is not fully known.
  - The shear stresses in reinforced concrete member cannot be computed from $v = VQ/Ib$, because this equation does not account for the influence of the reinforcement and also because concrete is not an elastic homogeneous material.

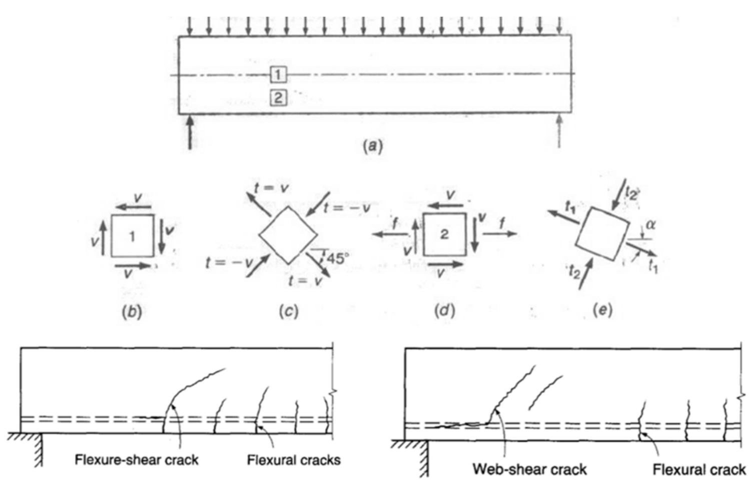
Shear Stresses in Rectangular Beams

- **RC Beams in Non-Linear Inelastic Range**
  - Tests have shown that the average shear stress prior to crack formation is:
    - v_{av} = V/ bd

The value computed from above equation must therefore be regarded merely as a measure of the average intensity of shear stresses in the section. The maximum value, which occurs at the neutral axis, will exceed this average by an unknown but moderate amount.

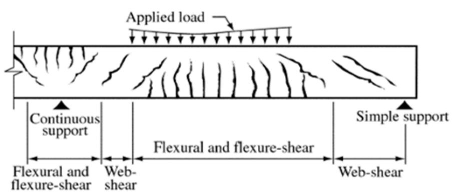


Diagonal Tension in RC Beams subjected to Flexure and Shear Loading



Types of Cracks in Reinforced Concrete Beam

- **These are of two types**
  1. Flexure Cracks
  2. Diagonal Tension Cracks
     I. **Web-shear cracks:** These cracks are formed at locations where flexural stresses are negligibly small.
     II. **Flexure shear cracks:** These cracks are formed where shear force and bending moment have large values.



Types of Cracks in Reinforced Concrete Beam

Diagonal Tension in RC Beams subjected to Flexure and Shear Loading

- **Conclusions from Previous Discussion**
  - The tensile stresses are not confined to horizontal bending stresses that are caused by bending alone.
  - Tensile stresses of various inclinations and magnitudes resulting from shear alone (at the neutral axis) or from the combined action of shear and bending, exist in all parts of a beam and can impair its integrity if not adequately provided for.
  - It is for this reason that the inclined tensile stresses, known as *diagonal tension stress* must be carefully considered in reinforced concrete design.

Shear Strength of Concrete

- **Tensile Strength of Concrete**
  - The direct tensile strength of concrete ranges from 3 to $5\sqrt{f_c'}$.
- **Shear Strength of Concrete in Presence of Cracks**
  - A large number of tests on beams have shown that in regions where small moment and large shear exist (web shear crack location) the nominal or average shear strength is taken as $3.5\sqrt{f_c'}$.
  - However in the presence of large moments (for which adequate longitudinal reinforcement has been provided), the nominal shear strength corresponding to formation of diagonal tension cracks can be taken as: $$v_{cr} = 2\sqrt{f_c'}$$

Shear Strength of Concrete

- **Shear Strength of Concrete in Presence of Cracks**
  - The same has been adopted by ACI code (refer to ACI 22.5.2.1).
  - This reduction of shear strength of concrete is due to the pre-existence of flexural cracks.
  - It is important to mention here that this value of shear strength of concrete exists at the ultimate i.e., just prior to the failure condition.

