ROADBED SOILS -B

Phase Relationship, Shear Strength & Soil Structure

PHASE RELATIONSHIP

Density and Unit Weight

- Mass is a measure of a body's inertia, or its "quantity of matter". Mass is not changed at different places.
- Weight is force, the force of gravity acting on a body. The value is different at various places (Newton's second law F = ma) (Giancoli, 1998)
- The unit weight is frequently used than the density is (e.g. in calculating the overburden pressure).

$$\text{Density, } \rho = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Unit weight, } \gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{\text{Mass} \cdot g}{\text{Volume}}$$

g : acceleration due to gravity

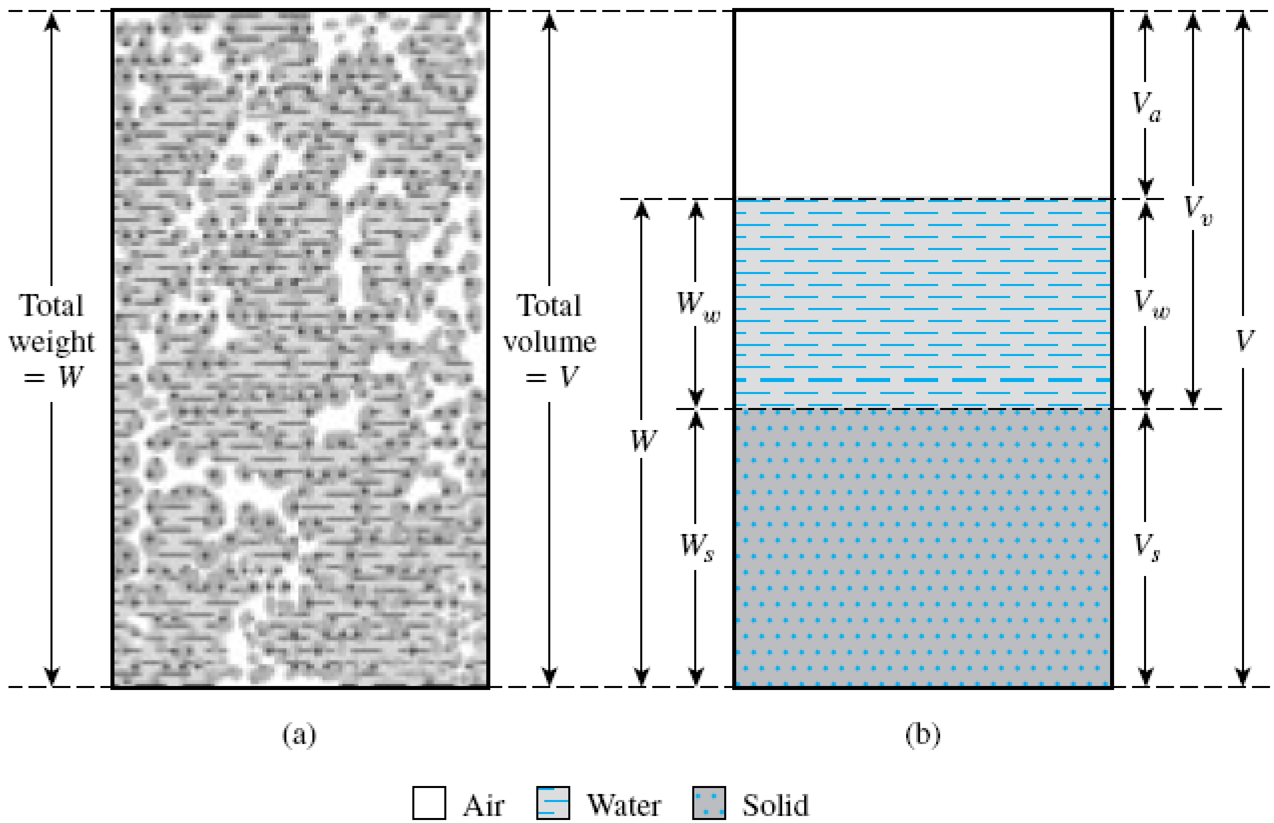
$$\gamma = \rho \cdot g = \rho \cdot 9.8\ {}^{m}/_{sec^2}$$

