Lecture 06

Time Value of Money

By

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CHAPTER 7
Time Value of Money

- Rates of return
- Amortization

The Power of Compound Interest

A 20-year old student wants to start saving for retirement. She plans to save $3 a day. Every day, she puts $3 in her drawer. At the end of the year, she invests the accumulated savings ($1,095) in an online stock account. The stock account has an expected annual return of 12%.

How much money by the age of 65?

	N	I/YR	PV	PMT	FV
INPUTS	45	12	0	-1095	
OUTPUT					1,487,261.89

If she begins saving today, and sticks to her plan, she will have $1,487,261.89 by the age of 65.

How much would a 40-year old investor accumulate by this method?

INPUTS	25	12	0	-1095	
	N	I/YR	PV	PMT	FV
OUTPUT					146,000.59

Waiting until 40, the investor will only have $146,000.59, which is over $1.3 million less than if saving began at 20. So it pays to get started early.

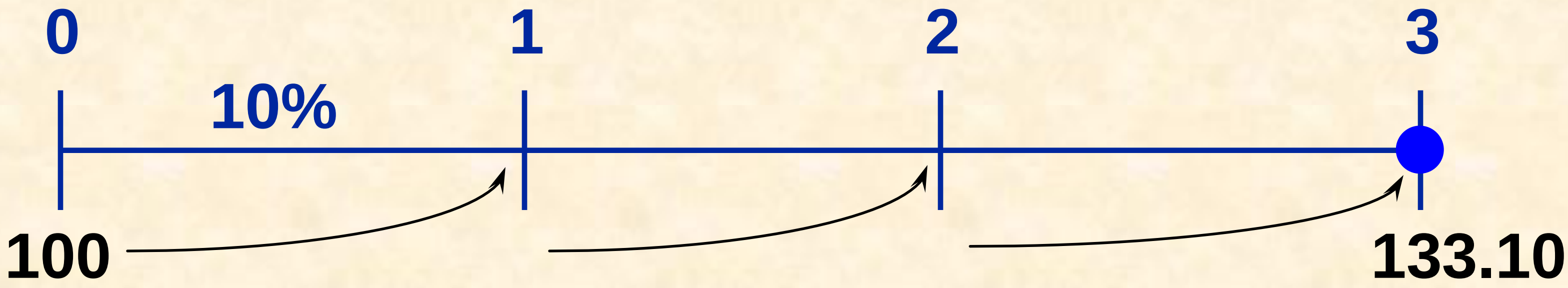
How much would the 40-year old investor need to save to accumulate as much as the 20-year old?

	N	I/YR	PV	PMT	FV
INPUTS	25	12	0		1487261.89
OUTPUT				-11,154.42	

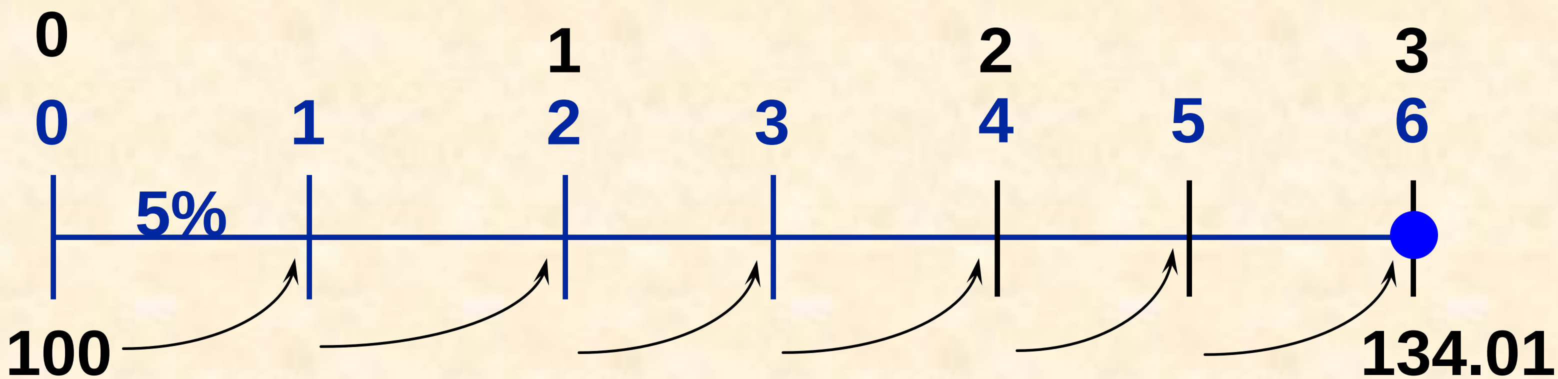
The 40-year old investor would have to save $11,154.42 every year, or $30.56 per day to have as much as the investor beginning at the age of 20.

Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated I% constant? Why?

LARGER! If compounding is more frequent than once a year--for example, semiannually, quarterly, or daily--interest is earned on interest more often.



Annually: $FV_3 = \$100(1.10)^3 = \$133.10.$



Semiannually: $FV_6 = \$100(1.05)^6 = \$134.01.$

We will deal with 3 different rates:

i_{Nom} = nominal, or stated, or quoted, rate per year.

i_{Per} = periodic rate.

EAR = EFF% = effective annual rate.

- i_{Nom} is stated in contracts. Periods per year (m) must also be given.
- Examples:
  - 8%; Quarterly
  - 8%, Daily interest (365 days)

- **Periodic rate = i_{Per} = i_{Nom}/m, where m is number of compounding periods per year. m = 4 for quarterly, 12 for monthly, and 360 or 365 for daily compounding.**
- **Examples:**

8% quarterly: i_{Per} = 8%/4 = 2%.

8% daily (365): i_{Per} = 8%/365 = 0.021918%.

- **Effective Annual Rate (EAR = EFF%):** The annual rate that causes PV to grow to the same FV as under multi-period compounding.

Example: EFF% for 10%, semiannual:

$$FV = (1 + i_{Nom}/m)^m$$

$$= (1.05)^2 = 1.1025.$$

$$EFF\% = 10.25\% \text{ because}$$

$$(1.1025)^1 = 1.1025.$$

Any PV would grow to same FV at 10.25% annually or 10% semiannually.

- **An investment with monthly payments is different from one with quarterly payments. Must put on EFF% basis to compare rates of return. Use EFF% only for comparisons.**
- **Banks say “interest paid daily.” Same as compounded daily.**

How do we find EFF% for a nominal rate of 10%, compounded semiannually?

$$EFF = \left(1 + \frac{i_{Nom}}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.10}{2}\right)^2 - 1.0$$

$$= (1.05)^2 - 1.0$$

$$= 0.1025 = 10.25\%.$$

Or use a financial calculator.

EAR = EFF% of 10%

$$EAR_{Annual} = \mathbf{10\%.}$$

$$EAR_{Q} = (1 + 0.10/4)^{4} - 1 = \mathbf{10.38\%.}$$

$$EAR_{M} = (1 + 0.10/12)^{12} - 1 = \mathbf{10.47\%.}$$

$$EAR_{D(365)} = (1 + 0.10/365)^{365} - 1 = \mathbf{10.52\%.}$$

Can the effective rate ever be equal to the nominal rate?

- **Yes, but only if annual compounding is used, i.e., if m = 1.**
- **If m > 1, EFF% will always be greater than the nominal rate.**

When is each rate used?

i_{Nom}: **Written into contracts, quoted by banks and brokers. Not used in calculations or shown on time lines.**

i_{Per}: **Used in calculations, shown on time lines.**

If i_{Nom} has annual compounding, then $i_{Per} = i_{Nom}/1 = i_{Nom}$.

EAR = EFF%: **Used to compare returns on investments with different payments per year.**

(Used for calculations if and only if dealing with annuities where payments don’t match interest compounding periods.)

FV of $100 after 3 years under 10% semiannual compounding? Quarterly?

