Lecture 03

Design of Doubly Reinforced Beam in Flexure

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Topics Addressed

- Background
- Flexural Capacity
- Maximum Reinforcement
- Design Steps
- Examples
- References

Background

- The problem in increasing the capacity of the beam is the restriction that A_s should not exceed A_{smax}. This places a restriction on the maximum flexural capacity of the beam.
- If A_s exceeds A_{smax}, the strain in concrete will reach to a value of 0.003 before ε_s reaches to 0.005, thus violating the ACI code recommenation for ensuring ductile behavior.
- However, If either the strength of concrete is increased or some reinforcement is placed on compression side, the load at which strain will reach to a value of 0.003 will be increased, When this happens A_s on tension side can be increaseed without compromising ductility, which will also increase the flexural capacity of the beam.

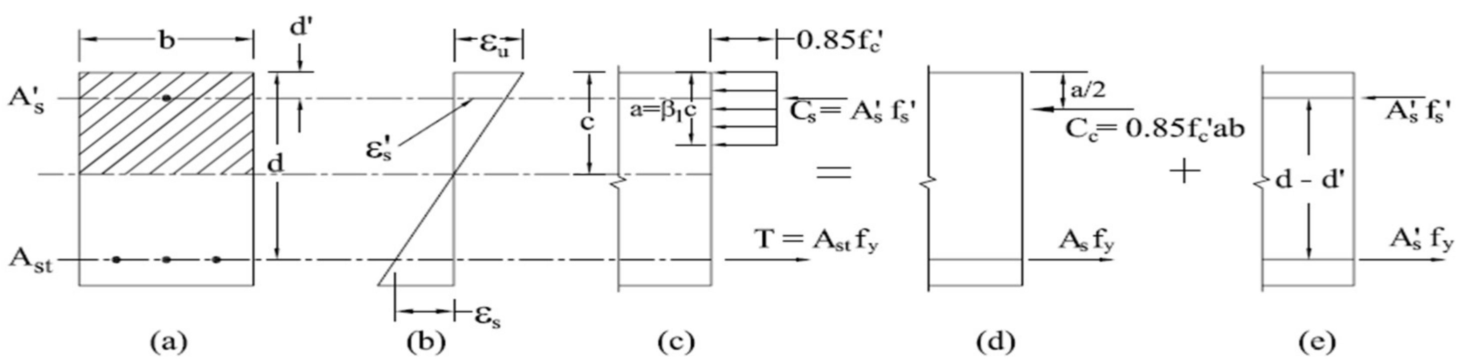
Background

- Practically this can achieved simply by placing some amount of additional reinforcement A_s' on both faces (tension and compression) of the beam. This will increase the range of A_{smax}
- In this case the beam is called as doubly reinforced beam.

Flexural Capacity

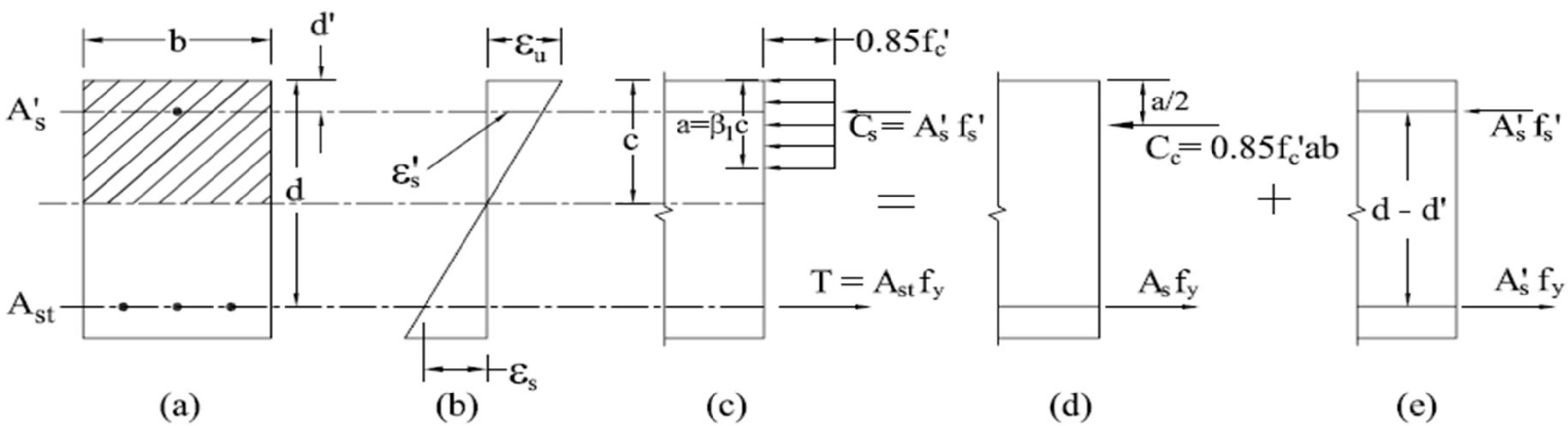
- Consider figure d and e, the flexural capacity of doubly reinforced beam consists of two couples:
- The forces $A_s f_y$ and $0.85f_c'ab$ provides the couple with lever arm (d – a/2).
- $M_{n1} = A_s f_y (d – a/2)$ ……..……………………… (c)
- The forces $A_s' f_y$ and $A_s' f_s'$ provide another couple with lever arm (d – d′).

