Lecture 02

Design of Singly Reinforced Beam in Flexure

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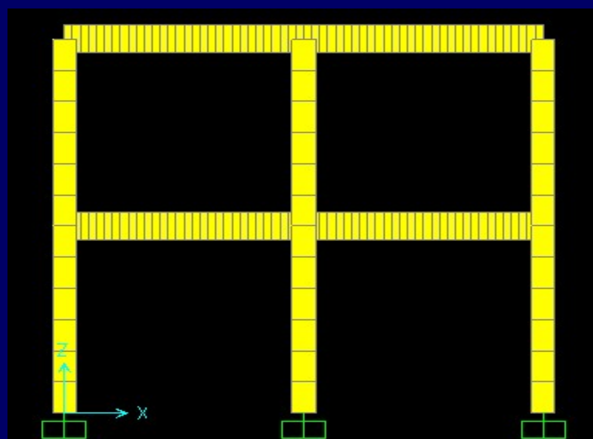
Topics Addressed

- Behavior of RC Beams under gravity load
- Mechanics of RC Beams under gravity load
- ACI Code Recommendations
- Design Steps
- Example

Behavior of RC Beams Under Gravity Load



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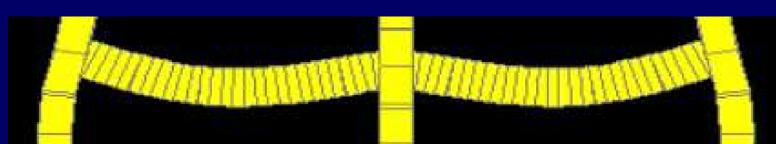
Behavior of RC Beams Under Gravity Load



Behavior of RC Beams Under Gravity Load

- Beam Test

In order to clearly understand the behavior of RC members subjected to flexure load only, the response of such members at three different loading stages is discussed.

Mechanics of RC Beams Under Gravity Load

1. Un-cracked Concrete – Elastic Stage:

- At loads much lower than the ultimate, concrete remains un-cracked in compression as well as tension and the behavior of steel and concrete both is elastic.

2. Cracked Concrete (tension zone) – Elastic Stage

- With increase in load, concrete cracks in tension but remains un-cracked in compression. Concrete in compression and steel in tension both behave in elastic manner.

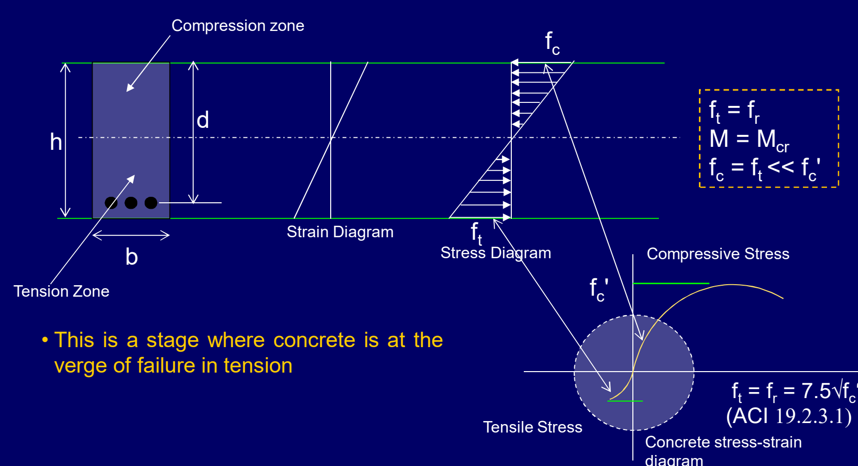
Mechanics of RC Beams Under Gravity Load

3. Cracked Concrete (tension zone) – Inelastic (Ultimate Strength) Stage

- Concrete is cracked in tension. Concrete in compression and steel in tension both enters into inelastic range. At collapse, steel yields and concrete in compression crushes.