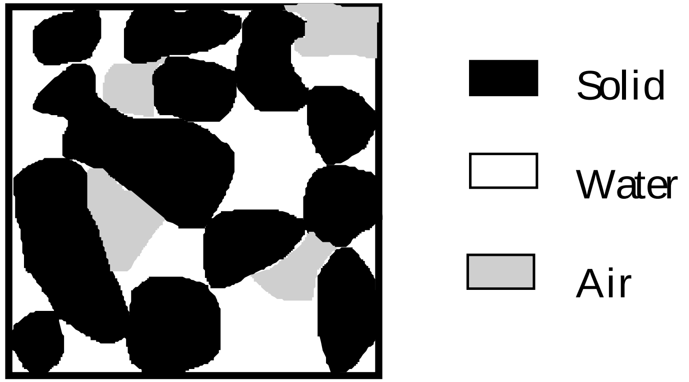
$$\text{Water, } \gamma = 9.8\ {}^{kN}/_{m^3}$$

$$G_s = \frac{\rho_s}{\rho_w} = \frac{\rho_s \cdot g}{\rho_w \cdot g} = \frac{\gamma_s}{\gamma_w}$$

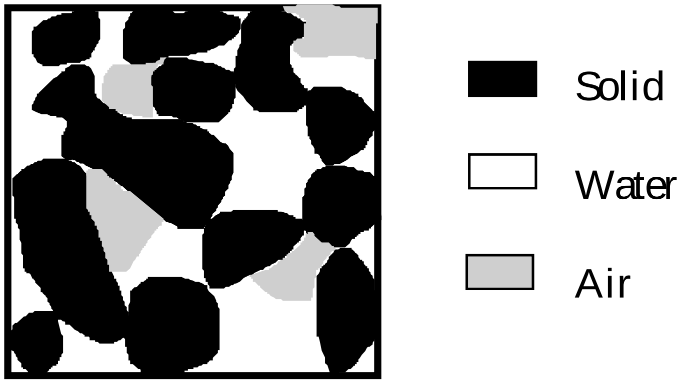
RELATIONSHIPS

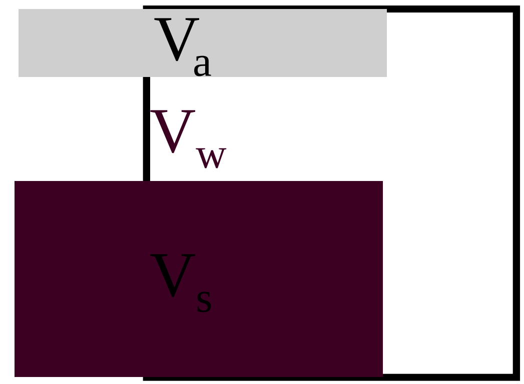
- e
- n
- S
- ω
- ρ
- ρ_s
- ρ_d
- ρ_{sat}
- ρ’





- Soil is generally a three phase material
- Contains solid particles and voids
- Voids can contain liquid and gas phases





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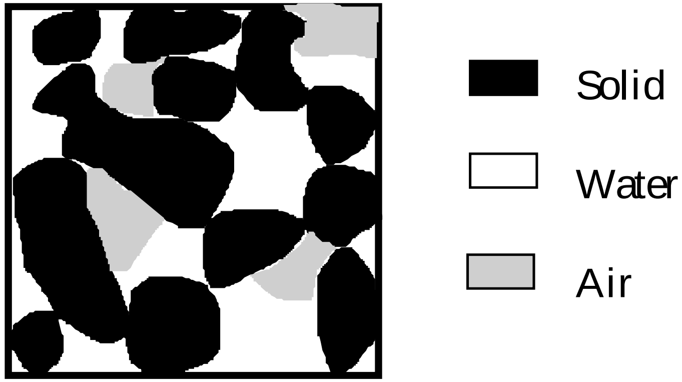

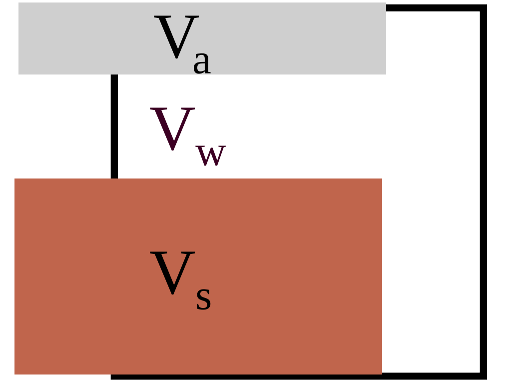

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Phase	Volume	Mass	Weight
Air	V_a	0	0
Water	V_w	M_w	W_w
Solid	V_s	M_s	W_s