Web Reinforcement Requirement in Reinforced Concrete Beams

- **Nominal Shear Capacity (V_n) of Reinforced Concrete Beam**
  - Now general expression for shear capacity of reinforced concrete beam is given as:

    $V_n = V_c + V_s$ (ACI 22.5.1.1)

    Where,

    V_c = Nominal shear capacity of concrete,

    V_s = Nominal shear capacity of shear reinforcement.

    Note: in case of flexural capacity $M_n = M_c + M_s$ ($M_c = 0$, at ultimate load)

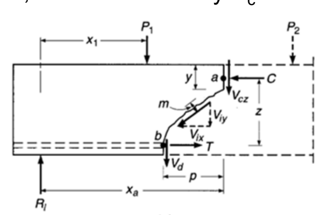
Web Reinforcement Requirement in Reinforced Concrete Beams

- **Nominal Shear Capacity (V_n) of Reinforced Concrete Beam**
  - Unlike moment, in expression for shear, the term $V_c \neq 0$ because test evidence have led to the conservative assumption that just prior to failure of a web-reinforced beam, the sum of the three internal shear components is equal to the cracking shear V_{cr}.

Web Reinforcement Requirement in Reinforced Concrete Beams

- **Nominal Shear Capacity (V_n) of Reinforced Concrete Beam**
  - This sum is generally (somewhat loosely) referred to as the contribution of the concrete to the total shear resistance, and is denoted by V_c. Thus

    $V_c = V_{cz} + V_d + V_{iy}$

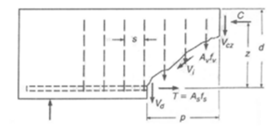


Where,

V_{iy} = Vertical component of sizable interlock forces.
V_{cz} = Internal vertical forces in the uncracked portion of the concrete.
V_d = Internal vertical forces across the longitudinal steel, acting as a dowel.

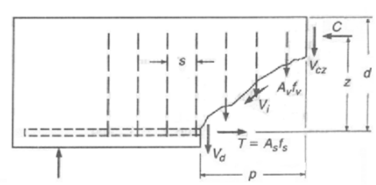
Web Reinforcement Requirement in Reinforced Concrete Beams

- **Nominal Shear Capacity (V_n) of Reinforced Concrete Beam**
  - $\mathbf{V_c}$ = $V_{cz} + V_d + V_{iy}$ = **2√ (f_c') $b_w d$** [ACI 22.5.5.1]
  - $V_n = V_c + V_s$
  - where $V_s = n A_v f_y$
  - The number of stirrups 'n' spaced a distance 's' apart depend on the length 'p' of the horizontal projection of the diagonal crack, as shown, hence, n = p/s. This length 'p' is conservatively assumed to be equal to the effective depth 'd' of the beam; thus **n = d/s**
  - Therefore; $V_s = A_v f_y d / s$ (ACI 22.5.10.5.3)



Web Reinforcement Requirement in Reinforced Concrete Beams

- **Nominal Shear Capacity (V_n) of Reinforced Concrete Beam**
  - $\Phi V_n = \Phi V_c + \Phi Vs$
  - To avoid shear failure $\Phi V_n \geq V_u$
  - So, $V_u = \Phi V_c + \Phi Vs$
  - $V_u = \Phi V_c + \Phi A_v f_y d / s$
  - $s = \dfrac{\Phi \, Av f_y d}{(Vu - \Phi \, V_c)}$



- **Strength Reduction Factor**
  - The strength reduction factor Φ is to be taken equal to 0.75 for shear. (ACI 21.2.2)