Mechanics of RC Beams Under Gravity Load

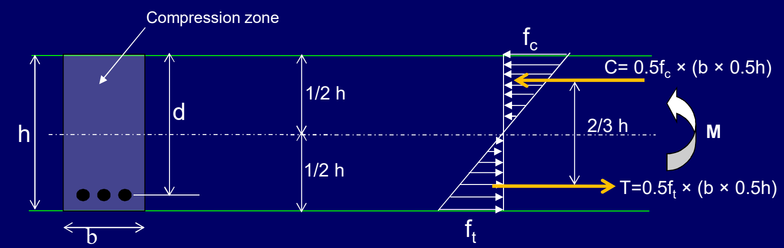
Stage-1: Behavior



- This is a stage where concrete is at the verge of failure in tension

Mechanics of RC Beams Under Gravity Load

Stage-1: Calculation of Forces



$C = T$; $f_c = f_t$

$M = 0.5f_c \times (b \times 0.5h) \times (2/3\ h)$

$\quad = 1/6\ f_c \times b \times h^2$

$f_c = f_t = 6M/(bh^2)$

OR

$f_c = f_t = Mc/I_g$

where $c = 0.5h$

$I_g = bh^3/12$

$f_c = f_t = 6M/(bh^2)$

At $f_t = f_r$, where modulus of rupture, $f_r = 7.5 \sqrt{f_c'}$

Cracking Moment Capacity, $M_{cr} = f_r \times I_g/(0.5h) = (f_r \times b \times h^2)/6$

The contribution of steel is ignored for simplification.

If there is no reinforcement, member will fail in tension.

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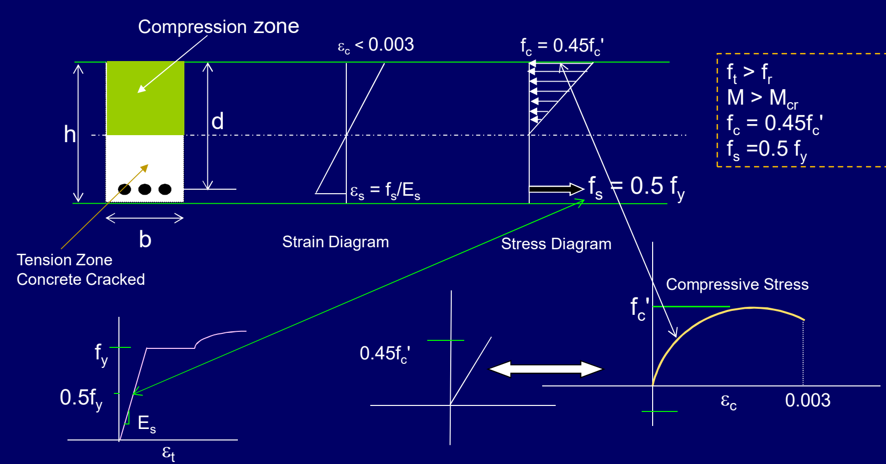
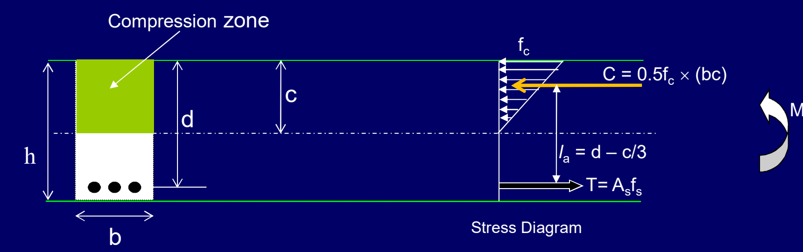
Mechanics of RC Beams Under Gravity Load

Stage-2: Behavior



Mechanics of RC Beams Under Gravity Load

Stage-2: Calculation of Forces



In terms of moment couple ($\sum M = 0$)

