Lecture 02

Design of Singly Reinforced Beam in Flexure

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Topics Addressed

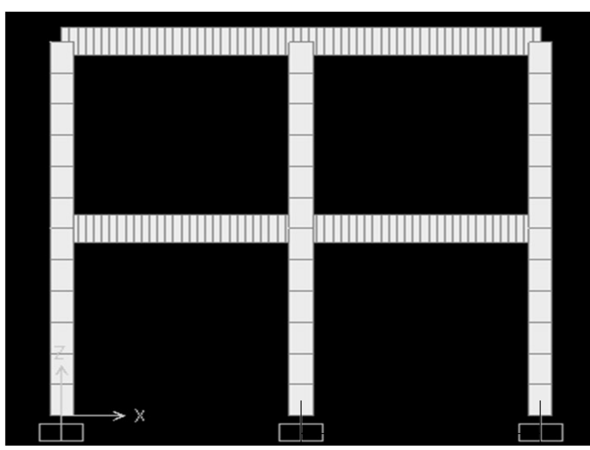
- Behavior of RC Beam under gravity load
- Mechanics of RC Beam under gravity load
- ACI Code Recommendations
- Design Steps
- Example

Objectives

At the end of this lecture, students will be able to;

- Explain behavior of RC Beam under gravity load
- Identify Relevant ACI 318 code recommendations for RC Beam
- Analyze and Design RC Beam for flexure

Behavior of RC Beam Under Gravity Load



Story

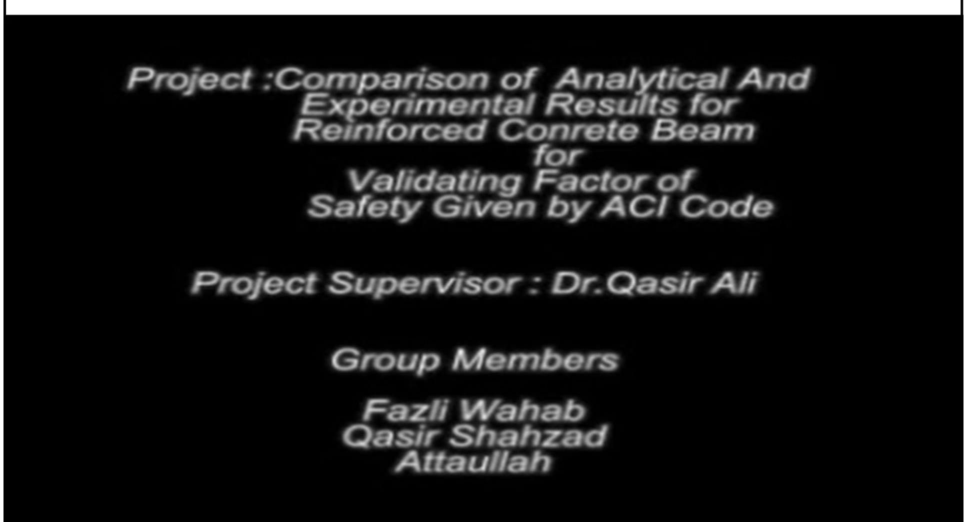
Behavior of RC Beam Under Gravity Load

- Beam Test

  In order to clearly understand the behavior of RC members subjected to flexure load only, the response of such members at three different loading stages is discussed.

Behavior of RC Beam Under Gravity Load

- Beam test video



Mechanics of RC Beam Under Gravity Load

1. Un-cracked Concrete – Elastic Stage:

   - At loads much lower than the ultimate, concrete remains un-cracked in compression as well as in tension and the behavior of steel and concrete both is elastic.

2. Cracked Concrete (tension zone) – Elastic Stage

   - With increase in load, concrete cracks in tension but remains un-cracked in compression. Concrete in compression and steel in tension both behave in elastic manner.

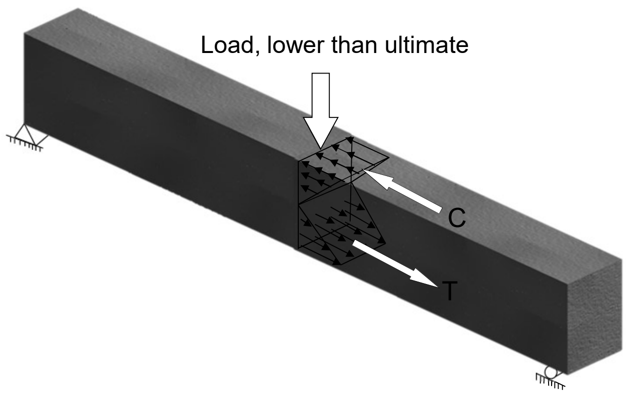
Mechanics of RC Beam Under Gravity Load

3. Cracked Concrete (tension zone) – Inelastic (Ultimate Strength) Stage

   - Concrete is cracked in tension. Concrete in compression and steel in tension both enters into inelastic range. At collapse, steel yields and concrete in compression crushes.