  $M_{n2} = A_s' f_s' (d – d')$ …………………………………….. (d)



Flexural Capacity

- The total nominal capacity of doubly reinforced beam is therefore,
- $M_n = M_{n1} + M_{n2} = A_s f_y (d – a/2) + A_s' f_s' (d – d')$



Flexural Capacity

- Factored flexural capacity is given as,

  $\Phi M_n = \Phi A_s f_y (d – a/2) + \Phi A_s' f_s' (d – d')$ …………….. (e)

- To avoid failure, $\Phi M_n \geq M_u$. For $\Phi M_n = M_u$, we have from equation (e),

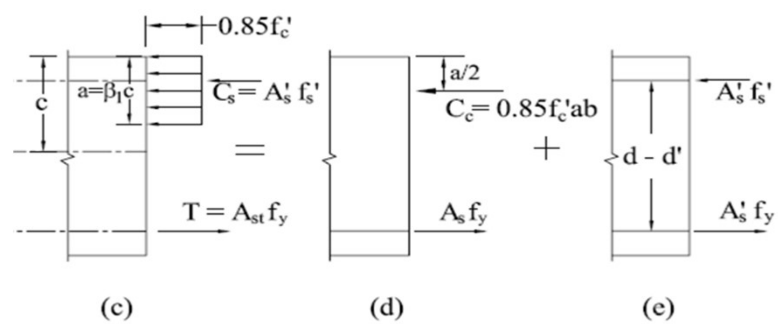
  $M_u = \Phi A_s f_y (d – a/2) + \Phi A_s' f_s' (d – d')$ ……………..… (f)

- Where, $\Phi A_s f_y (d – a/2)$ is equal to $\Phi M_{nmax\ (singly)}$ for $A_s = A_{smax}$
- Therefore, $M_u = \Phi M_{nmax\ (singly)} + \Phi A_s' f_s' (d – d')$
- $\{M_u – \Phi M_{nmax\ (singly)}\} = \Phi A_s' f_s' (d – d')$
- $A_s' = \{M_u – \Phi M_{nmax\ (singly)}\} / \{\Phi f_s' (d – d')\}$ …………….. (g) ; where, $f_s' \leq f_y$

Maximum Reinforcement

- $C_c + C_s = T$ [$\sum F_x = 0$]
- $0.85f_c'ab + A_s'f_s' = A_{st}f_y$
- For $A_{max} = \beta_1 c = 0.85 \times 0.375d$; A_{st} will become A_{stmax}
- $0.85f_c'\beta_1 0.375db + A_s'f_s' = A_{stmax}f_y$
- $A_{stmax} = \beta_1 0.31875bdf_c'/f_y + A_s'f_s'/f_y$
- $A_{stmax} = A_{smax\ (singly)} + A_s'f_s'/f_y$

C_c = Compression force due to concrete in compression region,
C_s = Compression force in steel in compression region needed to balance the tension force in addition to the tension force provided by $A_{smax\ (singly)}$.

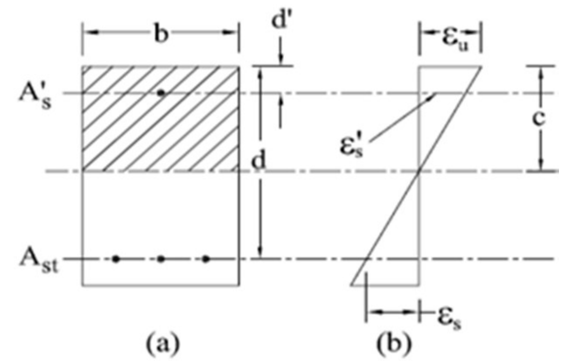


Maximum Reinforcement

- $A_{stmax} = A_{smax\ (singly)} + A_s'f_s'/f_y$
- The total steel area actually provided A_{st} as tension reinforcement must be less than A_{stmax} in above equation i.e. $A_{st} \leq A_{stmax}$
- $A_{stmax\ (singly)}$ is a fixed number, whereas A_s' is steel area actually placed on compression side. (For more clarification, see example)
  - Note that Compression steel in the above equation may or may not yield when tension steel yields.