Units

• Length	metres
• Mass	tonnes (1 tonne = 10^3 kg)
• Density	t/m^3
• Weight	kilonewtons (kN)
• Stress	kilopascals (kPa) 1 kPa= 1 kN/m^2
• Unit weight	kN/m^3
• Accuracy	Density of water, ρ_w = 1 t/m^3
	Stress/Strength to 0.1 kPa

Weight and Unit weight

- Force due to mass (weight) more important than mass
- $W = M\,g$
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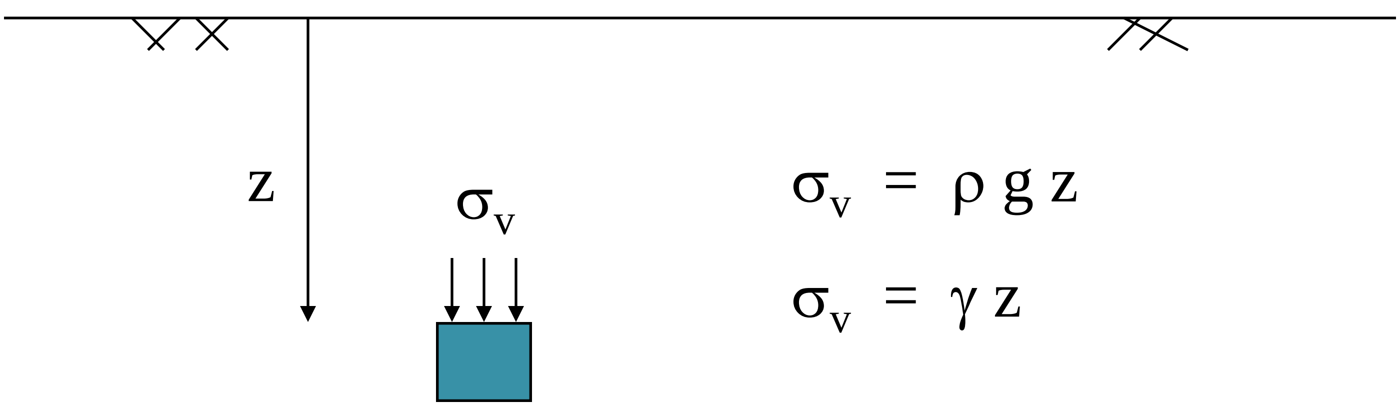

$$\sigma_v = \rho\,g\,z$$

$$\sigma_v = \gamma\,z$$

- Unit weight

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s\left[1 + \left(\frac{W_w}{W_s}\right)\right]}{V} = \frac{W_s(1 + w)}{V}$$

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_d = \frac{\gamma}{1 + w}$$

Note that unit weight of water (γ_w) is equal to 9.81 kN/m^3 or 62.4 lb/ft^3 or 1000 kgf/m^3.

Specific Gravity

This is defined by

$$G = \frac{Density\ of\ Material}{Density\ of\ Water} = \frac{\rho}{\rho_w}$$

$$G = \frac{Unit\ Weight\ of\ Material}{Unit\ Weight\ of\ Water} = \frac{\gamma}{\gamma_w}$$

- $G_s \cong 2.65$ for most soils
- G_s is useful because it enables the volume of solid particles to be calculated from mass or weight

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$$V_s = \frac{M_s}{\rho_s} = \frac{M_s}{G_s\,\rho_w} = \frac{W_s}{\gamma_s} = \frac{W_s}{G_s\,\gamma_w}$$

Voids ratio

- It is not the actual volumes that are important but rather the ratios between the volumes of the different phases. This is described by the voids ratio, e, or porosity, n, and the degree of saturation, S.

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The relation between these quantities can be simply determined as follows