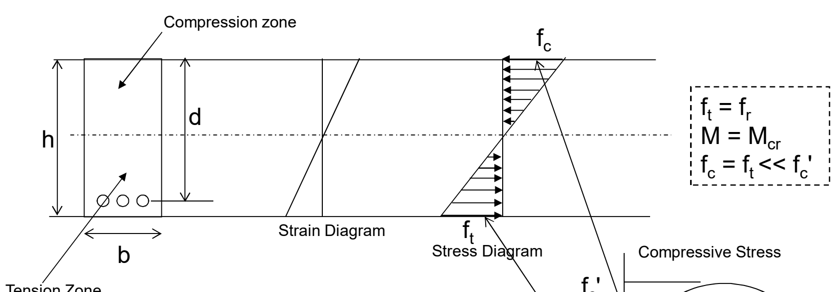
Mechanics of RC Beam Under Gravity Load

Stage-1: Behavior

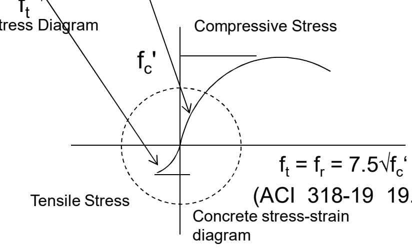


Mechanics of RC Beam Under Gravity Load

Stage-1: Behavior



- This is a stage where concrete is at the verge of failure in tension

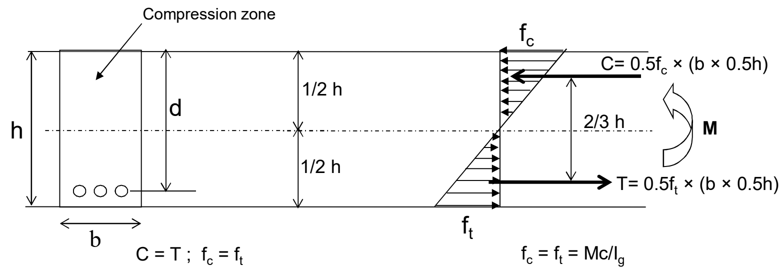


$f_t = f_r = 7.5\sqrt{f_c'}$

(ACI 318-19 19.2.3.1)

Mechanics of RC Beam Under Gravity Load

Stage-1: Calculation of Forces



C = T ; $f_c = f_t$

M = 0.5f_c × (b × 0.5h) × (2/3 h)

= 1/6 f_c × b × h^2

$f_c = f_t = 6M/(bh^2)$

OR

$f_c = f_t = Mc/I_g$

where c = 0.5h

$I_g = bh^3/12$

$f_c = f_t = 6M/(bh^2)$

At $f_t = f_r$, where modulus of rupture, $f_r = 7.5 \sqrt{f_c'}$

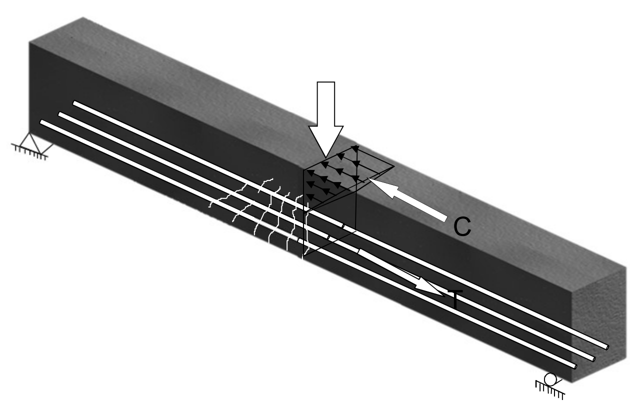
Cracking Moment Capacity, $M_{cr} = f_r \times I_g/(0.5h) = (f_r \times b \times h^2)/6$

The contribution of steel is ignored for simplification.

If there is no reinforcement, member will fail in tension.

Mechanics of RC Beam Under Gravity Load

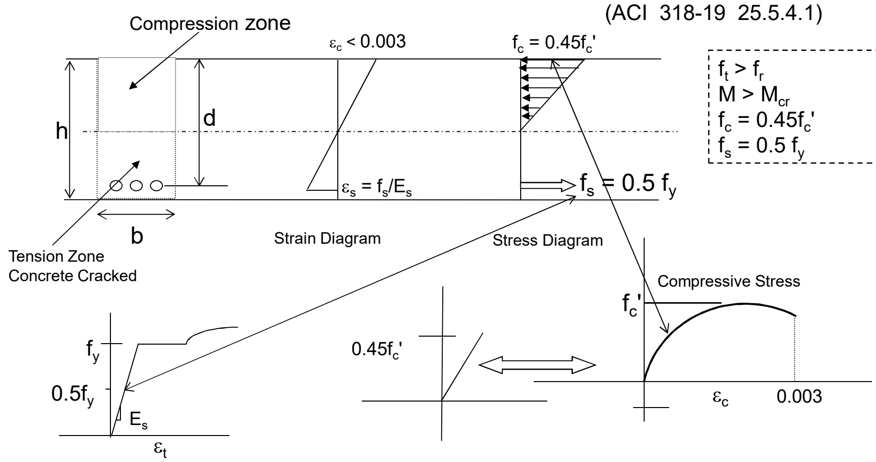
Stage-2: Behavior



Mechanics of RC Beam Under Gravity Load

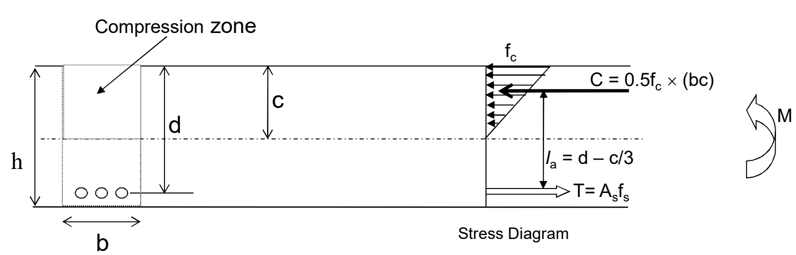
Stage-2: Behavior

(ACI 318-19 25.5.4.1)



Mechanics of RC Beam Under Gravity Load

Stage-2: Calculation of Forces



In terms of moment couple ($\sum M = 0$)