Maximum Reinforcement

- Conditions at which $f_s' = f_y$ when tension steel yields.



  - By similarity of triangle (fig b),

    compression steel strain (ε_s') is,
  - $\varepsilon_s' = \varepsilon_u (c – d')/ c$ …………………………….. (h)
  - For tensile steel strain (ε_s) = ε_t = 0.005 (for under reinforced behavior):
  - $c = 0.375d$
  - Substituting the value of c in eqn. (h), we get,
  - $\varepsilon_s' = \varepsilon_u (0.375d – d')/ 0.375d = (0.003 – 0.008d'/d)$ ……………….... (i)
  - Equation (i) gives the value of ε_s' for the condition at which reinforcement on tension side is at strain of 0.005 ensuring ductility.

Maximum Reinforcement

- Conditions at which $f_s' = f_y$ when tension steel yields.
  - $\varepsilon_s' = \{0.003 – 0.008d'/d\}$ ……..…………………. (i) OR
  - $d'/d = (0.003 - \varepsilon_s')/0.008$ ………………………. (j)
  - Substituting $\varepsilon_s' = \varepsilon_y$,in equation (j).
  - $d'/d = (0.003 - \varepsilon_y)/0.008$ ………….………..…. (k)
  - Equation (k) gives the value of d′/d that ensures that when tension steel is at a strain of 0.005 (ensuring ductility), the compression steel will also be at yield.
  - Therefore for compression to yield, d′/d should be less than the value given by equation (k).

Maximum Reinforcement

- Conditions at which $f_s' = f_y$ when tension steel yields.
  - Table 3 gives the ratios (d′/d) and minimum beam effective depths (d) for compression reinforcement to yield.
  - For grade 40 steel, the minimum depth of beam to ensure that compression steel will also yields at failure is 12.3 inch.

Table 3: Minimum beam depths for compression reinforcement to yield

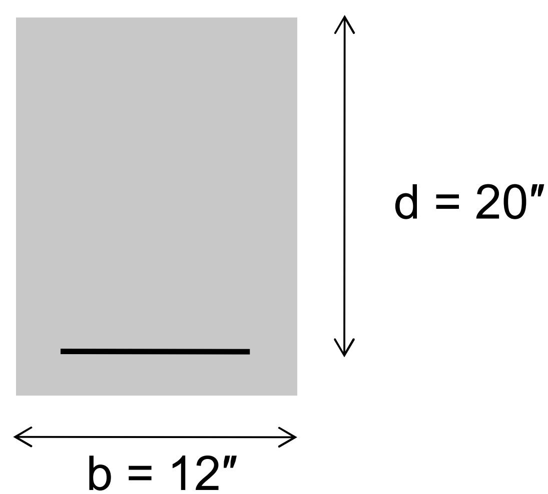
f_y, psi	Maximum d'/d	Minimum d for d' = 2.5" (in.)
40000	0.2	12.3
60000	0.12	21.5

Design Steps

- Step No. 01: Calculation of $\Phi M_{nmax\ (singly)}$
- Step No. 02: Moment to be carried by compression steel
- Step No. 03: Find ε_s' and f_s'
- Step No. 04: Calculation of A_s' and A_{st}.
- Step No. 05: Ensure that $d'/d < 0.2$ (for grade 40) so that selection of bars does not create compressive stresses lower than yield.
- Step No. 06: Ductility requirements: $A_{st} \leq A_{stmax}$
- Step No. 07: Drafting

Example

- Design a doubly reinforced concrete beam for an ultimate flexural demand of 4500 in-kip. The beam sectional dimensions are restricted. Material strengths are $f_c' = 3$ ksi and $f_y = 40$ ksi.



Example

- Solution:
  - **Step No. 01: Calculation of $\Phi M_{nmax\ (singly)}$**

    $\rho_{max\ (singly)} = 0.0203$

    $A_{smax\ (singly)} = \rho_{max\ (singly)} bd = 4.87\ in^2$

    $\Phi M_{nmax\ (singly)} = 2948.88$ in-kip

  - **Step No. 02: Moment to be carried by compression steel**

    $M_{u\ (extra)} = M_u - \Phi M_{nmax\ (singly)}$

    $= 4500 - 2948.88 = 1551.12$ in-kip

Example

- Solution:
  - **Step No. 03: Find ε_s' and f_s'**

    From table 2, d = 20″ > 12.3″, and for d′ = 2.5″, d′/d is 0.125 < 0.20 for grade 40 steel. So compression steel will yield.