$V_s = V - V_v = (1 - n)V$

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1 - n}$$

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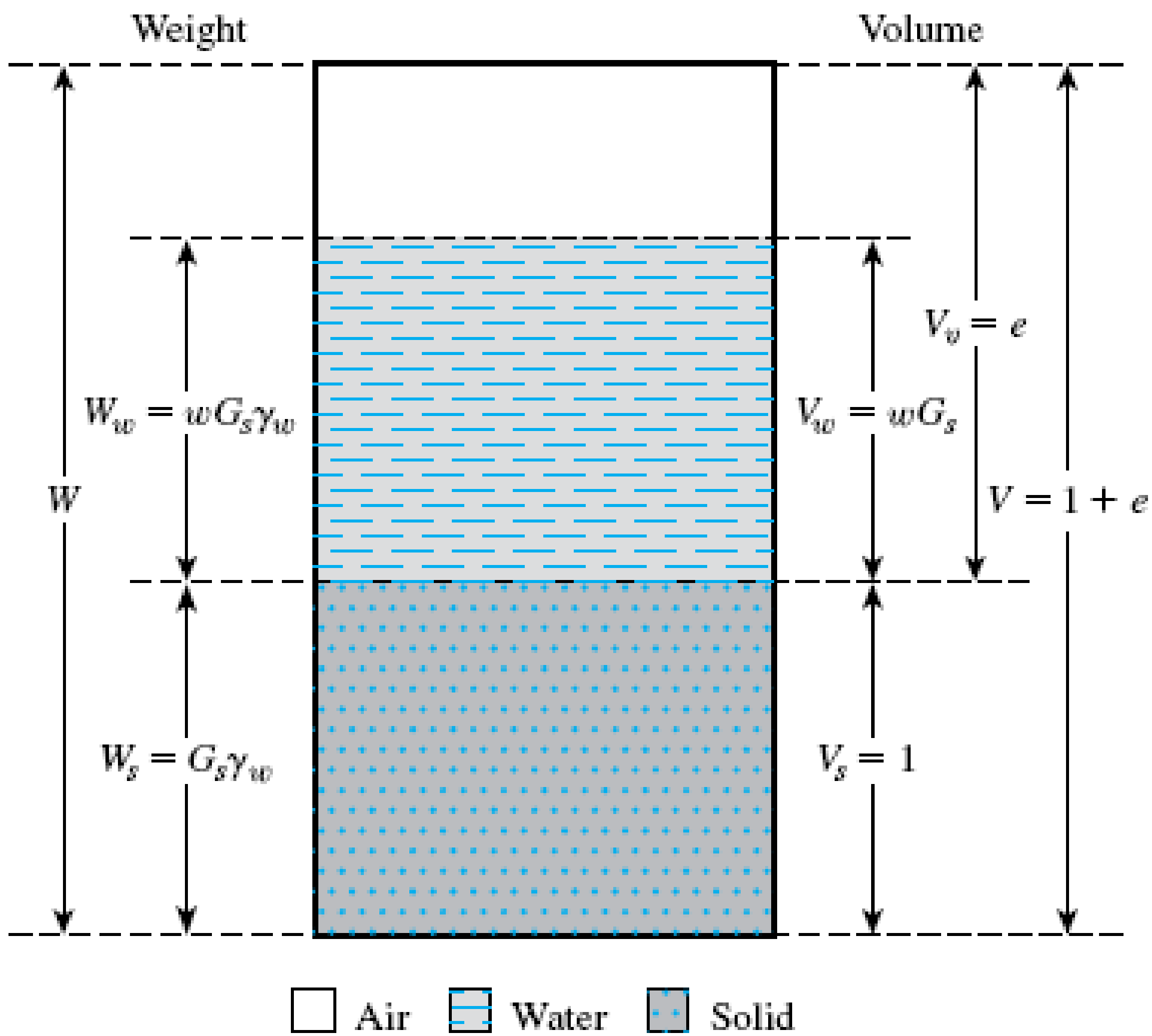
Hence

$$e = \frac{V_v}{V_s} = \frac{V_v}{(1 - n)V} = \frac{n}{1 - n}$$

Relationship among Unit weight, moisture content , & Sp. gravity



$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + wG_s\gamma_w}{1 + e} = \frac{(1 + w)G_s\gamma_w}{1 + e}$$