ACI Code Provisions for Shear Design

- **Location of Critical Section for Shear Design**

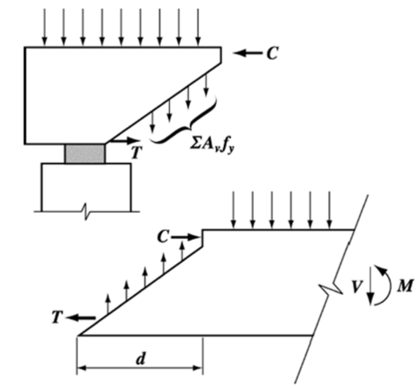


Fig. R11.1.3.1(a)—Free body diagrams of the end of a beam

In most of the cases, for the design of shear, critical shear is taken at a distance d from the support instead of maximum shear at the face of the support

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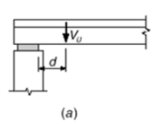
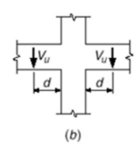
ACI Code Provisions for Shear Design

- **Location of Critical Section for Shear Design**

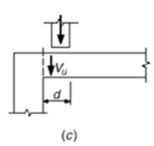
Typical support conditions

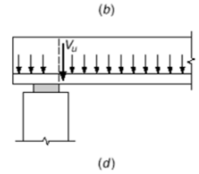




Typical support conditions

Concentrated load within distance “d”





A beam loaded near its bottom edge, e.g. inverted T-beam

End of a monolithic vertical element

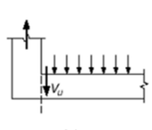


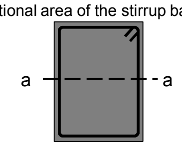
Fig a and b are the more frequent conditions while the fig c, d and e are the special conditions

ACI Code Provisions for Shear Design

1. When $\Phi V_c/2 > V_u$, no web reinforcement is required.
2. When $\Phi V_c/2 < V_u$ but $\Phi V_c \geq V_u$, theoretically no web reinforcement is required. However ACI 22.5 recommends that minimum web reinforcement in the form of Maximum spacing s_{max} shall be minimum of:
   - $A_v f_y/(50b_w)$,
   - d/2
   - 24 inches
   - $A_v f_y / \{0.75\sqrt{f'_c}\, b_w\}$

A_v is the cross sectional area of web reinforcement within a distance s, for single loop stirrups (2 legged), $A_v = 2A_s$

A_s = cross sectional area of the stirrup bar



At section a-a, if #3 bar is used $A_s = 0.11$ in^2,

$A_v = 2 \times 0.11 = 0.22$ in^2

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ACI Code Provisions for Shear Design

3. When $V_u > \Phi V_c$, web reinforcement is required:

Required Spacing, $s_d = \Phi A_v f_y d / (V_u - \Phi V_c)$

If $s_d > s_{max}$; use s_{max}

Maximum Spacing s_{max} is minimum of:

- $A_v f_y/(50b_w)$,
- d/2
- 24 inches
- $A_v f_y / \{0.75\sqrt{f'_c}\, b_w\}$

ACI Code Provisions for Shear Design

4. **Check for Depth of Beam:**
   - if $\Phi V_s \leq \Phi 8\sqrt{f'_c}b_w d$ (ACI 22.5.1.2), depth of beam ok

     If $\Phi V_s > \Phi 8\sqrt{f'_c}b_w d$, increase depth of beam.
5. **Check for Spacing:**
   - if $\Phi V_s \leq \Phi 4\sqrt{f'_c}\, b_w d$ (ACI 9.7.6.2)

     Spacing provided is ok.
   - if $\Phi V_s > \Phi 4\sqrt{f'_c}\, b_w d$

     Reduce spacing by one half.

