Lecture 04

Design of T and L Beams in Flexure

By: Prof Dr. Qaisar Ali

Civil Engineering Department

UET Peshawar

drqaisarali@uetpeshawar.edu.pk

Topics Addressed

- Introduction to T and L Beams
- ACI Code provisions for T and L Beams
- Design Cases
- Design of Rectangular T-beam
- Design of True T-beam
- References

Introduction to T and L Beam

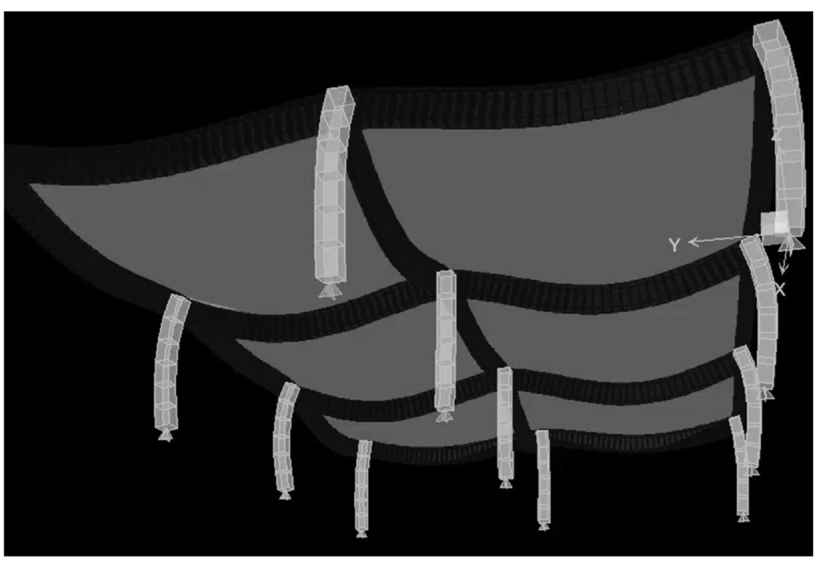
- The T or L Beam gets its name when the slab and beam produce the cross sections having the typical T and L shapes in a monolithic reinforced concrete construction.

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

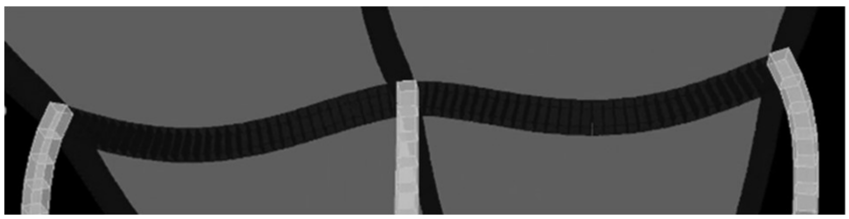
Introduction to T and L Beam

- In casting of reinforced concrete floors/roofs, forms are built for beam sides, the underside of slabs, and the entire concrete is mostly poured at once, from the bottom of the deepest beam to the top of the slab.

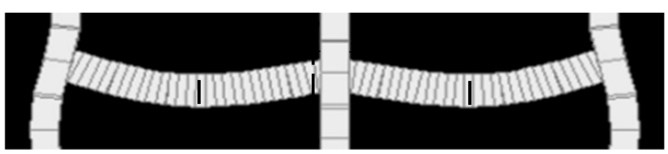
Introduction to T and L Beam

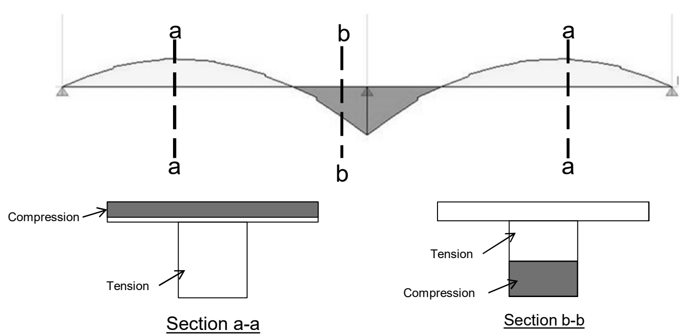


Introduction to T and L Beam



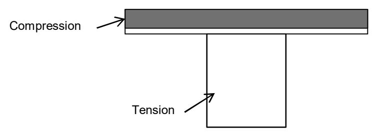
Introduction to T and L Beam





Introduction to T and L Beam

- Positive Bending Moment
  - In the analysis and design of floor and roof systems, it is common practice to assume that the monolithically placed slab and supporting beam interact as a unit in resisting the positive bending moment.
  - As shown, the slab becomes the compression flange, while the supporting beam becomes the web or stem.