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1 + e}$$

$$e = \frac{G_s\gamma_w}{\gamma_d} - 1$$

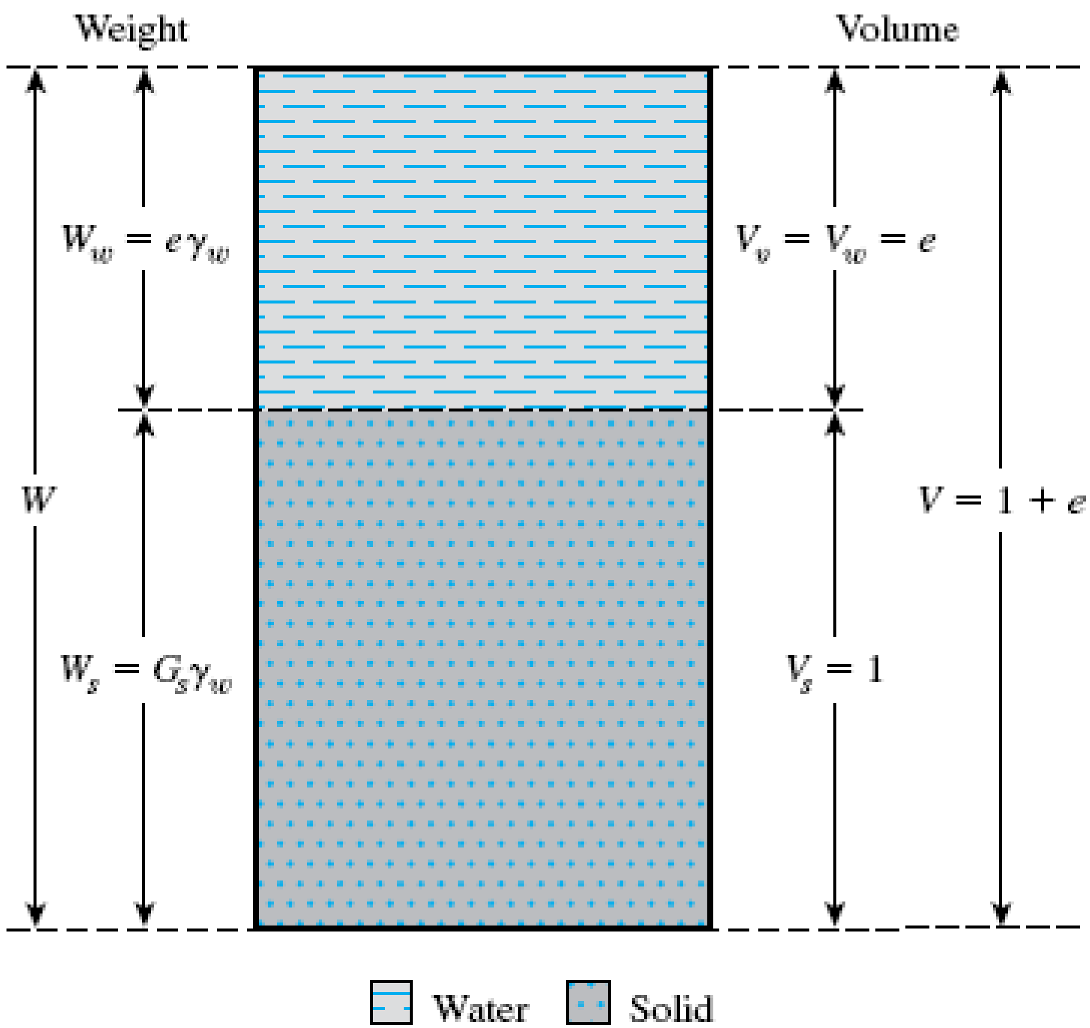
$$V_w = \frac{W_w}{\gamma_w} = \frac{wG_s\gamma_w}{\gamma_w} = wG_s$$