ACI Code Provisions for Shear Design

- **Spacing Requirements for Stirrups**

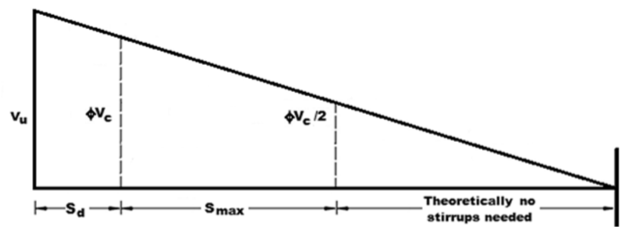
Table: Provisions for Shear Design

		$V_u \leq \phi V_c/2$	$\phi V_c \geq V_u > \phi V_c/2$	$V_u > \phi V_c$
Required area of stirrups, A_v		none	$0.75\sqrt{f'_c}\frac{b_w s}{f_y} \geq \frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{0.75\sqrt{f'_c}\, b_w} \leq \frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Maximum	—	$d/2 \leq 24$ in.	$d/2 \leq 24$ in. for $(V_u - \phi V_c) \leq \phi 4\sqrt{f'_c}\, b_w d$
				$d/4 \leq 12$ in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$

Note: $\Phi V_s \leq \Phi\ 8\ \sqrt{f'c}\ b_w d$; otherwise increase the depth

ACI Code Provisions for Shear Design

Placement of Shear Reinforcement

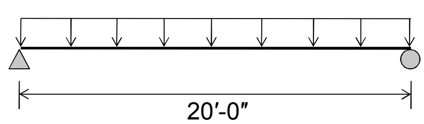


When $\Phi V_c < V_u$, use Design Spacing, 's_d'
When $\Phi V_c > V_u$, use Maximum Spacing, 's_{max}'
"V_u" is the shear force at distance "d" from the face of the support.
"ΦV_c" and "$\Phi V_c/2$" are plotted on shear force diagram.

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - Design the beam shown below as per ACI 318-14.

W_D = 0.75 kip/ft
W_L = 0.75 kip/ft



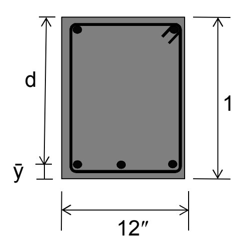
Take f'_c = 3 ksi & f_y = 40 ksi

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 01: Sizes.**
      - For 20′ length, h_{min} = l/16 = 20*12/16 = 15″
      - For grade 40, we have = h_{min} =15″ x (0.4 + 40,000/100,000) = 12″
      - This is the minimum requirement of the code for depth of beam.
      - However we select 18″ deep beam.
      - Generally the minimum beam width is 12″, therefore, width of the beam is taken as 12″
      - The final selection of beam size depends on several factors specifically the availability of formwork.

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 01: Sizes.**
      - Depth of beam, h = 18″
      - h = d + ȳ; ȳ is usually taken from 2.5 to 3.0 inches
      - For ȳ = 2.5 in; d = 18 – 2.5 = 15.5″
      - Width of beam cross section (b_w) = 12″
      - In RCD, Width of beam is usually denoted by b_w instead of b



Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 02: Loads.**
      - Self weight of beam = $\gamma_c b_w h$ = 0.15 × (12 × 18/144) = 0.225 kips/ft
      - $W_u = 1.2W_D + 1.6W_L$

        = 1.2 × (0.225 + 0.75) + 1.6 × 0.75 = 2.37 kips/ft

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 03: Analysis.**
      - Flexural Analysis:

        $M_u = W_u l^2/8$ = 2.37 × (20)^2 × 12/8 = 1422 in-kips
      - Analysis for Shear in beam:

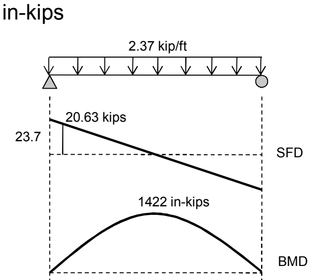
        V = 23.7 kips,

        To find V_u at a distance 'd' from face of Support, d = 15.5″ = 1.29′

        using similarity of triangles:

        V_u / (10 - 1.29) = 23.7 / 10

        V_u = 23.7 × (10 – 1.29) / 10 = 20.63 k



Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - $\Phi M_n \geq M_u$ (ΦM_n is M_{design} or $M_{capacity}$)
        - For $\Phi M_n = M_u$
        - $\Phi A_s f_y(d – a/2) = M_u$
        - $A_s = M_u / \{\Phi f_y (d – a/2)\}$
        - Calculate "A_s" by trial and success method.