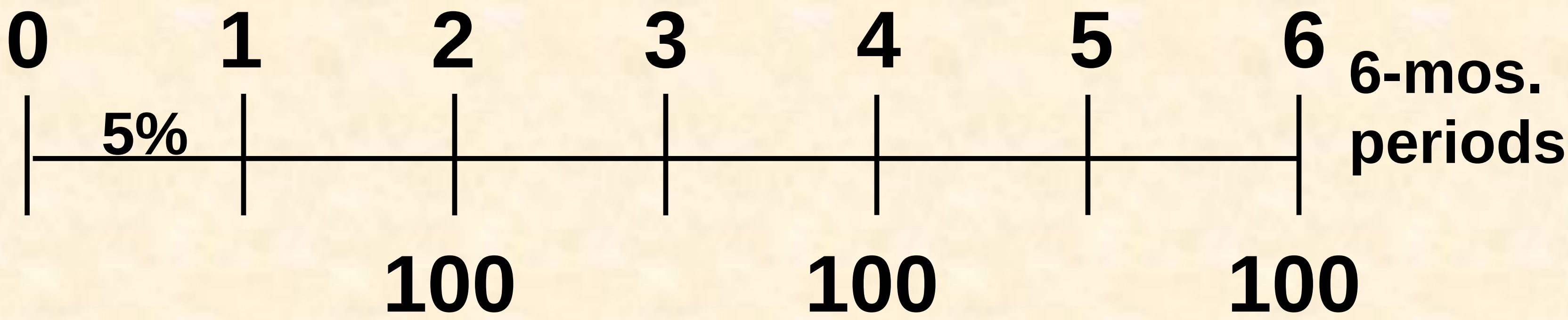
$$FV_n = PV\left(1 + \frac{i_{Nom}}{m}\right)^{mn}.$$

$$FV_{3S} = \$100\left(1 + \frac{0.10}{2}\right)^{2x3}$$

$$= \$100(1.05)^6 = \$134.01.$$

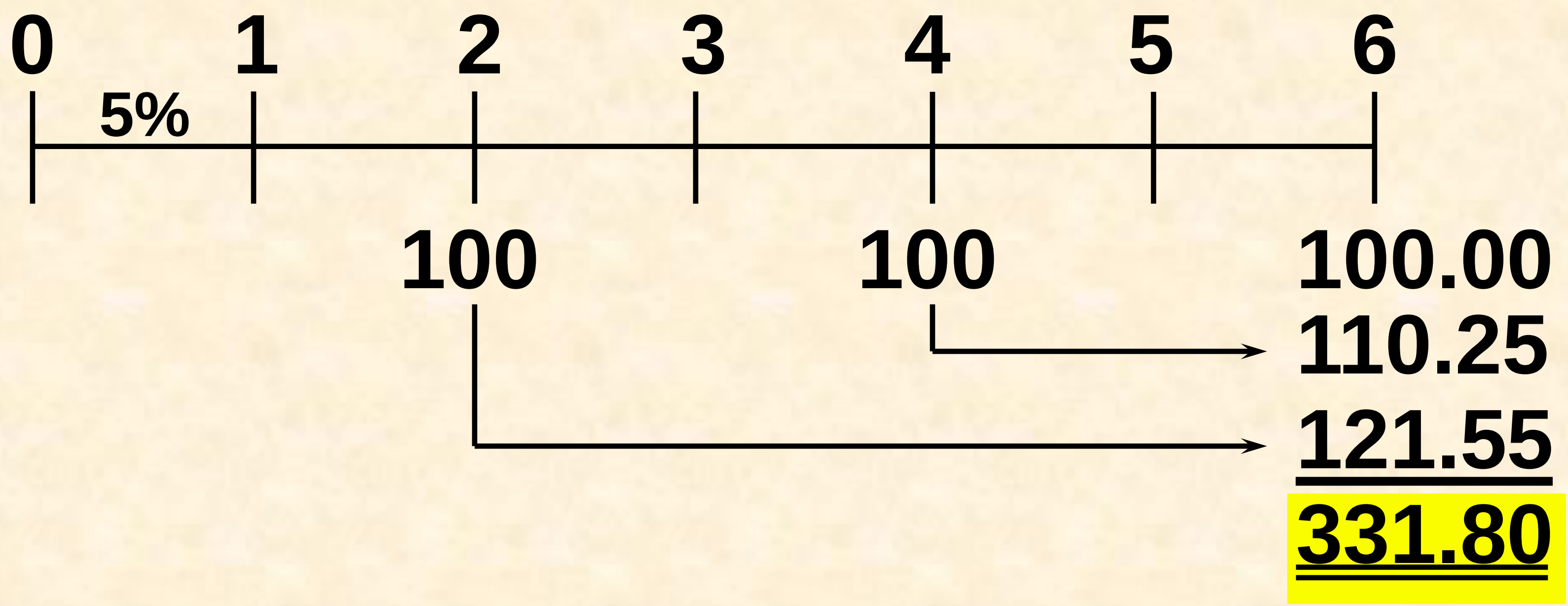
$$FV_{3Q} = \$100(1.025)^{12} = \$134.49.$$