Degree of Saturation

$$S = \frac{V_w}{V_v} = \frac{wG_s}{e}$$

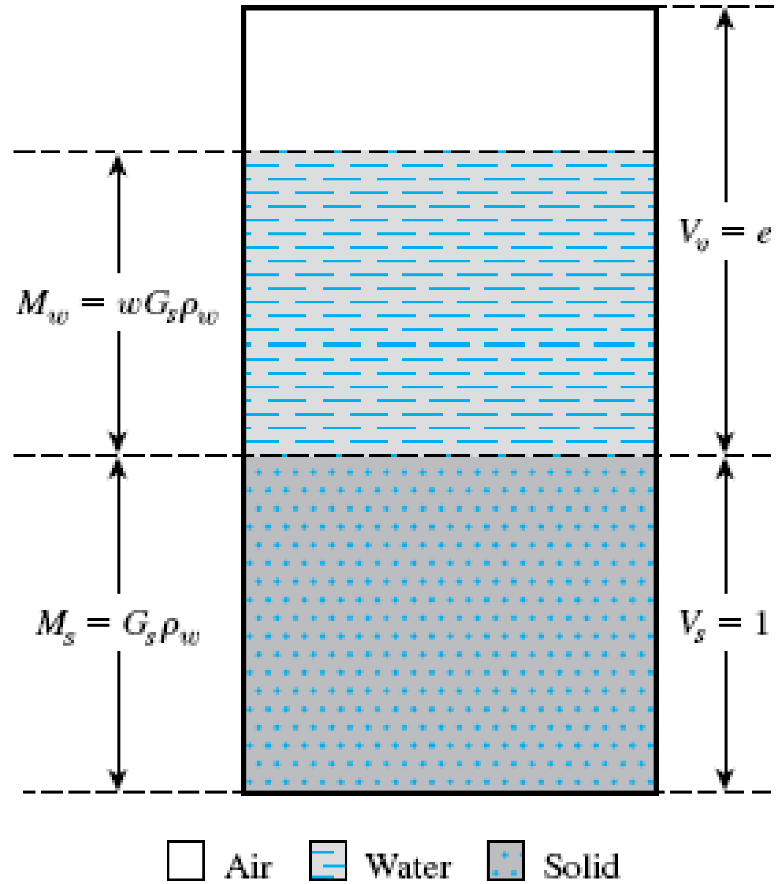
$$Se = wG_s$$

Saturated samples



$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + e\gamma_w}{1 + e} = \frac{(G_s + e)\gamma_w}{1 + e}$$

$$e = wG_s$$



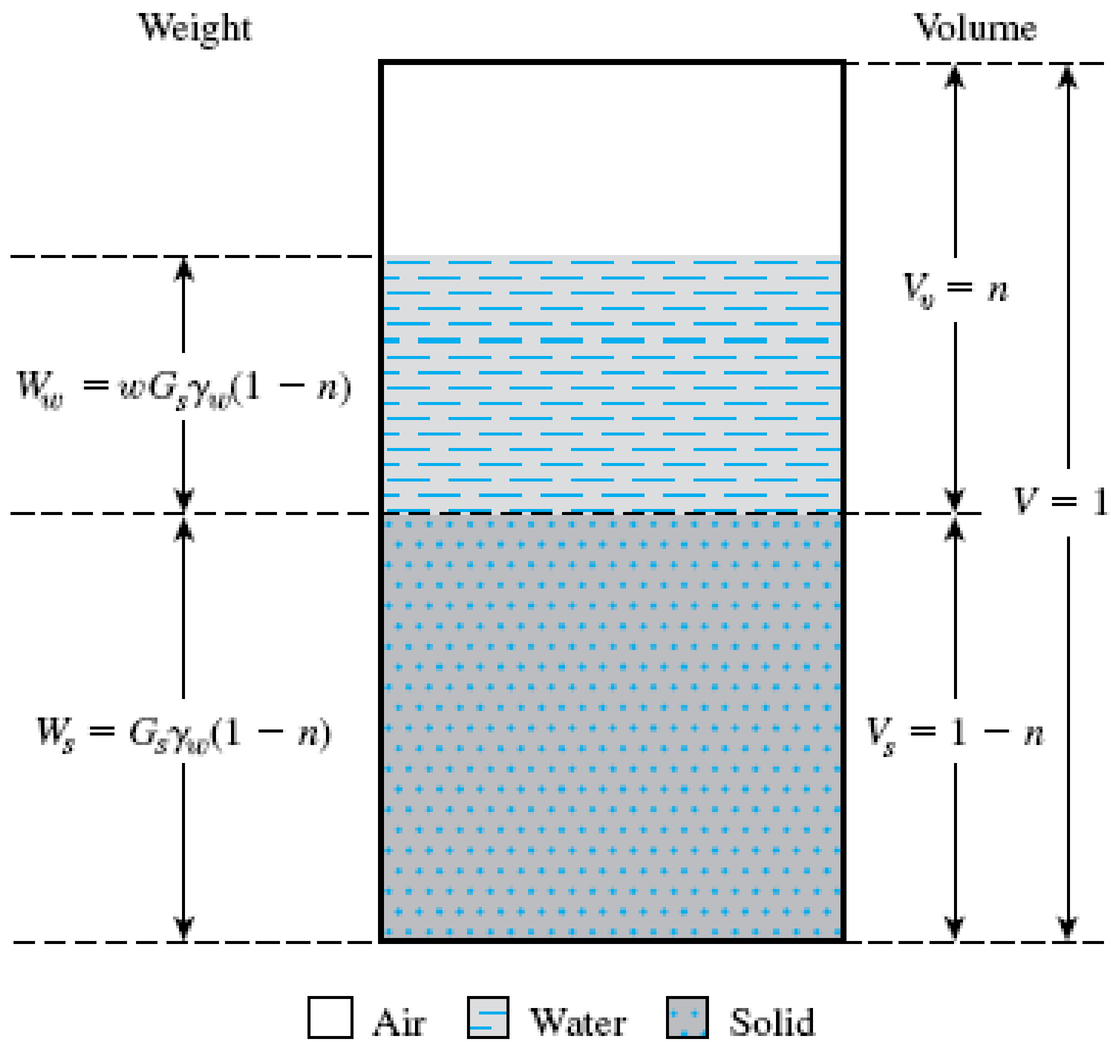
$$\text{Density} = \rho = \frac{(1 + w)G_s\rho_w}{1 + e}$$

$$\text{Dry density} = \rho_d = \frac{G_s\rho_w}{1 + e}$$

$$\text{Saturated density} = \rho_{sat} = \frac{(G_s + e)\rho_w}{1 + e}$$

where ρ_w = density of water = 1000 kg/m^3.