$M = T/_a = A_sf_s\ (d - c/3)$

$A_s = M/f_s(d - c/3)$

$C = T \quad (\sum F_x = 0)$

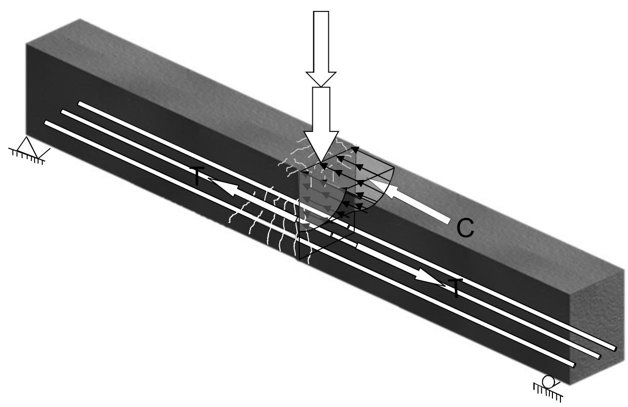
$(½)f_cbc = A_sf_s$

$c = 2A_sf_s\ /\ f_cb$ {where $f_s = nf_c$ and $n = E_s/E_c$}

$c = 2A_sn/b$

Mechanics of RC Beam Under Gravity Load

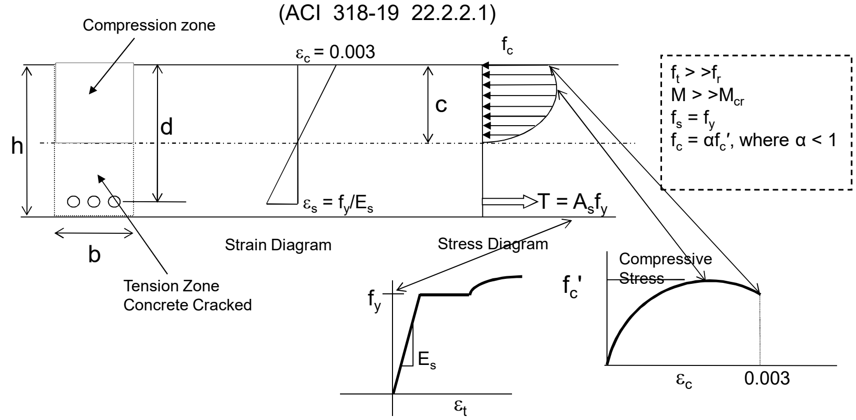
Stage-3: Behavior



Mechanics of RC Beam Under Gravity Load

Stage-3: Behavior

(ACI 318-19 22.2.2.1)

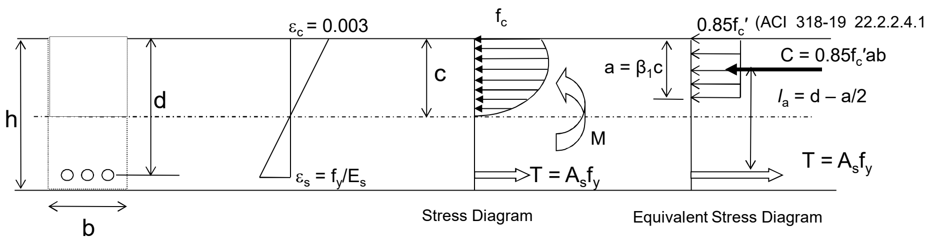


Stress-Strain Diagram for Reinforcing Steel in Tension

Stress-Strain Diagram for Concrete in Compression

Mechanics of RC Beam Under Gravity Load

Stage-3: Calculation of Forces



In terms of moment couple ($\sum M = 0$)

$M = T l_a = A_s f_y (d – a/2)$

$A_s = M/f_y(d – a/2)$

$C = T \quad (\sum F_x = 0)$

$0.85 f_c' ab = A_s f_y$

$a = A_s f_y / 0.85 f_c' b$

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Mechanics of RC Beam Under Gravity Load

- Basic Assumptions: (ACI 318-19, 22.2)
  - A plane section before bending remains plane after bending.
  - Stresses and strain are approximately proportional up to moderate loads (concrete stress ≤ $0.5f_c'$). When the load is increased, the variation in the concrete stress is no longer linear.
  - Tensile strength of concrete is neglected in the design of reinforced concrete beams.
  - The bond between the steel and concrete is perfect and no slip occurs.
  - Strain in concrete and reinforcement shall be assumed proportional to the distance from neutral axis.

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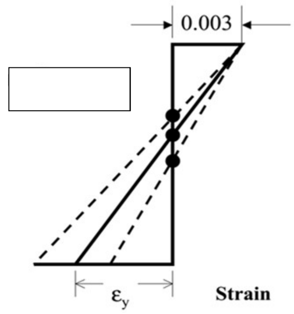
Mechanics of RC Beam Under Gravity Load

- Basic Assumptions: (ACI 318-19, 22.2)
  - The maximum usable concrete compressive strain at the extreme fiber is assumed to be 0.003.



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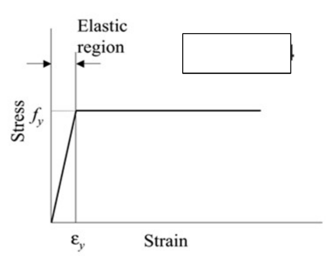
Mechanics of RC Beam Under Gravity Load

- Basic Assumptions: (ACI 318-19, 22.2)
  - The steel is assumed to be uniformly strained to the strain that exists at the level of the centroid of the steel. Also if the strain in the steel ε_s is less than the yield strain of the steel ε_y, the stress in the steel is $E_s\varepsilon_s$. If $\varepsilon_s \geq \varepsilon_y$, the stress in steel will be equal to f_y.

Figure 9



Idealized Stress-Strain Curve

ACI Code Recommendations

1. Strength Design Method (ACI 318-19, 4.6)

- According to the ACI Code, the RC Members shall be designed using the strength design method.
- In the strength design method, the loads are amplified and the capacities are reduced.

Figure 9

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ACI Code Recommendations

1. Strength Design Method

- The loads are amplified in the following manner.
- Load combinations (ACI 318-19, 5.3)
- $W_U = 1.2\ W_D + 1.6\ W_L$
- $M_U = 1.2\ M_D + 1.6\ M_L$
- Where; W_D = Service Dead load and W_L = Service Live load
- W_U = Amplified load or Ultimate load
- M_U = Amplified moment or Ultimate moment

Figure 9

ACI Code Recommendations

1. Strength Design Method

- According to strength design method the resisting member flexural capacity calculated from specified dimension (size of members) and specified material strength called as the nominal flexural capacity $M_n = A_s f_y (d - a/2)$ shall be reduced by multiplying it with strength reduction factor $\Phi = 0.9$, to get the design flexural capacity (M_d).

