(*) The Per Unit System:-

⇒ An interconnected Power systems typically consists of many different voltage levels usually in KV, MV etc given a system containing several transformers and rotating machines.

⇒ The Per unit system simplify the analysis of complex power systems by choosing a common set of base parameters in terms of which all system quantities are defined.

⇒ Per unit systems are used by Engineers to quantify equipment parameters in terms of their ratings.

e.g a motor with 100 Horse power (HP) rating which delivers 90 HP to a load is said to be 90% loaded or 0.9 Per unit.

In this case, the per unit base is Horse Power rating of 100. when the Motor delivers 90 HP.

$$90/100 = 0.9 \text{ per unit}$$

In general.

$$\text{Per Unit} = \frac{\text{Present Value}}{\text{Base Value}}$$

⇒ Voltage, current, Kilovolt amperes and Impedence are so related that selection of base values for any two of them determines the base values of the remaining two.

If we specify the base value of current and voltage, base impedence and base Kilovolt amperes can be determined.

i.e
$$I_{base} = \frac{S_{base}}{V_{base}}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

⇒ We also assume that

① The value of S_{base} is constant for all points in the power system.

② The ratio of voltage base on either side of a transformer is chosen to equal the ratio of the transformer voltage rating, then the transformer per unit impedence remains unchanged when referred from one side of a transformer to the other.

This will allow us to eliminate the ideal transformer from the transformer model (i-e we will not reflect Impedence from one side of the transformer to other).

—— x —— x —— x ——

✪ Calculation of Per Unit Base Value for Single Phase system and Three Phase systems:-

Base Quantity	Single Phase Systems	Three Phase systems.
Power	$S_{base\,1\phi} = V_{base} \times I_{base}$	$S_{base\,3\phi} = \sqrt{3}\, V_{base} \times I_{base}$ (Where Vbase is a line to line voltage)

Base Quantity	Single Phase Systems	Three Phase Systems
Voltage	Base Voltage is almost always known. Use the nominal rating for the equipment or system. Otherwise $V_{base} = \frac{S_{base\,1\phi}}{I_{base}}$	Same comment as single phase. Otherwise $V_{base\,L\text{-}L} = \frac{S_{base\,3\phi}}{\sqrt{3}\,I_{base}}$
Current	$I_{base} = \frac{S_{base\,1\phi}}{V_{base}}$	$I_{base} = \frac{S_{base\,3\phi}}{\sqrt{3}\,V_{base\,L\text{-}L}}$
Impedence	$Z_{base} = \frac{V_{base}^2}{S_{base\,1\phi}}$	$Z_{base} = \frac{(\sqrt{3}\,V_{base\,L\text{-}L})^2}{S_{base\,3\phi}}$

Advantages:-

1. Device Parameters tend to fall into a relatively narrow range.

2. The method is defined in order to eliminate ideal transformers as circuit components.

Disadvantages:-

For actual value, extra calculations have to be done.

Key Points:-

① The base value and actual value has the same units.

② P.U value is dimensionless.

③ Base value is always a real number.

④ Actual value may be complex.

⑤ In polar form, the angle of the per unit value is the same as that of actual value.

Now Proof for finding the P.U values:-

Consider Complex Power

$$S = V I^* \longrightarrow (1)$$

or $$S\angle\theta = V\angle\alpha \cdot I\angle -\beta$$ (From Power Triangle)

V = Phasor Voltage (volts), I = Phasor Current (Amp)

Suppose we arbitrarily pick a value S_{base}. Then divide the both sides of eq (1) by S_{base}.

$$\frac{S\angle\theta}{S_{base}} = \frac{V\angle\alpha \cdot I\angle -\beta}{S_{base}}$$

Let us define

$$V_{base} \times I_{base} = S_{base} \longrightarrow (2)$$

$$\Rightarrow \frac{S\angle\theta}{S_{base}} = \frac{V\angle\alpha \cdot I\angle-\beta}{V_{base} \cdot I_{base}}$$

$$S_{p.u}\angle\theta = \left(\frac{V\angle\alpha}{V_{base}}\right)\left(\frac{I\angle-\beta}{I_{base}}\right)$$

$$S_{p.u} = V_{p.u}\angle\alpha \times I_{p.u}\angle-\beta$$

$$S_{p.u} = V_{p.u} \cdot I_{p.u}^{*} \longrightarrow (3)$$

Impedence - Z

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V^2_{base}}{S_{base}}$$

According to Ohm's Law

$$Z = \frac{V}{I} \rightarrow (4).$$

To find out Per Unit value,

divide the above eq (4) by Z_{base}.

$$\frac{Z}{Z_{base}} = \frac{V/I}{Z_{base}} = \frac{V/I}{\frac{V_{base}}{I_{base}}}$$

$$Z_{p.u} = \frac{V/V_{base}}{I/I_{base}}$$

$$\boxed{Z_{p.u} = \frac{V_{p.u}}{I_{p.u}}}$$

—— x —— x —— x ——