$M = Tl_a = A_sf_s (d – c/3)$

$A_s = M/f_s(d – c/3)$

$C = T \quad (\sum F_x = 0)$

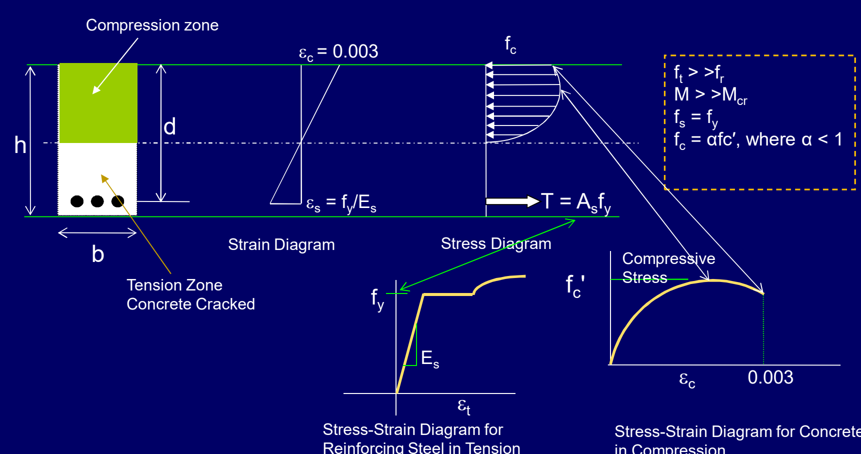
$(½)f_cbc = A_sf_s$

$c = 2A_sf_s / f_cb$ {where $f_s = nf_c$ and $n = E_s/E_c$}

$c = 2A_sn/b$

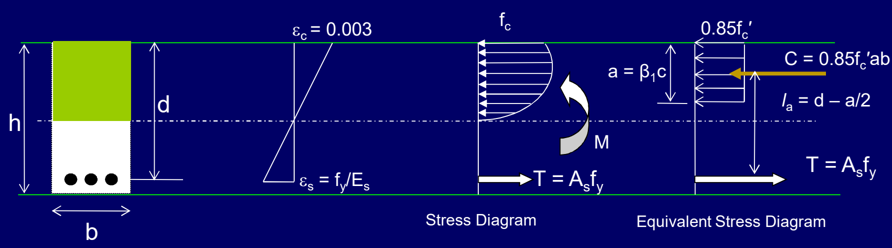
Mechanics of RC Beams Under Gravity Load

Stage-3: Behavior



Mechanics of RC Beams Under Gravity Load

Stage-3: Calculation of Forces



In terms of moment couple ($\sum M = 0$)

$M = T l_a = A_s f_y (d - a/2)$

$A_s = M/f_y(d - a/2)$

$C = T \quad (\sum F_x = 0)$

$0.85f_c'ab = A_s f_y$

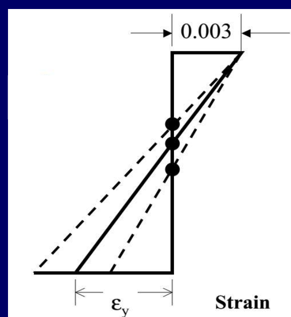
$a = A_s f_y / 0.85f_c' b$

Mechanics of RC Beams Under Gravity Load

- Basic Assumptions: (ACI 22.2)
  - A plane section before bending remains plane after bending.
  - Stresses and strain are approximately proportional up to moderate loads (concrete stress ≤ $0.5f_c'$). When the load is increased, the variation in the concrete stress is no longer linear.
  - Tensile strength of concrete is neglected in the design of reinforced concrete beams.
  - The bond between the steel and concrete is perfect and no slip occurs.
  - Strain in concrete and reinforcement shall be assumed proportional to the distance from neutral axis.

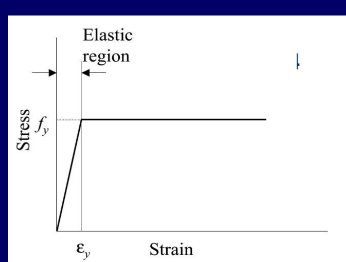
Mechanics of RC Beams Under Gravity Load

- Basic Assumptions: (ACI 22.2)
  - The maximum usable concrete compressive strain at the extreme fiber is assumed to be 0.003.