What's the value at the end of Year 3 of the following CF stream if the quoted interest rate is 10%, compounded semiannually?



- **Payments occur annually, but compounding occurs each 6 months.**
- **So we can't use normal annuity valuation techniques.**

1st Method: Compound Each CF



$$FVA_3 = \$100(1.05)^4 + \$100(1.05)^2 + \$100$$
$$= \$331.80.$$

2nd Method: Treat as an Annuity

Could you find FV with a financial calculator?

Yes, by following these steps:

a. Find the EAR for the quoted rate:

$$EAR = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 10.25\%.$$

Or, to find EAR with a calculator:

NOM% = 10.

P/YR = 2.

EFF% = 10.25.

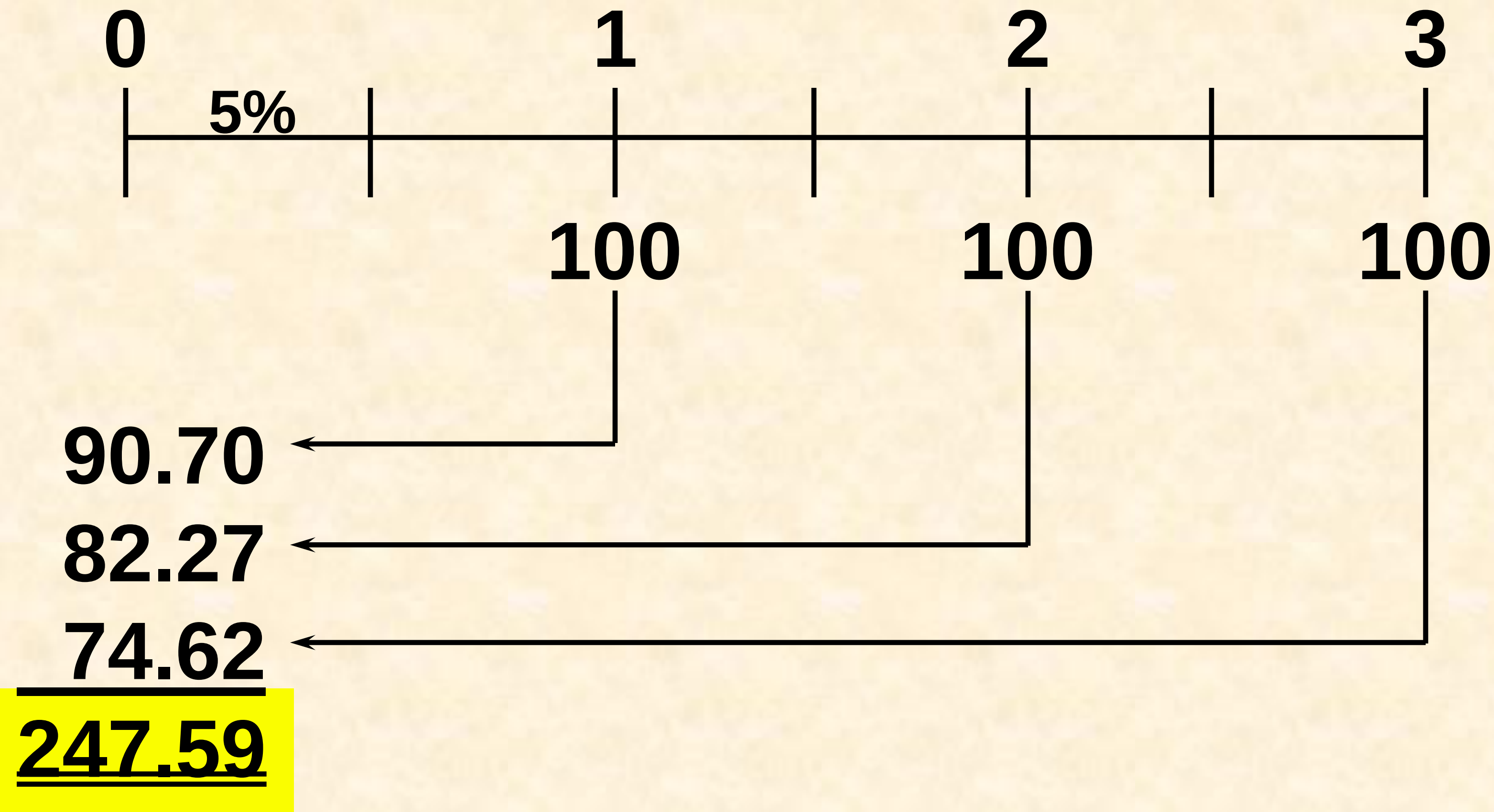
b. The cash flow stream is an annual annuity. Find k_{Nom} (annual) whose EFF% = 10.25%. In calculator,