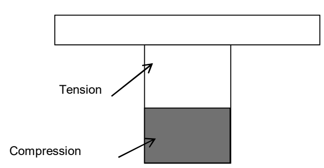


Section a-a

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Introduction to T and L Beam

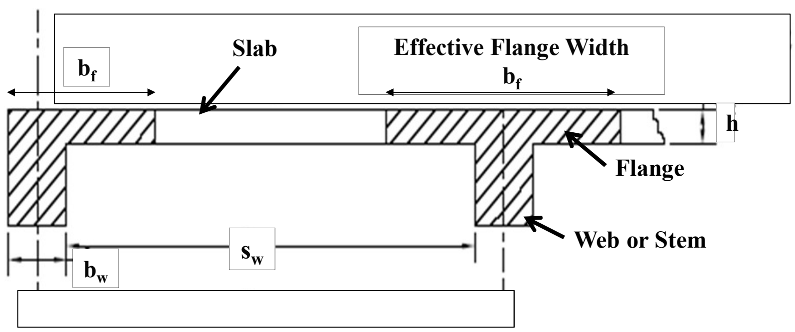
- Negative Bending Moment
  - In the case of negative bending moment, the slab at the top of the stem (web) will be in tension while the bottom of the stem is in compression. This usually occurs at interior support of continuous beam.



Section b-b

ACI Code Provisions for T and L Beams

- For T and L beams supporting monolithic or composite slabs, the effective flange width b_f shall include the beam web width b_w plus an effective overhanging flange width in accordance with ACI Table 6.3.2.1

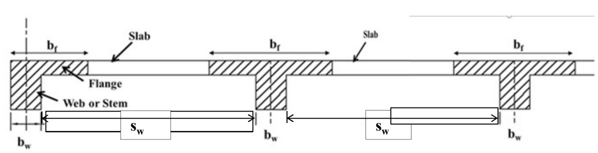


Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

ACI Code Provisions for T and L Beams

- Calculation of Effective Flange Width (b_f) (ACI 6.3.2.1)

	T - Beam
1	$b_w + 16h$
2	$b_w + s_w$
3	$b_w + \ell_n/4$



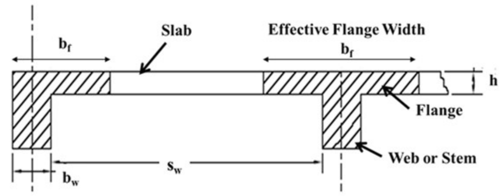
Least of the above values is selected

Where $\mathbf{b_w}$ is the width of the beam, **h** is the slab thickness, $\mathbf{s_w}$ is the clear distance to the adjacent beam and $\boldsymbol{\ell_n}$ is the clear length of beam.

ACI Code Provisions for T and L Beams

- Calculation of Effective Flange Width (b_f) (ACI 6.3.2.1)

L - Beam	
1	$b_w + 6h$
2	$b_w + s_w/2$
3	$b_w + \ell_n/12$



Least of the above values is selected

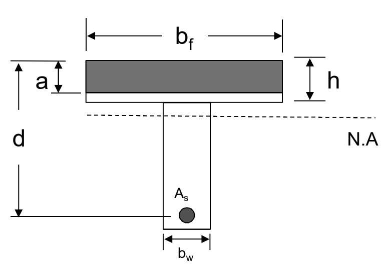
Where $\mathbf{b_w}$ is the width of the beam, **h** is the slab thickness, $\mathbf{s_w}$ is the clear distance to the adjacent beam and $\boldsymbol{\ell_n}$ is the clear length of beam.

Design Cases

- In designing a T-Beam for positive bending moment, there might exist two conditions:

Condition 1. The depth of the compression block may be less than or equal to the slab depth i.e. flange thickness ($a \le h$)

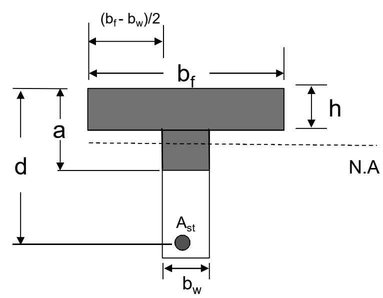
In such a condition the T-Beam is designed as rectangular beam for positive bending with the width of compression block equal to b_f.