Figure 9

$M_d = \Phi M_n$; $\Phi = 0.9$

For no failure; $\Phi M_n \geq M_u$

ACI Code Recommendations

2. Nominal Flexural Capacity of RC Member

- The nominal flexural capacity of RC Members shall be calculated from the conditions corresponding to stage 3.
  - **ACI 318-19, R21.2.2 —** *The Nominal Flexural Strength (M_n) of a RC member is reached when the strain in the extreme compression fiber reaches the assumed strain limit of 0.003,* (i.e. strains at stage 3.)

Figure 9

- In other words, the member finally fails by crushing of concrete, even if steel in tension has yielded well before crushing of concrete.

ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}): (ACI 318-19, 21.2.2)

- When concrete crushes at $\varepsilon_c = 0.003$, depending on the amount of steel (A_s) present as tension reinforcement, following conditions are possible for steel strain (ε_s)

1. $\varepsilon_s = \varepsilon_y$ Balanced Failure Condition, Brittle Failure
2. $\varepsilon_s < \varepsilon_y$ Over reinforced condition, Brittle failure
3. $\varepsilon_s > \varepsilon_y$ Under Reinforced Condition, Ductile Failure

- For relatively high amount of tension reinforcement, failure may occur under conditions 1 & 2, causing brittle failure. It is for this reason that ACI code restricts maximum amount of reinforcement in member subjected to flexural load only.

ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}): (ACI 318-19, 21.2.2)

- To ensure ductile failure & hence to restrict the maximum amount of reinforcement, the ACI code recommends that for tension controlled sections (Beams) $\varepsilon_s = \varepsilon_t = \varepsilon_{ty} + 0.003$.

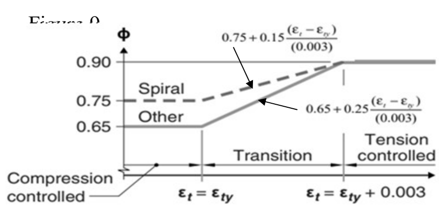
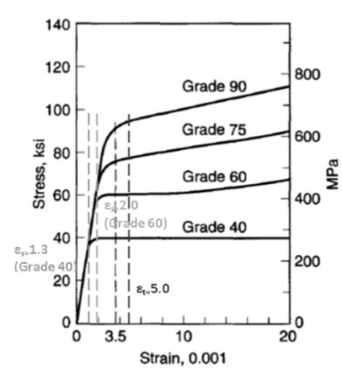


Fig. R21.2.2b—Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t.



ACI Code Recommendations

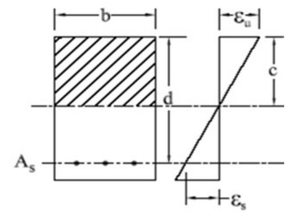
3. Maximum Reinforcement (A_{smax}): (ACI 318-19, 21.2.2)

- From equilibrium of internal forces,
- $\sum F_x = 0 \;\rightarrow\; C = T$
- $0.85f_c'ab = A_sf_y$ …………(a)

Figure 9

- From similarity of triangles,

  in strain diagram at failure condition,
- $c/\varepsilon_u = (d - c)/\varepsilon_s$
- $c = d\varepsilon_u/(\varepsilon_u + \varepsilon_s)$



ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}): (ACI 318-19, 21.2.2)

- For ductility in Tension Controlled sections (Beams)
- $\varepsilon_s = \varepsilon_t = \varepsilon_{ty} + 0.003$ (ACI Table 21.2.2)
- and at failure $\varepsilon_u = 0.003$ (ACI R21.2.2),

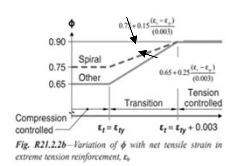


Figure 9

- For $f_y = 40$ ksi
- $c = d\varepsilon_u/(\varepsilon_u + \varepsilon_s) \rightarrow c = 0.41d$ and, $a = \beta_1c = \beta_10.41d$
- For $f_y = 60$ ksi
- $c = d\varepsilon_u/(\varepsilon_u + \varepsilon_s) \rightarrow c = 0.37d$ and, $a = \beta_1c = \beta_10.37d$
- Therefore, when $a = \beta_10.41d$ ($f_y = 40$ ksi), $a = \beta_10.37d$ ($f_y = 60$ ksi), and $A_s = A_{smax}$ in equation (a). Hence equation (a) becomes,
- $0.85f_c'\beta_10.41db = A_{smax}f_y$ ($f_y = 40$ ksi)
- $0.85f_c'\beta_10.37db = A_{smax}f_y$ ($f_y = 60$ ksi)

ACI Code Recommendations

3. Maximum Reinforcement (A_{smax}):
   - For β_1 = 0.85 when $f'_c \le$ 4000 psi
   - $A_{smax} = 0.3 \frac{f_c'}{f_y} bd$ (f_y = 40 ksi)
   - $A_{smax} = 0.27 \frac{f_c'}{f_y} bd$ (fy = 60 ksi)
   - For f_c' = 3 ksi ; and f_y = 40 ksi ; A_{smax} = 0.0225 bd;
     - Let ρ = Reinforcement ratio = Area of steel / Effective area of concrete
     - ρ= A_s / bd
     - ρ_{max} = A_{smax} / bd = 0.0222; which means 2.22 % of effective area of concrete.
   - For f_c' = 3 ksi ; and f_y = 60 ksi
     - A_{smax} = 0. 0135 bd; which means 1.35 % of effective area of concrete.

- **ACI** 318-19, 22.2.2.4.3 — Factor β_1 shall be taken as 0.85 for concrete strengths f_c' up to and including 4000 psi. For strengths above 4000 psi, β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65.