Relationship among Unit weight, moisture content , & Porosity



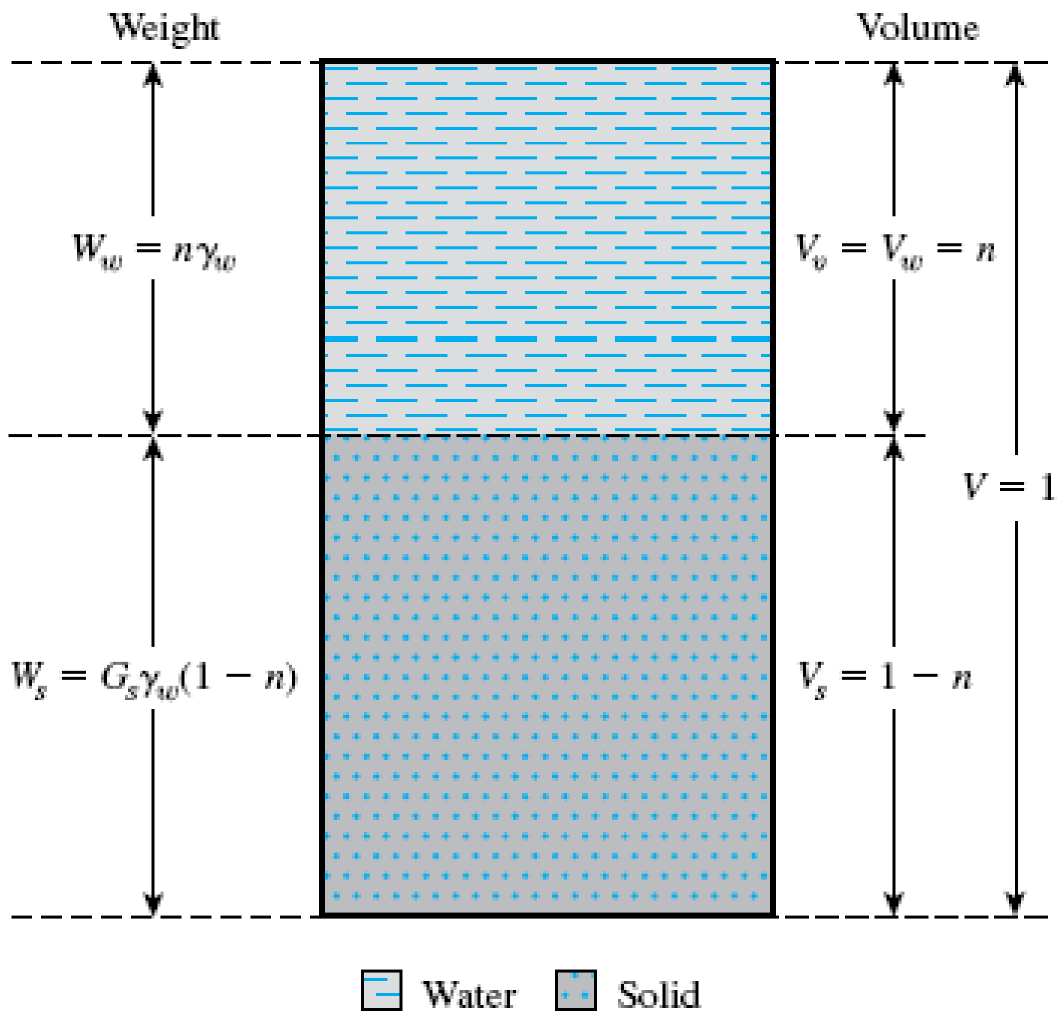
$$W_s = G_s\gamma_w(1 - n)$$

$$W_w = wW_s = wG_s\gamma_w(1 - n)$$

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w(1 - n)}{1} = G_s\gamma_w(1 - n)$$

$$\gamma = \frac{W_s + W_w}{V} = G_s\gamma_w(1 - n)(1 + w)$$

Saturated samples



$$\gamma_{\text{sat}} = \frac{W_s + W_w}{V} = \frac{(1-n)G_s\gamma_w + n\gamma_w}{1} = [(1-n)G_s + n]\gamma_w$$

$$w = \frac{W_w}{W_s} = \frac{n\gamma_w}{(1-n)\gamma_w G_s} = \frac{n}{(1-n)G_s}$$