Design Cases

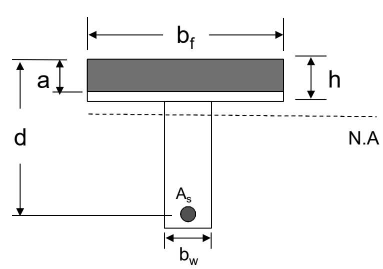
Condition 2. The compression block may cover the flange and extend into the web (a > h)

In such condition the T-Beam is designed as true T-beam.



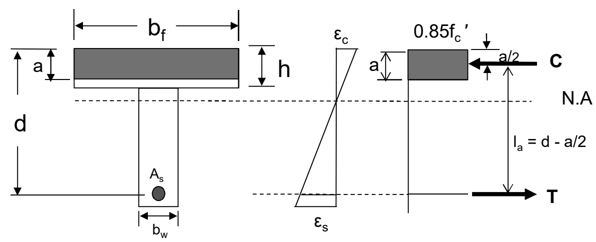
Design of Rectangular T-beam

- Flexural Capacity
  - When $a \le h$



Design of Rectangular T-beam

- Flexural Capacity



$(\sum F_x = 0)$

$0.85f_c' ab_f = A_s f_y$

$a = A_s f_y / 0.85 f_c' b_f$

$(\sum M = 0)$

$M_n = T l_a = A_s f_y (d – a/2)$

As $\Phi M_n = M_u$; $\Phi A_s f_y (d – a/2) = M_u$

Therefore, $A_s = M_u / \Phi f_y (d – a/2)$

The other checks remains as that of the rectangular beam design.

Note: In calculating A_{smax}, use b_w , not b_f.

Design of Rectangular T-beam

- Example 01
  - The roof of a hall has a 5″ thick slab supported on beam having 30 feet c/c and 28.5 feet clear span. Adjacent beams are having 9 feet clear distance and have been cast monolithically with slab. Overall depth of beam (including slab thickness) being 24 in and width of beam web being 14 in. Calculate the steel reinforcement area for the beam for a total factored load (including self weight of beam) of 3 k/ft. Use f_c' = 3 ksi and f_y = 60 ksi.

Design of Rectangular T-beam

- Example Solution:
  - Span length ($l_{c/c}$) = 30′ ; clear length (l_n) = 28.5′
  - w_u = 3 k/ft
  - d = 24-2.5 = 21.5″, b_w = 14″; h = 5″
  - Effective flange width (b_f) is minimum of,
    - b_w + 16h = 14 + 16 × 5 = 94″
    - b_w + s_w = 14 + 9 × 12 = 122″
    - b_w + l_n/4 + = 14 + 28.5 × 12/4 = 99.5″
  - Therefore, b_f = 94″

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Design of Rectangular T-beam

- Example Solution:
  - Check if the beam behaviour is T or rectangular.
  - Mu = $w_u l^2/8 = 3 \times 30^2 \times 12 / 8$ = 4050 in-kips
  - Trial # 01
    - Let a = h = 5″

      $A_s = M_u/\Phi f_y(d - a/2)$ = 4050/{0.90 × 60 × (21.5 – 5/2)} = 3.94 in^2
  - Trial # 02
    - a = $A_s f_y/(0.85 f_c' b_f)$ = 3.94 × 60/ (0.85 × 3 × 94) = 0.98″ < h = 5″

      Therefore, design as Rectangular beam.

Design of Rectangular T-beam

- Example Solution:

  $A_s = M_u/\Phi f_y(d – a/2) = 4050/\{0.90 \times 60 \times (21.5 – 0.98/2)\} = 3.56$ in^2

  - Trial # 03
    - $a = A_s f_y/(0.85 f_c' b_f) = 3.56 \times 60/ (0.85 \times 3 \times 94) = 0.89''$

      $A_s = M_u/\Phi f_y(d – a/2) = 4050/\{0.90 \times 60 \times (21.5 – 0.89/2)\} = 3.56$ in^2

  Therefore $A_s = 3.56$ in^2

  Try #8 bars, No of Bars = 3.56 / 0.79 = 4.50, say 05 bars

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Design of Rectangular T-beam

- Example Solution:

  Now check ρ_{max} and ρ_{min}

  $\rho_{max} = 0.85\beta_1(f_c'/f_y)\{\varepsilon_u/(\varepsilon_u + \varepsilon_t)\}$

  $\rho_{max} = 0.85 \times 0.85 \times (3/60)\{0.003/(0.003 + 0.004)\} = 0.0155$

  $\rho_{min} = 3\sqrt{f'_c}/f_y \geq 200/f_y$

  $3\sqrt{f'_c}/f_y = 3 \times \sqrt{3000}/60000 = 0.0027$

  $200/f_y = 200/60000 = 0.0033$

  Therefore, $\rho_{min} = 0.0033$

  $\rho_{provided} = A_s/b_w d = 5 \times 0.79/14 \times 21.5 = 0.0131$

  $\rho_{min} < \rho_{provided} < \rho_{max}$

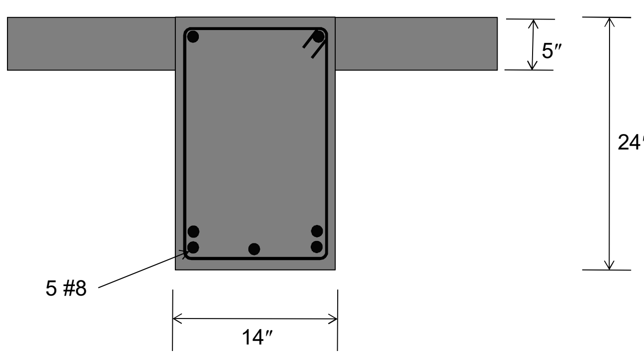
Design of Rectangular T-beam

- Example Solution:

  Check design capacity your self.

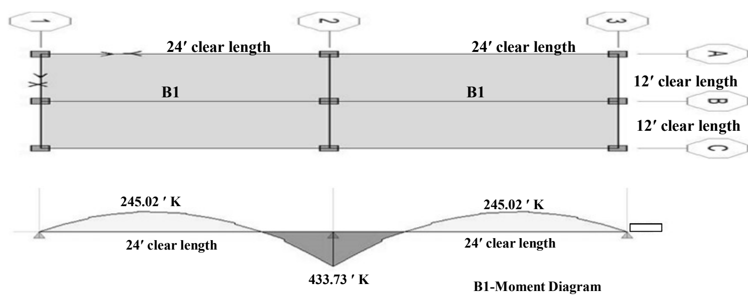
Design of Rectangular T-beam

- Example Solution:



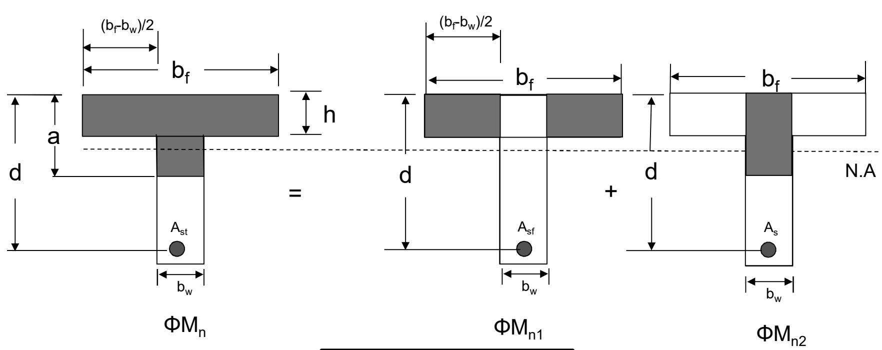
Design of Rectangular T-beam

- Example 2: Design the Beam B1 for the following moments
  - f_c' = 4 ksi, f_y = 60 ksi, beam width = 12″,
    overall beam depth (including slab thickness) = 18″, Slab thickness = 6″



Design of True T-beam

- Flexural Capacity
  - When a > h



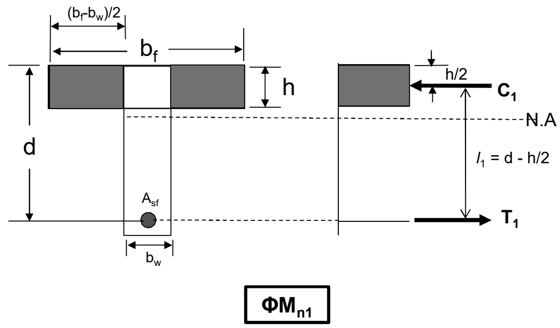
$\Phi M_n = \Phi M_{n1} + \Phi M_{n2}$

Design of True T-beam

- Flexural Capacity

  ΦM_{n1} Calculation

  From stress diagram



  - $T_1 = C_1$
  - $C_1 = 0.85 f_c' (b_f - b_w)h$
  - $T_1 = A_{sf} f_y$
  - $A_{sf} f_y = 0.85 f_c' (b_f - b_w)h$