ACI Code Recommendations

- Video of beam having reinforcement more than maximum reinforcement

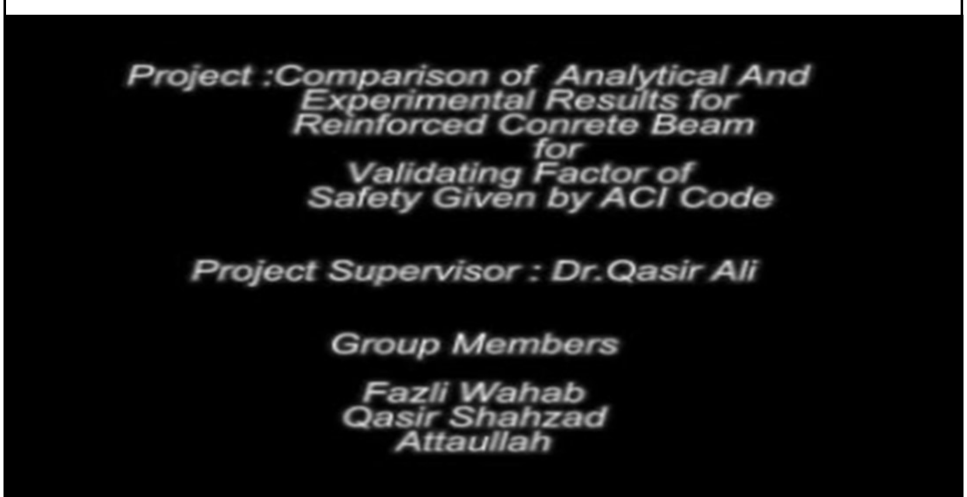
ACI Code Recommendations

4. Minimum Reinforcement (A_{smin}): (ACI 9.6.1.1 & 9.6.1.2)

- *A minimum area of flexural reinforcement, $A_{s,min}$, shall be provided at every section where tension reinforcement is required by analysis.*
- *$A_{s,min}$ shall be the greater of (a) and (b), for fy ≤ 80 ksi*
- (a) $3 \frac{\sqrt{f_c'}}{f_y}$ b d
- (b) $\frac{200}{f_y}$ b d

ACI Code Recommendations

- Video of beam having reinforcement less than minimum reinforcement



ACI Code Recommendations

- ρ_{max} and ρ_{min} for various values of f_c' and f_y

Table 01: Maximum & Minimum Reinforcement Ratios

f_c' (psi)	3000		4000		5000	
f_y (psi)	40000	60000	40000	60000	40000	60000
ρ_{min}	0.005	0.0033	0.005	0.0033	0.0053	0.0035
ρ_{max}	0.0222	0.0134	0.0296	0.018	0.0348	0.021

Design Steps

- The design involves the following steps:
  - Selection of Sizes
  - Calculation of Loads
  - Analysis
  - Design
  - Drafting

Design Steps

1. Selection of Sizes
   - Minimum depth of beams as per ACI 9.3.1

Support Conditions	Minimum h (f_y = 60 ksi)
Simply supported	l /16
One end continuous	l /18.5
Both ends continuous	l /21
Cantilever	l /8

Where l is the span length of the beam

For f_y other than 60 ksi, the expressions in Table shall be multiplied by $(0.4 + \frac{f_y}{100,000})$.

Design Steps

2. Calculation of Loads
   - Loads are calculated as follows:

     $W_u = 1.2W_D + 1.6W_L$

3. Analysis
   - The analysis of the member is carried out for ultimate load including self weight obtained from size of the member and the applied dead and live loads.
   - The maximum bending moment value is used for flexural design.

Design Steps

4. Design

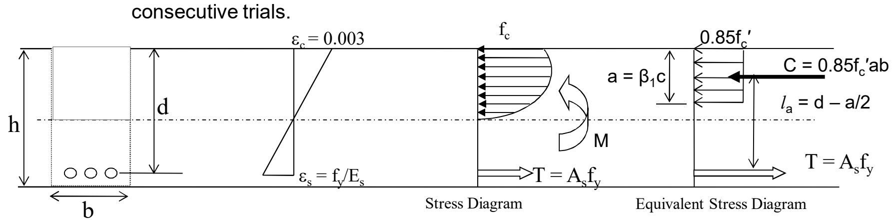
- Assume "a"
- Then calculate area of steel using the equation,

  $A_s = M_u / \{\Phi f_y (d - a/2)\}$

- Confirm the 'a' value using the equation,

  $a = A_s f_y / 0.85 f_c' b$

- Perform trial and success procedure until same A_s value is obtained from two consecutive trials.



Design Steps

4. Design

- Check for A_{smax} and A_{smin}
- $A_{smax} = 0.3 \frac{f_c'}{f_y} bd$ (f_y = 40 ksi)
- $A_{smax} = 0.27 \frac{f_c'}{f_y} bd$ (fy = 60 ksi)

(for $f'_c \leq 4000$ psi)

- $A_{smin} = 3 \frac{\sqrt{f_c'}}{f_y} b\, d \geq \frac{200}{f_y} b\, d$ (whichever is greater) for *fy ≤ 80 ksi*

Design Steps

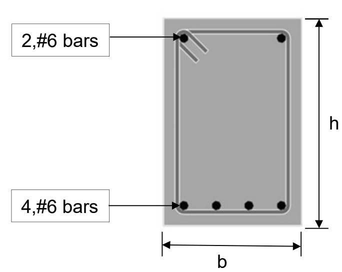
4. Design

- No of Bars Calculation
  - No of bars = A_s / A_b (A_b = Area of one bar to be used)
- The calculated no. of bars must be placed according to the ACI code criteria which is discussed next.