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Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - First Trial:
        - Assume a = 4″
        - $A_s = 1422 / [0.9 \times 40 \times \{15.5 – (4/2)\}] = 2.92$ in^2
        - $a = A_s f_y / (0.85 f_c' b_w)$
        - $= 2.92 \times 40 / (0.85 \times 3 \times 12) = 3.81$ inches

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Second Trial:
          - A_s = 1422 / [0.9 × 40 × {15.5 – (3.82/2)}] = 2.90 in^2
          - a = 2.90 × 40/ (0.85 × 3 × 12) = 3.79 inches
        - Third Trial:
          - A_s = 1422 / [0.9 × 40 × {15.5 – (3.79/2)}] = 2.90 in^2
          - a = 4.49 × 40/ (0.85 × 3 × 12) = 3.79 inches
        - "Close enough to the previous value of "a" so that A_s = 2.90 in^2 O.K

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Check for maximum and minimum reinforcement allowed by ACI:
        - $A_{smin} = 3 (\sqrt{f'_c} / f_y) b_w d \geq (200/f_y) b_w d$
        - $3 (\sqrt{f'_c} / f_y) b_w d = 3 \times (\sqrt{3000} / 40000) b_w d = 0.004 \times 12 \times 15.5 =$ **0.744 in^2**
        - $(200/f_y) b_w d = (200/40000) \times 12 \times 15.5 =$ **0.93 in^2**
        - **A_{smin} = 0.93 in^2**

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - $A_{smax} = 0.27 (f_c' / f_y) b_w d = 0.27 \times (3/40) \times 12 \times 15.5 =$ **3.76 in^2**
        - **A_{smin} (0.93) < A_s (2.90) < A_{smax} (3.76) O.K**

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Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Bar Placement: 5 #7 bars will provide 3.0 in^2 of steel area which is slightly greater than required.
        - Other options can be explored. For example,
          - 7 #6 bars (3.08 in^2),
          - 4 #8 bars (3.16 in^2),
          - or combination of two different size bars.

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for Shear:
        - V_u = 20.63 kips
        - ΦV_c = (Capacity of concrete in shear) = $\Phi 2 \sqrt{f'_c}\, b_w d$

          = 0.75×2× $\sqrt{3000}$ ×12×15.5/1000 = 15.28 kips

        As $\Phi V_c < V_u$, Shear reinforcement is required.

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Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for Shear:
        - Assuming #3, 2 legged (0.22 in^2), vertical stirrups.
        - Spacing required (S_d) = $\Phi A_v f_y d/ (V_u - \Phi V_c)$

          = 0.75×0.22×40×15.5/ (20.63–15.28) ≈ 19.12″

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for Shear:
        - Maximum spacing and minimum reinforcement requirement as permitted by ACI is minimum of:
          - $s_{max} = A_v f_y/(50b_w)$ =0.22 × 40000/(50 × 12) = 14.66″
          - s_{max} = d/2 = 15.5/2 = 7.75″
          - s_{max} = 24″
          - $A_v f_y/ 0.75\sqrt{(f_c')} b_w$ = 0.22×40000/ {(0.75×√(3000)×12} =17.85″
        - Therefore s_{max} = 7.75″

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for Shear:
        - Other checks:
        - Check for depth of beam:

          $\Phi V_s \le \Phi 8\sqrt{f'_c}\, b_w d$

          $\Phi 8\sqrt{f'_c}\, b_w d = 0.75 \times 8 \times \sqrt{3000} \times 12 \times 15.5/1000 = 61.12$ k