  Everything in the equation is known except A_{sf}

  - Therefore, $A_{sf} = 0.85 f_c' (b_f - b_w)h / f_y$
  - $\Phi M_{n1} = T_1 \times l_1 = \Phi A_{sf} f_y (d - h/2)$

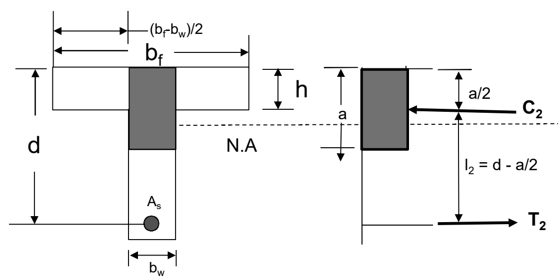
Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Design of True T-beam

- Flexural Capacity

  ΦM_{n2} Calculation

  From stress diagram



  - $T_2 = C_2$
  - $C_2 = 0.85 f_c' a b_w$
  - $T_2 = A_s f_y$
  - $A_s f_y = 0.85 f_c' a b_w$
  - $a = A_s f_y / (0.85 f_c' b_w)$
  - $\Phi M_{n2} = T_2 \times l_2 = \Phi A_s f_y (d - a/2)$

Design of True T-beam

- Flexural Capacity

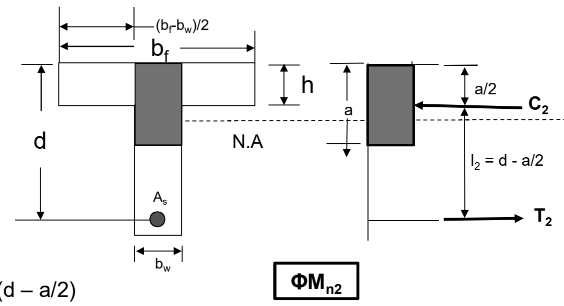
ΦM_{n2} Calculation

We know that $\Phi M_n = M_u$

$\Phi M_{n1} + \Phi M_{n2} = M_u$

ΦM_{n1} is already knows to us,

Therefore $\Phi M_{n2} = M_u - \Phi M_{n1}$



And as, $\Phi M_{n2} = T_2 \times l_2 = \Phi A_s f_y (d - a/2)$

Also $\Phi M_{n2} = M_u - \Phi M_{n1}$

Therefore, $A_s = (M_u - \Phi M_{n1}) / \Phi f_y (d - a/2)$; and $a = A_s f_y / (0.85 f_c' b_w)$

Calculate A_s by trial and success method.

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Design of True T-beam

- Ductility Requirements
  - $T = C_1 + C_2$ [$\sum F_x = 0$]

    $A_{st} f_y = 0.85 f_c' a b_w + 0.85 f_c' (b_f - b_w) h$

    $A_{st} f_y = 0.85 f_c' a b_w + A_{sf} f_y$
  - For ductility $\varepsilon_s = \varepsilon_t = 0.005$ (ACI table 21.2.2)
  - For $a = \beta_1 c = \beta_1 0.375d$, A_{st} will become A_{stmax}, Therefore,

    $A_{stmax} f_y = 0.85 f_c' \beta_1 0.375 d b_w + A_{sf} f_y$

    $A_{stmax} f_y = 0.85 f_c' \beta_1 0.375 d b_w + A_{sf}$

    $A_{stmax} = 0.31875\ \beta_1 (f_c'/f_y) d b_w + A_{sf}$ OR $A_{stmax} = A_{smax\ (singly)} + A_{sf}$
  - So, for T-beam to behave in a ductile manner $A_{st,\ provided} \le A_{stmax}$

Design of True T-beam

- Example 03
  - Design a simply supported T beam to resist a factored positive moment equal to 6500 in-kip. The beam is 12″ wide and is having 20″ effective depth including a slab thickness of 3 inches. The centre to centre and clear lengths of the beam are 25.5′ and 24′ respectively. The clear spacing between the adjacent beams is 3 ft.
  - Material strengths are f_c' = 3 ksi and f_y = 40 ksi.