Mechanics of RC Beams Under Gravity Load

- Basic Assumptions: (ACI 22.2)
  - The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel. Also if the strain in the steel ε_s is less than the yield strain of the steel ε_y, the stress in the steel is $E_s\varepsilon_s$. If $\varepsilon_s \geq \varepsilon_y$, the stress in steel will be equal to f_y



Idealized Stress-Strain Curve

ACI Code Recommendations

1. Strength Design Method (ACI 4.6)

- According to the ACI Code, the RC Members shall be designed using the strength design method.
- In the strength design method, the loads are amplified and the capacities are reduced.

ACI Code Recommendations

1. Strength Design Method

- The loads are amplified in the following manner.
- Load combinations (ACI 5.3)

$W_U = 1.2\, W_D + 1.6\, W_L$

$M_U = 1.2\, M_D + 1.6\, M_L$

Where; W_D = Dead load and W_L = Service Live load

W_U = Amplified load or Ultimate load

M_U = Amplified moment or Ultimate moment

ACI Code Recommendations

1. Strength Design Method

- According to strength design method the resisting member flexural capacity calculated from specified dimension (size of members) and material strength called as the nominal flexural capacity $M_n = A_s f_y (d - a/2)$ shall be reduced by multiplying it with strength reduction factor $\Phi = 0.9$, to get the design flexural capacity (M_d).

$M_d = \Phi M_n$; $\Phi = 0.9$

For no failure; $\Phi M_n = M_u$

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ACI Code Recommendations

2. Nominal Flexural Capacity of RC Member

- The nominal flexural capacity of RC Members shall be calculated from the conditions corresponding to stage 3.
  - **ACI-R21.2.2 —** *The Nominal Flexural Strength (ΦM_n) of a RC member is reached when the strain in the extreme compression fiber reaches the assumed strain limit of 0.003,* (i.e. strains at stage 3.)
- In other words, the member finally fails by crushing of concrete, even if steel in tension has yielded well before crushing of concrete.

ACI Code Recommendations

3. Maximum Reinforcement (Asmax): (ACI 21.2.2)

- When concrete crushes at $\varepsilon_c = 0.003$, depending on the amount of steel (A_s) present as tension reinforcement, following conditions are possible for steel strain (ε_s)

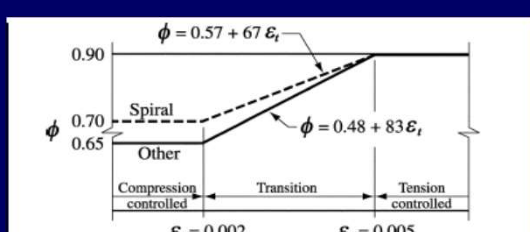
1. $\varepsilon_s = \varepsilon_y$ Balanced Failure Condition, Brittle Failure
2. $\varepsilon_s < \varepsilon_y$ Over reinforced condition, Brittle failure
3. $\varepsilon_s > \varepsilon_y$ Under Reinforced Condition, Ductile Failure

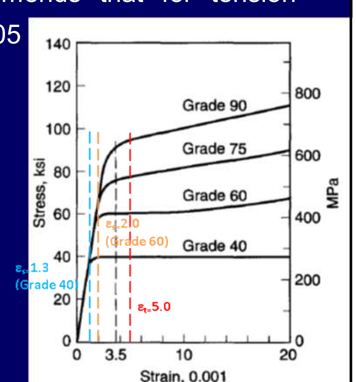
- For relative high amount of tension reinforcement, failure may occur under conditions 1 & 2, causing brittle failure. It is for this reason that ACI code restricts maximum amount of reinforcement in member subjected to flexural load only.