    Stress in compression steel $f_s' = f_y$

    Alternatively,

    $\varepsilon_s' = (0.003 – 0.008d'/d)$ ………………….. (i)

    $\varepsilon_s' = (0.003 – 0.008 \times 2.5/20) = 0.002 > \varepsilon_y = 40/29000 = 0.00137$

    As ε_s' is greater than ε_y, so the compression steel will yield.

Example

- Solution:
  - **Step No. 04: Calculation of A_s' and A_{st}.**

    $A_s' = M_{u(extra)}/\{\Phi f_s'(d - d')\}=1551.12/\{0.90\times40\times(20–2.5)\}= 2.46\ in^2$

  - Total amount of tension reinforcement (A_{st}) is,

    $A_{st} = A_{smax\ (singly)} + A_s'= 4.87 + 2.46 = 7.33\ in^2$

  - Using #8 bar, with bar area $A_b = 0.79\ in^2$

    No. of bars to be provided on tension side = A_{st}/ A_b= 7.33/ 0.79 = 9.28

    No. of bars to be provided on compression side=A_s'/A_b = 2.46/ 0.79 = 3.11

    Provide 10 #8 (7.9 in^2 in 3 layers) on tension side and

    4 #8 (3.16 in^2 in 1 layer) on compression side.

Example

- Solution:
  - **Step No. 05: Ensure that d′/d < 0.2 (for grade 40) so that selection of bars does not create compressive stresses lower than yield.**

    With tensile reinforcement of 10 #8 bars in 3 layers and compression reinforcement of 4 #8 bars in single layer, d = 19.625″ and d′ = 2.375

    d′/d = 2.375/ 19.625 = 0.12 < 0.2, OK

Example

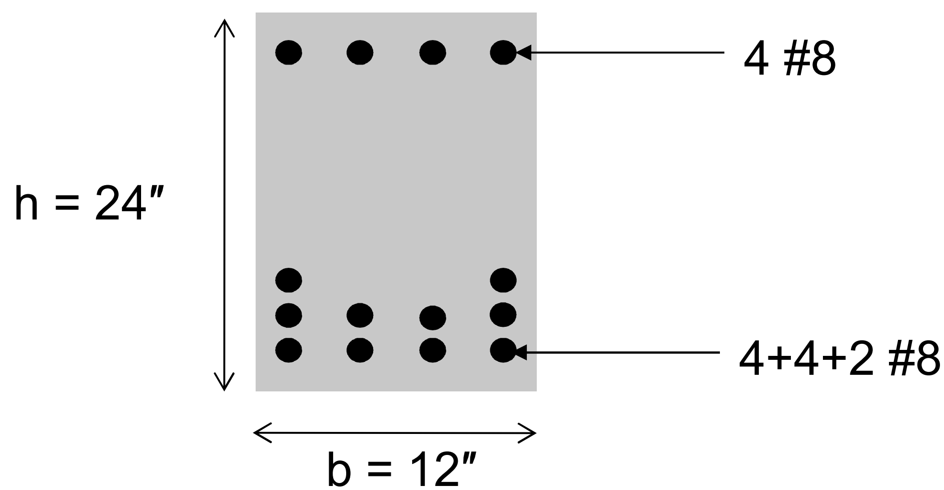
- Solution:
  - **Step No. 06: Ductility requirements:** $A_{st} \leq A_{stmax}$
  - A_{st} , which is the total steel area actually provided as tension reinforcement must be less than A_{stmax} .
    - $A_{stmax} = A_{smax\ (singly)} + A_s'f_s'/f_y$
    - $A_{stmax\ (singly\ )}$ is a fixed number for the case under consideration and A_s' is steel area actually placed on compression side.
  - $A_{smax\ (singly)} = 4.87\ in^2$; $A_s' = 4 \times 0.79 = 3.16\ in^2$
  - $A_{stmax} = 4.87 + 3.16 = 8.036\ in^2$

    $A_{st} = 7.9\ in^2$

    Therefore $A_{st} = 7.9\ in^2 < A_{stmax}$ OK.

Example

- Solution:
  - **Step No. 07: Drafting**
  - **Provide 10 #8 (7.9 in^2 in 3 layers) on tension side and 4 #8 (3.16 in^2 in 1 layer) on compression side.**



References

- Design of Concrete Structures 14th Ed. by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-14)