Various Forms of relationships

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)

Given	Relationship
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$

Dry unit weight (γ_d)

Given	Relationship
γ, w	$\frac{\gamma}{1+w}$
G_s, e	$\frac{G_s\gamma_w}{1+e}$
G_s, n	$G_s\gamma_w(1-n)$
G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$
e, w, S	$\frac{eS\gamma_w}{(1+e)w}$
γ_{sat}, e	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$
γ_{sat}, n	$\gamma_{sat}-n\gamma_w$
γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$

Saturated unit weight (γ_{sat})

Given	Relationship
G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
G_s, n	$[(1-n)G_s+n]\gamma_w$
G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
γ_d, e	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
γ_d, n	$\gamma_d+n\gamma_w$
γ_d, S	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Typical values in natural state

Table 3.2 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d	
			lb/ft^3	kN/m^3
Loose uniform sand	0.8	30	92	14.5
Dense uniform sand	0.45	16	115	18
Loose angular-grained silty sand	0.65	25	102	16
Dense angular-grained silty sand	0.4	15	121	19
Stiff clay	0.6	21	108	17
Soft clay	0.9–1.4	30–50	73–93	11.5–14.5
Loess	0.9	25	86	13.5
Soft organic clay	2.5–3.2	90–120	38–51	6–8
Glacial till	0.3	10	134	21

Example 1

- Distribution by mass and weight

Phase	Trimmings Mass (g)	Sample Mass, M (g)	Sample Weight, Mg (kN)
Total	55	290	2845×10^{-6}
Solid	45	237.3	2327.9×10^{-6}
Water	10	52.7	517×10^{-6}

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Air Volume $V_a = V - V_s - V_w$

Moisture content $m = \frac{W_w}{W_s} = \frac{10}{45} = 0.222 = 22.2\,\%$

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Degree of Saturation $S = \frac{V_w}{V_v} = \frac{V_w}{V_a + V_w} = 0.780 = 78.0\,\%$

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Saturated unit weight $\gamma_{sat} = \frac{(W + 14.9 \times 10^{-6} \times 9.81)}{V} = 19.04\,kN/m^3$

Note that $\gamma_{dry} < \gamma_{bulk} < \gamma_{sat}$

Example 2

Volume and weight distributions

Phase	Volume (m^3)	Dry Weight (kN)	Saturated Weight (kN)
Voids	0.7	0	$0.7 \times 9.81 = 6.87$
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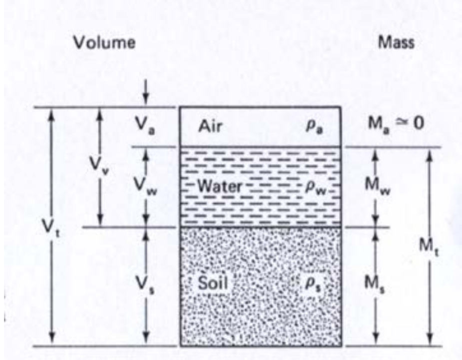
Moisture content (if saturated) $$m = \frac{6.87}{26.0} = 0.264 = 26.4\%$$

Relationships



(1)Water Content *w* **(100%)**

$$w = \frac{Mass\ of\ water\ (M_w)}{Mass\ of\ soil\ solids\,(M_s)} \cdot 100\%$$

-
- For some organic soils *w*>100%, up to 500 %
- For quick clays, *w*>100%

(2)Density of water (slightly varied with temperatures)

$$\rho_w = 1\,g/cm^3 = 1000\,kg/m^3 = 1\,Mg/m^3$$

- (3) Density of soil
- **a. Dry density**

$$\rho_d = \frac{\text{Mass of soil solids}(M_s)}{\text{Total volume of soil sample}(V_t)}$$

- **b. Total, Wet, or Moist density** (0%<S<100%, Unsaturated)

- **c. Saturated density** (S=100%, V_a =0)

$$\rho = \frac{\text{Mass of soil sample}(M_s + M_w)}{\text{Total volume of soil sample}(V_t)}$$

- **d. Submerged density** (Buoyant density)

$$\rho_{sat} = \frac{\text{Mass of soil solids + water}(M_s + M_w)}{\text{Total volume of soil sample}(V_t)}$$

$$\rho' = \rho_{sat} - \rho_w$$

Thanks