ACI Code Recommendations

3. Maximum Reinforcement (Asmax): (ACI 21.2.2)

- To ensure ductile failure & hence to restrict the maximum amount of reinforcement, the ACI code recommends that for tension controlled sections (Beams) $\varepsilon_s = \varepsilon_t = 0.005$



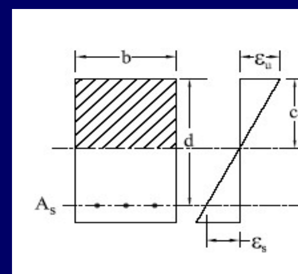


ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}): (ACI 21.2.2)

- From equilibrium of internal forces,
- $\sum F_x = 0 \quad \rightarrow \quad C = T$
- $0.85f_c'ab = A_sf_y$ …………(a)
- From similarity of triangles,

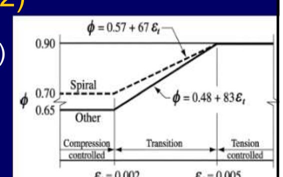
  in strain diagram at failure condition,
- $c/\varepsilon_u = (d - c)/\varepsilon_s$
- $c = d\varepsilon_u/(\varepsilon_u + \varepsilon_s)$



ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}): (ACI 21.2.2)

- For ductility in Tension Controlled sections (Beams)
- $\varepsilon_s = \varepsilon_t = 0.005$ (ACI 10.3.5)
- and at failure $\varepsilon_u = 0.003$ (ACI R10.3.3),
- $c = d\varepsilon_u/(\varepsilon_u + \varepsilon_s) \rightarrow c = 0.375d$ and, $a = \beta_1 c = \beta_1 0.375d$
- Therefore, when $a = \beta_1 0.375d$, $A_s = A_{smax}$ in equation (a). Hence equation (a) becomes,
- $0.85f_c'\beta_1 0.375db = A_{smax}f_y$
- $A_{smax} = 0.31875\beta_1 bd\, f_c'/f_y$ … (b)



318-11, 10.2.7.3 — Factor β_1 shall be taken as 0.85 for concrete strengths f_c' up to and including 4000 psi. For strengths above 4000 psi, β_1 shall be reduced continuously at a rate of 0.05 for each1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65.

ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}): (ACI 21.2.2)
   - $A_{smax} = 0.31875\ \beta_1 bd\ f_c'/f_y$ … (b)
   - For $\beta_1 = 0.85$; $f_c' = 3$ ksi ; and $f_y = 40$ ksi
     - $A_{smax} = 0.0203$ bd;
     - $\rho_{max} = A_{smax}$ / bd = 0.0203 which means 2 % of effective area of concrete.

     ρ = Reinforcement ratio = Area of steel / Effective area of concrete
   - β1 = 0.85; fc′ = 3 ksi ; and $f_y = 60$ ksi
     - A_{smax} = 0. 0135 bd; which means 1.35 % of gross area of concrete

ACI Code Recommendations

4. Minimum Reinforcement (A_{smin}): (ACI 9.6.1.2)
   - At every section of a flexural member where tensile reinforcement is required by analysis, the area A_s provided shall not be less than that given by $\rho_{min} b_w d$ where, ρ_{min} is equal to $3\sqrt{(f_c')}/f_y$ and not less than $200/f_y$.

For a statically determinate beam, this reinforcement shall be doubled.

ACI Code Recommendations

- ρ_{max} and ρ_{min} for various values of f_c' and f_y

Table 01: Maximum & Minimum Reinforcement Ratios

f_c' (psi)	3000		4000		5000	
f_y (psi)	40000	60000	40000	60000	40000	60000
ρ_{min}	0.005	0.0033	0.005	0.0033	0.0053	0.0035
ρ_{max}	0.0203	0.0135	0.027	0.018	0.0319	0.0213

Design Steps

- The design involves the following steps:
  - Selection of Sizes
  - Calculation of Loads
  - Analysis
  - Design
  - Drafting

Design Steps

- Selection of Sizes
  - Minimum depth of beams as per ACI 9.3.1

Support Conditions	Minimum h
Simply supported	l /16
One end continuous	l /18.5
Both ends continuous	l /21
Cantilever	l /8

Where l is the span length of the beam

For f_y other than 60,000 psi, the expressions in Table shall be multiplied by (0.4 + f_y/100,000).