Design Steps

4. Design

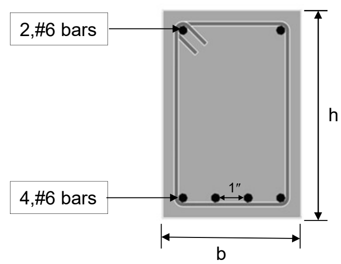
- Placement of bars:
  - Concrete clear cover
  - Minimum concrete clear cover for RC beams reinforcement shall be 1-1.5 in. (ACI 318-19, 20.5.1.3). Usually concrete clear cover is taken as 1.5 in.



Design Steps

4. Design

- Placement of bars:
  - <u>Distance between adjacent bars</u>
  - ACI 318-19, 25.2 specifies that the minimum clear distance between adjacent bars shall be at least the greatest of the nominal diameter of the bars, 1 in and $(4/3)d_{agg.}$



Design Steps

4. Design

- Placement of bars:
  - <u>Maximum number of bars in a single layer</u>
  - Table A.7 of Appendix A gives the maximum number of bars that can be placed in a single layer in beams, assuming 1.5 in. concrete cover and the use of No.4 stirrups.

TABLE A.7
Maximum number of bars as a single layer in beam stems

$\frac{3}{4}$ in. Maximum Size Aggregate, No. 4 (No. 13) Stirrups*

Bar No. Inch-Pound	Bar No. SI	Beam Width b_w, in. 8	10	12	14	16	18	20	22	24	26	28	30
5	16	2	4	5	6	7	8	10	11	12	13	15	16
6	19	2	3	4	6	7	8	9	10	11	12	14	15
7	22	2	3	4	5	6	7	8	9	10	11	12	13
8	25	2	3	4	5	6	7	8	9	10	11	12	13
9	29	1	2	3	4	5	6	7	8	9	9	10	11
10	32	1	2	3	4	5	6	6	7	8	9	10	10
11	36	1	2	3	3	4	5	5	6	7	8	8	9
14	43	1	2	2	3	3	4	5	5	6	6	7	8
18	57	1	1	2	2	3	3	4	4	4	5	5	6

1 in. Maximum Size Aggregate, No. 4 (No. 13) Stirrups*

Bar No. Inch-Pound	Bar No. SI	Beam Width b_w, in. 8	10	12	14	16	18	20	22	24	26	28	30
5	16	2	3	4	5	6	7	8	9	10	11	12	13
6	19	2	3	4	5	6	7	8	9	9	10	11	12
7	22	1	2	3	4	5	6	7	8	9	10	10	11
8	25	1	2	3	4	5	6	7	7	8	9	10	11
9	29	1	2	3	4	5	6	7	7	8	9	9	10
10	32	1	2	3	4	5	6	6	7	7	8	9	10

*Minimum concrete cover assumed to be $1\frac{1}{2}$ in. to the No. 4 (No. 13) stirrup.
Source: Adapted from Ref. 3.8. Used by permission of American Concrete Institute.

Table A.7 Appendix A
Ref: Design of concrete structures, 14th Ed, Nilson

Design Steps

4. Design

- Placement of bars:
  - Minimum number of bars in a single layer
  - There are also restrictions on the minimum number of bars that can be placed in a single layer, based on requirements for the distribution of reinforcement to control the width of flexural cracks. Table A.8 gives the minimum number of bars that will satisfy ACI Code requirements.
  - For beam size upto 15 inch, 2 bars are required.

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Design Steps

4. Design

- Placement of bars:
  - Table A.8

TABLE A.8
Minimum number of bars as a single layer in beam stems governed by crack control requirements of the ACI Code

(a) 2 in. clear cover, sides and bottom

Bar No.		Minimum Number of Bars as a Single Layer of a Beam Stem — Beam Stem Width b_w, in.														
Inch-Pound	SI	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36
3–14	10–43	1	1	2	2	3	3	3	3	3	4	4	4	4	4	5
18	57	1	1	2	2	2	3	3	3	3	3	4	4	4	4	4

(b) $1\frac{1}{2}$ in. clear cover, sides and bottom

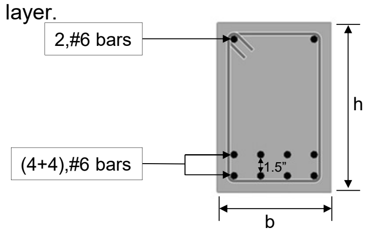
Bar No.		Minimum Number of Bars as a Single Layer of a Beam Stem — Beam Stem Width b_w, in.														
Inch-Pound	SI	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36
3–4	10–13	1	1	2	2	3	3	3	3	3	4	4	4	4	4	4
5–14	16–43	1	1	2	2	3	3	3	3	3	3	4	4	4	4	4
18	57	1	1	2	2	2	3	3	3	3	3	4	4	4	4	4

Table A.8 Appendix A, Ref: Design of concrete structures, 14th Ed, Nilson

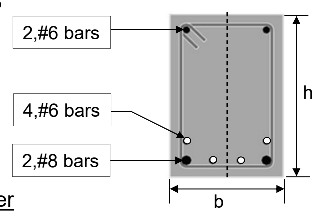
Design Steps

4. Design

- Placement of bars:
  - Distance between adjacent layers
  - Where beam reinforcement is placed in two or more layers, the clear distance between layers must not be less than the greatest of 1.5 in.,1.5d_b, and (4/3)d_{agg}, and the bars in the upper layer should be placed directly above those in the bottom layer.



Design Steps



4. Design

- Placement of bars:
  - Variation in diameter of bars in a single layer
  - The variation in diameter of bars in a single layer shall be limited to two bar sizes, using, say, No. 8 and No. 6 bars together, but not Nos. 5 and 8.
  - Symmetry of bars
  - Bars should be arranged symmetrically about the vertical centerline.

