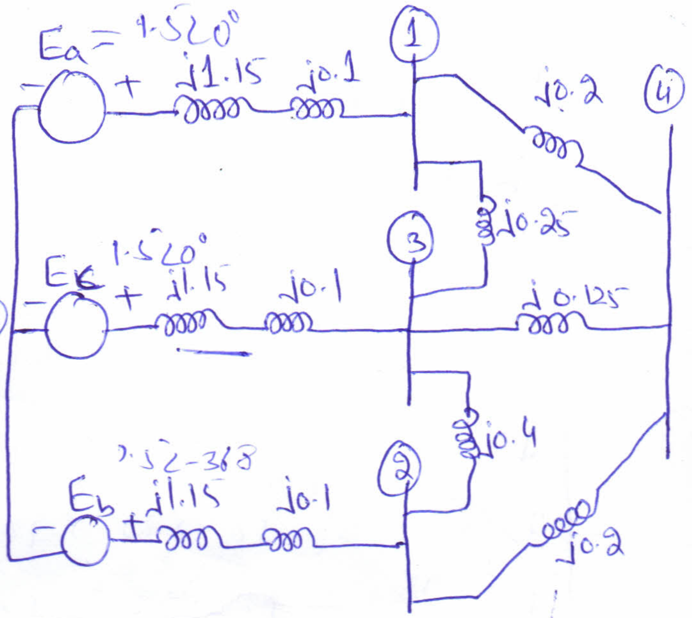
Example: Write in Matrix form the node equations necessary to solve for the voltages of the numbered buses of fig. The network is equivalent as follows. The emfs shown in fig are $E_a = 1.5\angle 0°$, $E_b = 1.5\angle -36.87°$ and $E_c = 1.5\angle 0°$, all in per unit.

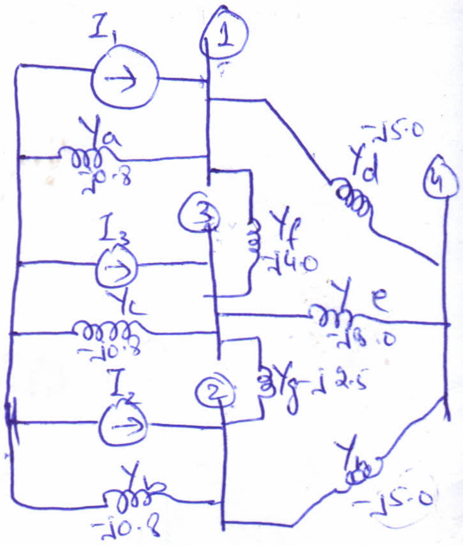


Sol. The Current Sources are

$$I_1 = I_3 = \frac{1.5\angle 0°}{j1.25} = 1.2\angle -90°$$

$$= 0 - j1.20 \text{ P.U}$$

$$I_2 = \frac{1.5\angle -36.87°}{j1.25} = 1.2\angle -126.87° = -0.72 - j0.96 \text{ P.U}$$



Self-admittances in P.U are.

$$Y_{11} = -j5.0 - j4.0 - j0.8 = -j9.8$$

$$Y_{22} = -j5.0 - j2.5 - j0.8 = -j8.3$$

$$Y_{33} = -j4.0 - j2.5 - j8.0 - j0.8 = -j15.3$$

$$Y_{44} = -j5.0 - j5.0 - j8.0 = -j18.0$$

and the mutual admittances in P.U are.

$$Y_{12} = Y_{21} = 0 \qquad Y_{23} = Y_{32} = +j2.5$$

$$Y_{13} = Y_{31} = -(-j4.0) = +j4.0 \qquad Y_{24} = Y_{42} = +j5.0$$

$$Y_{14} = Y_{41} = +j5.0 \qquad Y_{34} = Y_{43} = +j8.0$$

The Node equations in matrix form are.

$$\begin{bmatrix} 0 - j1.20 \\ -0.72 - j0.96 \\ 0 - j1.20 \\ 0 \end{bmatrix} = \begin{bmatrix} -j9.8 & j0.0 & j4.0 & j5.0 \\ j0.0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j15.3 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Multiplying b/s of above matrix by the Inverse of bus admittance Matrix gives.

i-e.
$$A^{-1} = \frac{1}{\det A} \cdot \operatorname{adj} A$$

$$\begin{bmatrix} j0.4774 & j0.376 & j0.4020 & j0.4142 \\ j0.3706 & j0.4872 & j0.3922 & j0.4126 \\ j0.4020 & j0.3922 & j0.4568 & j0.4232 \\ j0.4142 & j0.4126 & j0.4232 & j0.4733 \end{bmatrix} \begin{bmatrix} 0 - j1.20 \\ -0.72 - j0.96 \\ 0 - j1.20 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\begin{bmatrix} 1.4111 - j0.2668 \\ 1.3830 - j0.3508 \\ 1.4059 - j0.2824 \\ 1.4009 - j0.2971 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

And so, the voltages are.