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Design of True T-beam

- Example Solution:
  - Span length ($l_{c/c}$) = 25.5′ ; clear length (l_n) = 24′
  - d = 20″; b_w = 12″; h = 3″
  - Effective flange width (b_f) is minimum of,
    - b_w + 16h = 12 + 16 × 3 = 60″
    - b_w + s_w = 12 + 3 × 12 = 48″
    - b_w + l_n/4 = 12 + 24 × 12/4 = 84″
  - Therefore, b_f = 48″

Design of True T-beam

- Example Solution:
  - Check if the beam behaviour is T or rectangular.
  - Let a = h = 3″

    $A_s = M_u/\Phi f_y(d - a/2) = 6500/\{0.90 \times 40 \times (20 - 3/2)\} = 9.75\ in^2$

    $a = A_s f_y/(0.85 f_c' b_f) = 9.76 \times 40/\ (0.85 \times 3 \times 48) = 3.18'' > h$
  - Therefore, design as T beam.

Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan

Design of True T-beam

- Example Solution:
  - Design:
  - We first calculate A_{sf}

    $A_{sf} = 0.85 f_c' (b_f - b_w)\ h/f_y$

    $= 0.85 \times 3 \times (48 - 12) \times 3/40 = 6.885\ in^2$
  - The nominal moment resistance (ΦM_{n1}), provided by A_{sf} is,

    $\Phi M_{n1} = \Phi A_{sf} f_y \{d - h/2\} = 0.9 \times 6.885 \times 40 \times \{20 - 3/2\} = 4585.41$ in-kip

Design of True T-beam

- Example Solution:
  - Design:
  - The nominal moment resistance (ΦM_{n2}), provided by remaining steel A_s is,
    $\Phi M_{n2} = M_u - \Phi M_{n1} = 6500 - 4585.41 = 1914.45$ in-kip
  - Let a = 0.2d = 0.2 × 20 = 4″

    $A_s = \Phi M_{n2}/ \{\Phi f_y(d - a/2)\} = 1914.45 / \{0.9 \times 40 \times (20 - 4/2)\} = 2.95$ in^2

    $a = A_s f_y/(0.85 f_c' b_w) = 2.95 \times 40 / (0.85 \times 3 \times 12) = 3.85''$
  - This value is close to the assumed value of "a". Therefore,

    $A_{st} = A_{sf} + A_s = 6.885 + 2.95 = 9.84$ in^2 (13 #8 Bars)

Design of True T-beam

- Example Solution:
  - Ductility requirements, $(A_{st} = A_s + A_{sf}) \leq A_{stmax}$

    $A_{stmax} = A_{smax\ (singly)} + A_{sf}$

    $= 4.87 + 6.885 = 11.76$ in^2

    $A_{st} = A_s + A_{sf} = 13 \times 0.79 = 10.27$ in^2 < 11.76 O.K.

Design of True T-beam

- Example Solution:
  - Ensure that $A_{st} > A_{smin}$

    $A_{st} = 10.27\ in^2$
  - $A_{smin} = \rho_{min} b_w d$
  - $\rho_{min} = 3\sqrt{(f_c')}/f_y \geq 200/f_y$

    $3\sqrt{(f_c')}/f_y = 3 \times \sqrt{(3000)}/40000 = 0.004$

    $200/f_y = 200/40000 = 0.005$

    $\rho_{min} = 0.005$; $A_{smin} = 0.005 \times 12 \times 20 = A_{st} > 1.2\ in^2$ O.K.

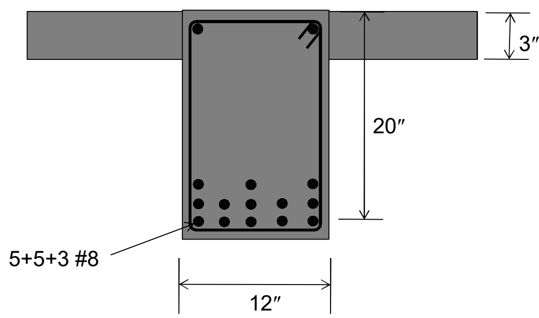
Design of True T-beam

- Example Solution:

  Check design capacity your self.

Design of True T-beam

- Example Solution:



References

- Design of Concrete Structures 14th Ed. by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-14)