Design Steps

- Calculation of Loads
  - Loads are calculated as follows:

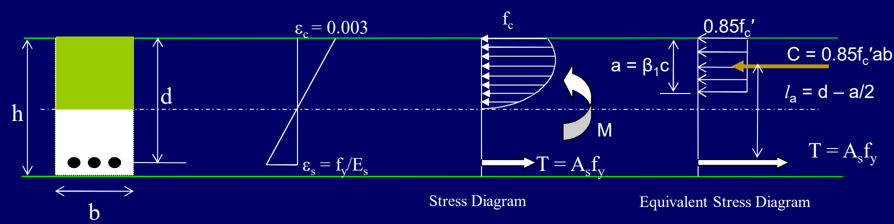
    $W_u = 1.2W_D + 1.6W_L$
- Analysis
  - The analysis of the member is carried out for ultimate load including self weight obtained from size of the member and the applied dead and live loads.
  - The maximum bending moment value is used for flexural design.

Design Steps

- Design
  - Assume "a"
  - Then calculate area of steel using the equation,

    $A_s = M_u / \{\Phi f_y (d – a/2)\}$
  - Confirm the 'a' value using the equation,

    $a = A_s f_y / 0.85 f_c' b$
  - Perform trial and success procedure until same A_s value is obtained from two consecutive trials



Design Steps

- Design
  - Flexure capacity check
  - $M_n = A_s f_y (d – a/2)$ [Nominal capacity]
  - $\Phi M_n = \Phi A_s f_y (d – a/2)$ [Design capacity]
  - To avoid failure, $\Phi M_n \geq M_u$

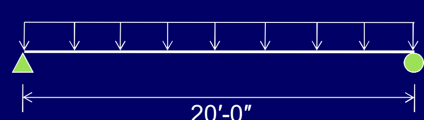
Design Steps

- Drafting
  - Based on the design, drawings of the structural members are prepared showing the dimensions of member and detail of reinforcing bars.

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - Design the beam shown below as per ACI 318-14.

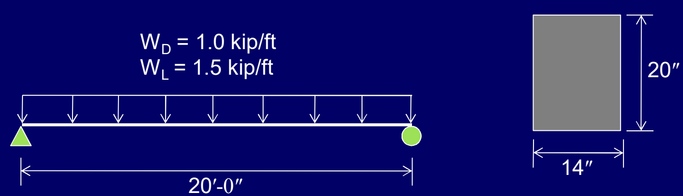
W_D = 1.0 kip/ft
W_L = 1.5 kip/ft



Take f'_c = 3 ksi & f_y = 40 ksi

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 01: Sizes.**
      - For 20′ length, h_{min} = l/16 = 20*12/16 = 15″
      - However we select 20″ deep beam
      - Width of beam cross section (b_w) = 14″ (assumption)



Beam section

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Example 2.1

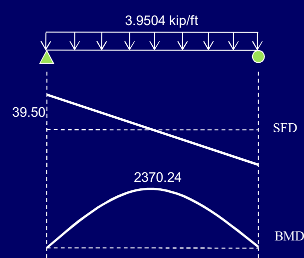
- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 02: Loads.**
      - Self weight of beam = $\gamma_c b_w h$ = 0.15 × (14 × 20/144) = 0.292 kips/ft
      - $W_u = 1.2W_D + 1.6W_L$

        = 1.2 × (1.0 + 0.292) + 1.6 × 1.5 = 3.9504 kips/ft

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 03: Analysis.**
      - Flexural Analysis:

        $M_u = W_u l^2/8 = 3.9504 \times (20)^2 \times 12/8 = 2370.24$ in-kips



Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - $\Phi M_n \geq M_u$ (ΦM_n is M_{design} or $M_{capacity}$)
        - For $\Phi M_n = M_u$
        - $\Phi A_s f_y (d - a/2) = M_u$
        - $A_s = M_u / \{\Phi f_y (d - a/2)\}$
        - Calculate "A_s" by trial and success method.