$V_1 = 1.4111 - j0.2668 = 1.436 \angle -10.71^\circ$ P.U

$V_2 = 1.3830 - j0.3508 = 1.427 \angle -14.24^\circ$ P.U

$V_3 = 1.4059 - j0.2824 = 1.434 \angle -11.36^\circ$ P.U

$V_4 = 1.4009 - j0.2971 = 1.432 \angle 11.97^\circ$ P.U

The Bus Admittance and Impedance Matrices:

In example we invert the bus admittance matrix Y_{bus} and called the resultant matrix Z_{bus} i-e Bus Impedance Matrix.

By definition.

$$Z_{bus} = Y_{bus}^{-1} \longrightarrow (1)$$

and for a network of three Independent nodes.

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \longrightarrow (2)$$

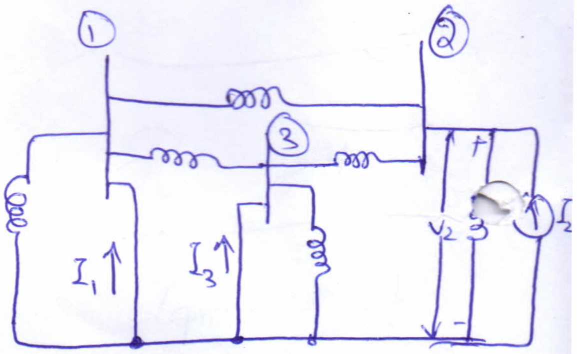
The Impedance elements of Z_{bus} on the Principal diognal are called "driving point Impedance of the nodes" and off diognal elements are called the "tranfer Impedance of the nodes".

The bus impedance matrix is important and very useful in making fault Calculations.

Starting with the node equation expressed in admittances are.

$$I = Y_{bus} \cdot V \longrightarrow (A)$$

Let we take a Circuit of three Independent nodes shown as follows.



At node 2 :-

$$I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3 \longrightarrow (3)$$

For ~~Mutual~~ Self Admittances:

If V_1 and V_3 are reduced to zero by shorting node (1) and (3) to the reference node and the Current I_2 is injected at node 2, the self admittance at node 2 is

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1 = V_3 = 0} \longrightarrow (4)$$

Thus, the self admittance of a particular node could be measured by shorting all other nodes to the reference node and then finding the ratio of the Current injected at the node to the voltage resulting at that node.

(7)

Mutual Admittances

Figure also serves to illustrate mutual admittances. At node 1. the equation obtained by expanding eq 7

$$I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3$$

from which we see that.

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1 = V_3 = 0} \longrightarrow (5)$$

Thus the mutual admittance is measured by shorting all nodes ~~exp~~ except node 2 to the reference node and injecting a current I_2 at node 2 as shown in figure.

We have made this detailed examinations of the node admittances in order to differentiate them clearly from the impedances of the bus Impedance Matrix.

Bus Impedance Matrix.

we solve eq (A) by premultiplying b/s of the equation by $Y^{-1}_{bus} = Z_{bus}$, which gives.

$$V = Z_{bus} . I \longrightarrow (6)$$

For a Network of three Independent Nodes

As we know

$I = Y_{bus} V.$

$I = \frac{1}{Z} V.$

so

$\boxed{V = Z_{bus} I}$

5

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

or

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 \longrightarrow (7)$$

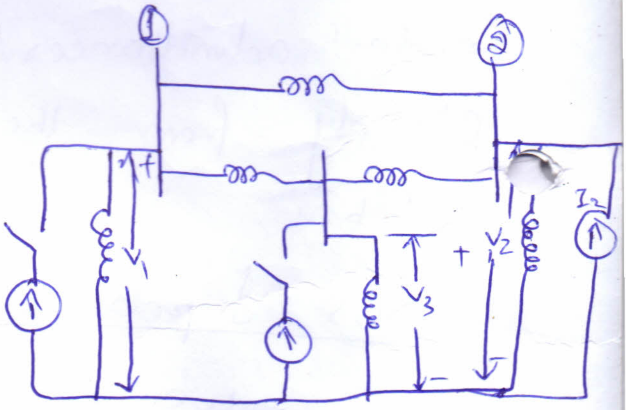
$$V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3 \rightarrow (8)$$

$$V_3 = Z_{31} I_1 + Z_{32} I_2 + Z_{33} I_3 \rightarrow (9)$$

From eq (8) we see that the driving point Impedance Z_{22} is determined by open-circuiting the Current Sources at node 1 and node 3 and injecting the Current I_2 at node 2. Then.

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = I_3 = 0}$$

Figure shows the Circuit described.



Since Z_{22} is defined by opening the Current Sources Connected to the other nodes whereas Y_{22} was found with the other nodes shorted, we must not expect any reciprocal relation b/w these two Quantities.

The Circuit also enables us to measure some transfer Impedances.

From eq (7) with Current Sources I_1 and I_3 open

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1 = I_3 = 0}$$

6

and from eq (9).

$$Z_{32} = \left.\frac{V_3}{I_2}\right|_{I_1 = I_3 = 0}$$

⇒ Thus we can measure the transfer Impedances Z_{12} and Z_{32} by injecting current at node 2 and finding the ratios of V_1 and V_3 to I_2 with the sources open at all nodes except at node 2.

⇒ We note that a mutual admittance is measured with all but one node short-circuit and that a transfer impedance is measured with all sources open-circuited except one.

Same for other nodes. By ~~or~~ injecting current at node (1) and make (2) and (3) node open.