EFF% = 10.25
P/YR = 1
NOM% = 10.25

c.

	N	I/YR	PV	PMT	FV
INPUTS	3	10.25	0	-100	
OUTPUT					331.80

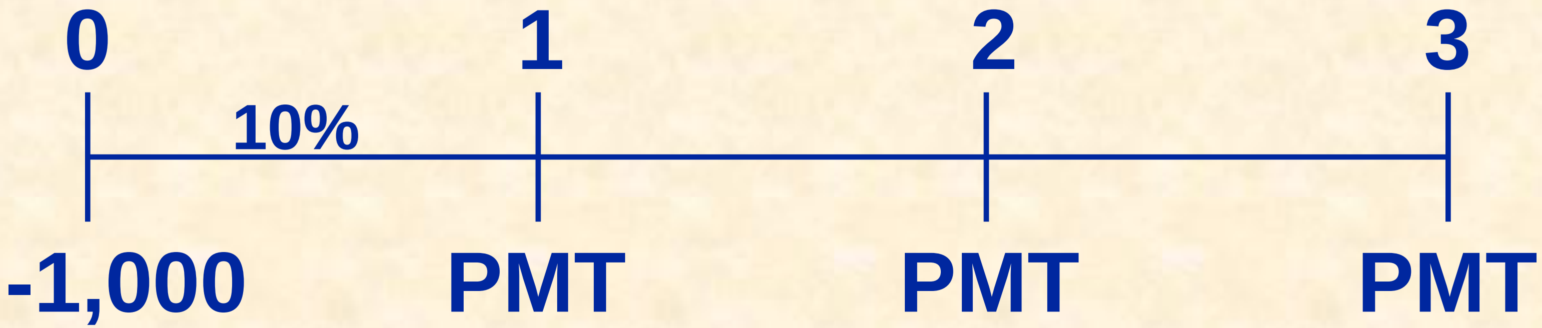
What’s the PV of this stream?



Amortization

Construct an amortization schedule for a $1,000, 10% annual rate loan with 3 equal payments.

Step 1: Find the required annual payments.



	N	I/YR	PV	PMT	FV
INPUTS	3	10	-1000		0
OUTPUT				402.11	

Step 2: Find the interest paid in Year 1.

$$INT_t = \text{Beg bal}_t\,(i)$$

$$INT_1 = \$1{,}000(0.10) = \boxed{\$100.}$$

Step 3: Find repayment of principal in Year 1.