Design Steps

4. Design

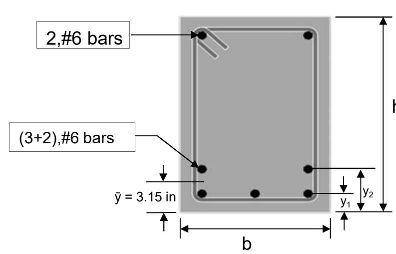
- Flexure capacity check
  - After placement of bars, check the flexural capacity from the actual 'd' and actually placed amount of reinforcement.
    - $\Phi M_n = \Phi A_s f_y(d – a/2)$ [Design capacity]
    - $\Phi M_n \geq M_u$
    - Initially effective depth, d is assumed and for flexure capacity check actual "d" is calculated as shown on next slide.

Design Steps

4. Design

- Flexure capacity check
  - If the assumed effective depth "d" is 15.5 in. The actual d can be calculated by calculating actual $\bar{y}$.
  - For h = 18" ;

    $d = h - \bar{y} = 18 – 3.15 = 14.85$ in



$y_1 = 1.5+3/8+1/2(6/8) = 2.25$ in

$y_2 = 1.5+ 3/8+6/8+1.5+1/2(6/8) = 4.5$ in

$5 \times 0.44 \times \bar{y} = 3 \times 0.44 \times y_1 + 2 \times 0.44 \times y_2$

$\bar{y} = 3.15$ in

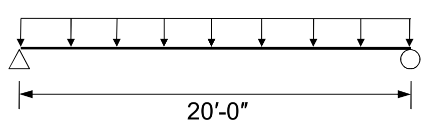
Design Steps

5. Drafting
   - Based on the design, drawings of the structural members are prepared showing the dimensions of member and detail of reinforcing bars.

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - Design the beam shown below as per ACI 318-19.

W_D = 0.5 kip/ft (Excluding self weight of beam)
W_L = 0.5 kip/ft



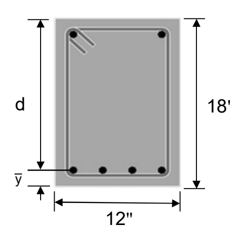
Take f_c' = 3 ksi & f_y = 40 ksi

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 01: Sizes.**
      - For 20′ length, h_{min} = *l* /16 = 20 x 12/16 = 15″
      - For grade 40, we have = h_{min} =15″ x (0.4 + 40,000/100,000) = 12″
      - This is the minimum requirement of the code for depth of beam.
      - However, we select 18″ deep beam.
      - Generally the minimum beam width is 12″, therefore, width of the beam is taken as 12″
      - The final selection of beam size depends on several factors specifically the availability of formwork.

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 01: Sizes.**
      - Depth of beam, h = 18″
      - h = d + ȳ; ȳ is usually taken from 2.5 to 3.0 inches
      - For ȳ = 2.5 in; d = 18 – 2.5 = 15.5″
      - Width of beam cross section (b_w) = 12″
      - In RCD, Width of beam is usually denoted by b_w instead of b



Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 02: Loads.**
      - Self weight of beam = $\gamma_c b_w h$

        $= 0.15 \times (12 \times 18/144)$

        $= 0.225$ kips/ft

      - $W_u = 1.2W_D + 1.6W_L$

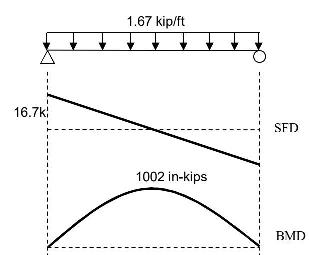
        $= 1.2 \times (0.5 + 0.225) + 1.6 \times 0.5 = 1.67$ kips/ft

- γ_c = density of concrete
- γ_c = 144 lb/ft^3 for plain concrete
- γ_c = 150 lb/ft^3 for Reinforced concrete

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 03: Analysis.**
      - Flexural Analysis:

        $M_u = W_u l^2/8 = 1.67 \times (20)^2 \times 12/8 = 1002$ in-kips



Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - $\Phi M_n \geq M_u$ (ΦM_n is M_{design} or $M_{capacity}$)
        - For $\Phi M_n = M_u$
        - $\Phi A_s f_y(d - a/2) = M_u$
        - $A_s = M_u / \{\Phi f_y (d - a/2)\}$
        - Calculate "A_s" by trial and success method.

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - First Trial:
        - Assume a = 4″
        - $A_s = 1002 / [0.9 \times 40 \times \{15.5 - (4/2)\}] = 2.06$ in^2
        - $a = A_s f_y / (0.85 f_c' b_w)$
        - $= 2.06 \times 40 / (0.85 \times 3 \times 12) = 2.69$ inches

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Second Trial:
          - A_s = 1002 / [0.9 × 40 × {15.5 – (2.69/2)}] = 1.97 in^2
          - a = 1.97 × 40/ (0.85 × 3 × 12) = 2.56 inches
        - Third Trial:
          - A_s = 1002 / [0.9 × 40 × {15.5 – (2.56/2)}] = 1.95 in^2
          - a = 1.95 × 40/ (0.85 × 3 × 12) = 2.55 inches
        - "Close enough to the previous value of "a" so that A_s = 1.95 in^2 O.K

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Check for maximum and minimum reinforcement allowed by ACI:
        - $A_{smin} = 3 (\sqrt{f_c'}/f_y) b_w d \geq (200/f_y) b_w d$
        - $3 (\sqrt{f_c'}/f_y) b_w d = 3 \times (\sqrt{3000}/40000) b_w d$ = 0.004 x 12 x15.5 = **0.744 in^2**
        - $(200/f_y) b_w d$ = (200/40000) x12 × 15.5 = **0.93 in^2**
        - **A_{smin} = 0.93 in^2**

Example 2.1

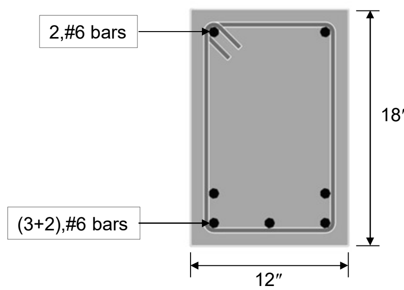
- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - $A_{smax} = 0.3 (f_c' / f_y) b_w d = 0.3 \times (3/40) \times 12 \times 15.5 =$ **4.2 in^2**
        - **A_{smin} (0.93) < A_s (1.95) < A_{smax} (4.20) O.K**

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 04: Design.**
      - Design for flexure:
        - Bar Placement: 5, #6 bars will provide 2.20 in^2 of steel area which is slightly greater than required.
        - Other options can be explored. For example,
          - 4, #7 bars (2.4 in^2),
          - 3, #8 bars (2.37 in^2),
          - or combination of two different size bars.

Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 05: Drafting**



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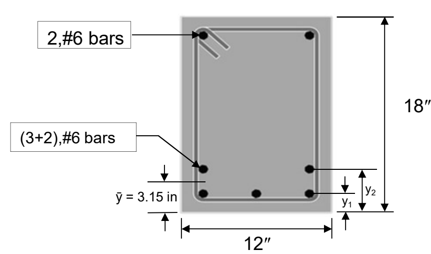
Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 06: Flexure capacity check**
    - Effective depth "d" was initially assumed as 15.5 in. At this stage actual d can be calculated by calculating actual ȳ.
    - d= h- ȳ=18 – 3.15=14.85 in
    - A_s= 5 x 0.44=2.2 in^2
    - a= A_sf_y/0.85 x f_c' x b_w
    - a= 2.2 x 40/0.85 x 3 x 12 =2.87 in



y_1 =1.5+3/8+1/2(6/8)=2.25 in

y_2= 1.5+ 3/8+6/8+1.5+1/2(6/8) = 4.5

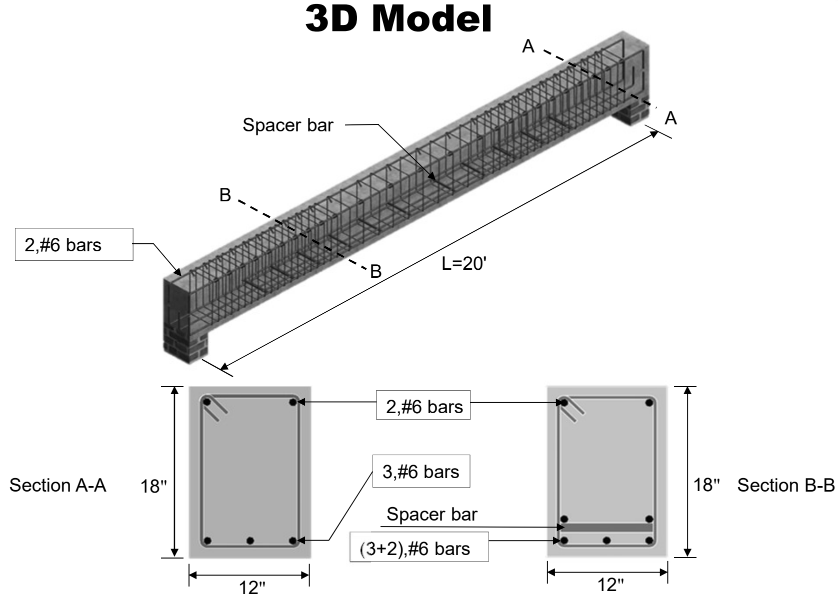
5 x 0.44 x ȳ=3 x 0.44 x y_1+2 x 0.44 x y_2

ȳ = 3.15 in

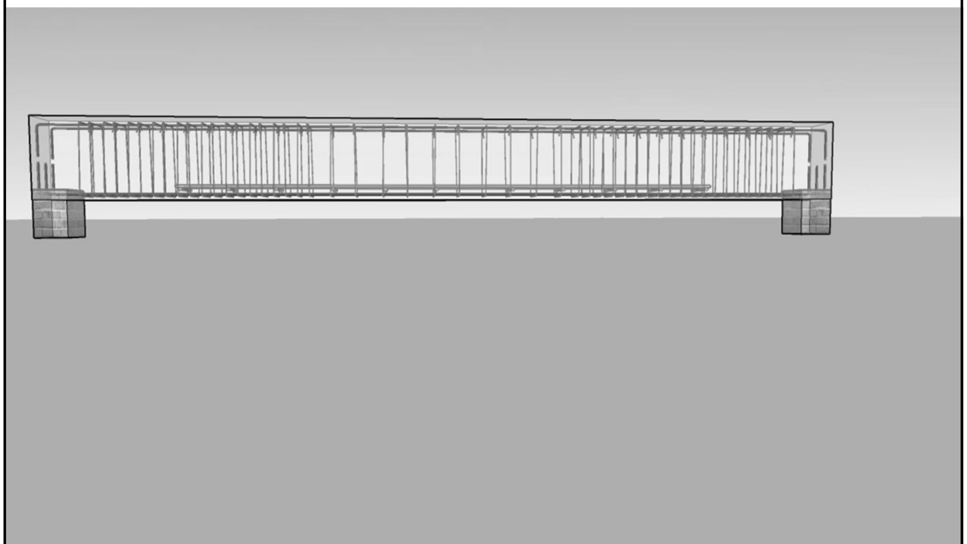
Example 2.1

- **Flexural Design of Beam as per ACI:**
  - **Solution:**
    - **Step No. 06: Flexure capacity check**
    - $\Phi M_n = \Phi A_s f_y(d - a/2)$
    - $\Phi M_n = 0.9 \times 2.2 \times 40 \times (14.85 - 2.87/2)$
    - $\Phi M_n = 1062.5$ in-kip
    - $\Phi M_n (1062.5 \text{ in-kip}) > M_u (1002 \text{ in-kip})$
    - Design is ok!

3D Model



3D Model



Example 2.2

- **Class Activity**

  Design a reinforced concrete simply supported beam having a span length of 30 ft and supporting a service dead load of 1.1 kip/ft and a uniform service live load of 1.1 kip/ft in addition to its self weight. Take f_c' = 4 ksi and f_y = 60 ksi.

References

- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)