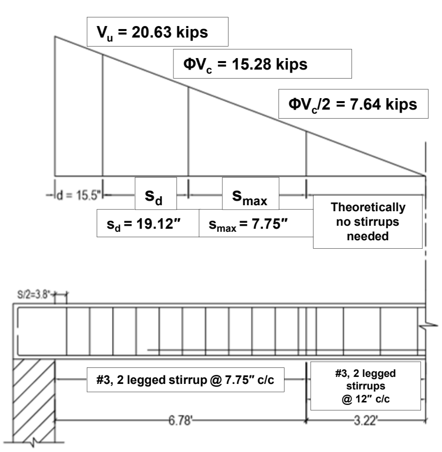
          $\Phi V_s = V_u - \Phi V_c = 20.63 - 15.28 = 5.35$ k < 61.12 k, O.K.
        - Therefore depth is O.K. If not, increase depth of beam.

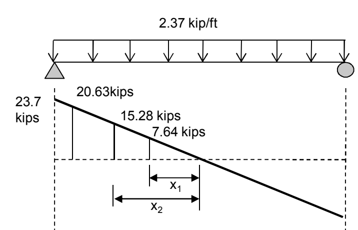
Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for Shear:
        - Other checks:
        - Check if "$\Phi V_s \leq \Phi 4 \sqrt{f'_c}\, b_w d$
        - 5.35 kips < 30.56 kips O.K.
        - "$\Phi V_s \leq \Phi 4 \sqrt{f'_c}\, b_w d$, the maximum spacing ($s_{max}$) is O.K. Otherwise reduce spacing by one half.

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Step 05: Drafting (Shear Reinforcement)**



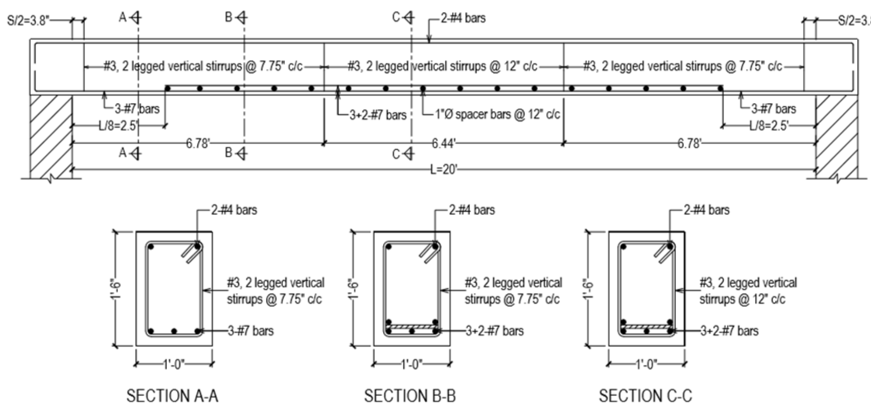


Note:

As S_d S_{max}, we will provide S_{max} from the support up to 6.78 ft. Beyond this point, theoretically no reinforcement is required, however, we will provide #3 2-legged stirrups @ 12 in c/c.

Example 6.1

- **Flexural and shear Design of Beam as per ACI:**
  - **Step 05: Drafting (Flexural Reinforcement)**



Practice Example

- **Example 01**
  - Design the 12″ x 18″ beam for shear using the following data:

S.No.	Concrete Compressive Strength f'_c (ksi)	Rebar Tensile Strength f_y (ksi)	Shear force V_u (kips)
1.	3	60	35
2.	4	60	45
3.	4	40	30

Practice Example

- **Example 02**
  - Design the 12″ x 24″ beam for shear using the following data:

S.No.	Concrete Compressive Strength f'_c (ksi)	Rebar Tensile Strength f_y (ksi)	Shear force V_u (kips)
1.	3	60	35
2.	4	60	45
3.	4	40	30

References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- ACI 318-14