$$\text{Repmt} = PMT - INT$$
$$= \$402.11 - \$100$$
$$= \boxed{\$302.11.}$$

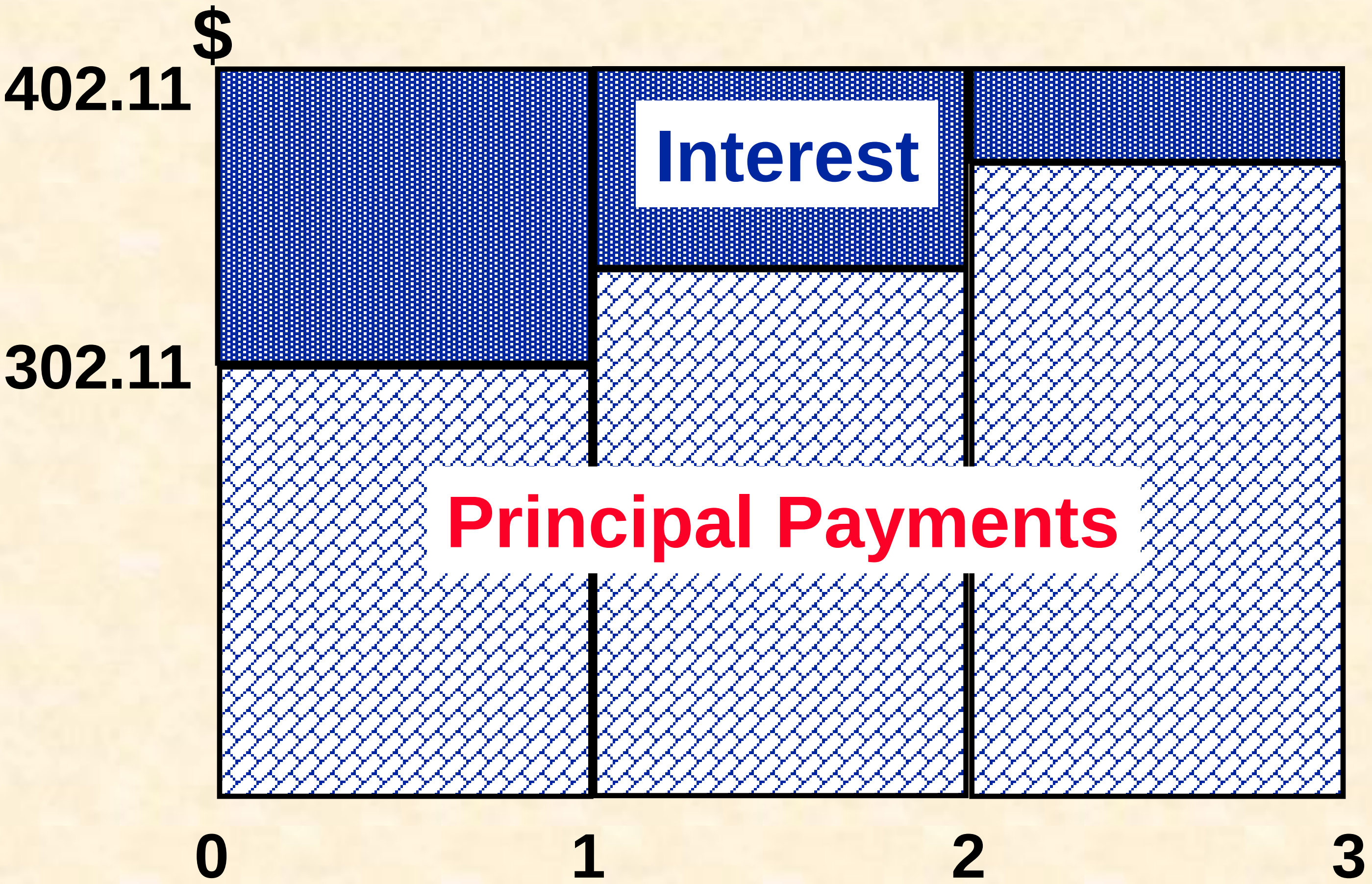
Step 4: Find ending balance after Year 1.

End bal = Beg bal – Repmt
= $1,000 – $302.11 = $697.89.

Repeat steps 2-4 for Years 2 and 3 to complete the amortization table.

YR	BEG BAL	PMT	INT	PRIN PMT	END BAL
1	$1,000	$402	$100	$302	$698
2	698	402	70	332	366
3	366	402	37	366	0
TOT		1,206.34	206.34	1,000	

Interest declines. Tax implications.



Level payments. Interest declines because outstanding balance declines. Lender earns 10% on loan outstanding, which is falling.

- **Amortization tables are widely used--for home mortgages, auto loans, business loans, retirement plans, etc. They are very important!**
- **Financial calculators (and spreadsheets) are great for setting up amortization tables.**