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - First Trial:
        - Assume a = 4″
        - A_s = 2370.24 / [0.9 × 40 × {17.5 – (4/2)}] = 4.25 in^2
        - a = $A_s f_y$/ (0.85f_c′b_w)
        - = 4.25 × 40/ (0.85 × 3 × 14) = 4.76 inches

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Second Trial:
          - A_s = 2370.24 / [0.9 × 40 × {17.5 – (4.76/2)}] = 4.35 in^2
          - a = 4.35 × 40/ (0.85 × 3 × 14) = 4.88 inches
        - Third Trial:
          - A_s = 2370.24 / [0.9 × 40 × {17.5 – (4.88/2)}] = 4.37 in^2
          - a = 4.37 × 40/ (0.85 × 3 × 14) = 4.90 inches
        - “Close enough to the previous value of “a” so that A_s = 4.37 in^2 O.K

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Check for maximum and minimum reinforcement allowed by ACI:
        - $\rho_{min} = 3\sqrt{f'_c}/f_y \geq 200/f_y$
        - $3 \times \sqrt{3000}/40000 = 0.004$
        - $200/40000 = 0.005$
        - Therefore, $\rho_{min} = 0.005$
        - $A_{smin} = \rho_{min} b_w d = 0.005 \times 14 \times 17.5 = 1.225$ in^2

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Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - $\rho_{max} = 0.85\beta_1(f_c'/f_y)\{\varepsilon_u/(\varepsilon_u + \varepsilon_t)\}$
        - ε_t = Net tensile strain. When $\varepsilon_t = 0.005$, $\Phi = 0.9$ for flexural design.
        - $\beta_1 = 0.85$ (for $f_c' \leq 4000$ psi
        - $\rho_{max} = 0.85 \times 0.85 \times (3/40) \times (0.003/(0.003+0.005) = 0.0204 = 2$ % of area of concrete.
        - $A_{smax} = 0.0204 \times 14 \times 17.5 = 4.998$ in^2
        - A_{smin} (1.225) < A_s (4.37) < A_{smax} (4.998) O.K

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Bar Placement: 10 #6 bars will provide 4.40 in^2 of steel area which is slightly greater than required.
        - Other options can be explored. For example,
          - 8 #7 bars (4.80 in^2),
          - 6 #8 bars (4.74 in^2),
          - or combination of two different size bars.

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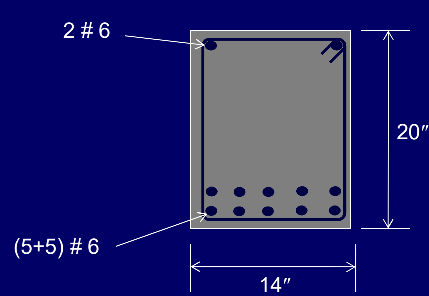
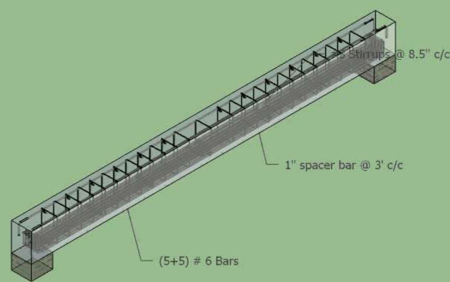
Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 05: Drafting**



3D Model



Example 2.2

- A reinforced concrete simply supported beam has a span of 30 ft and supports service dead load of 1100 lb/ft and a uniform service live load of 1100 lb/ft in addition to its self weight. Design a beam section to resist the factored external bending moment. Given $fc' = 4000\ psi, fy = 60{,}000\ psi$.

References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-14)