Lecture 05

Time Value of Money

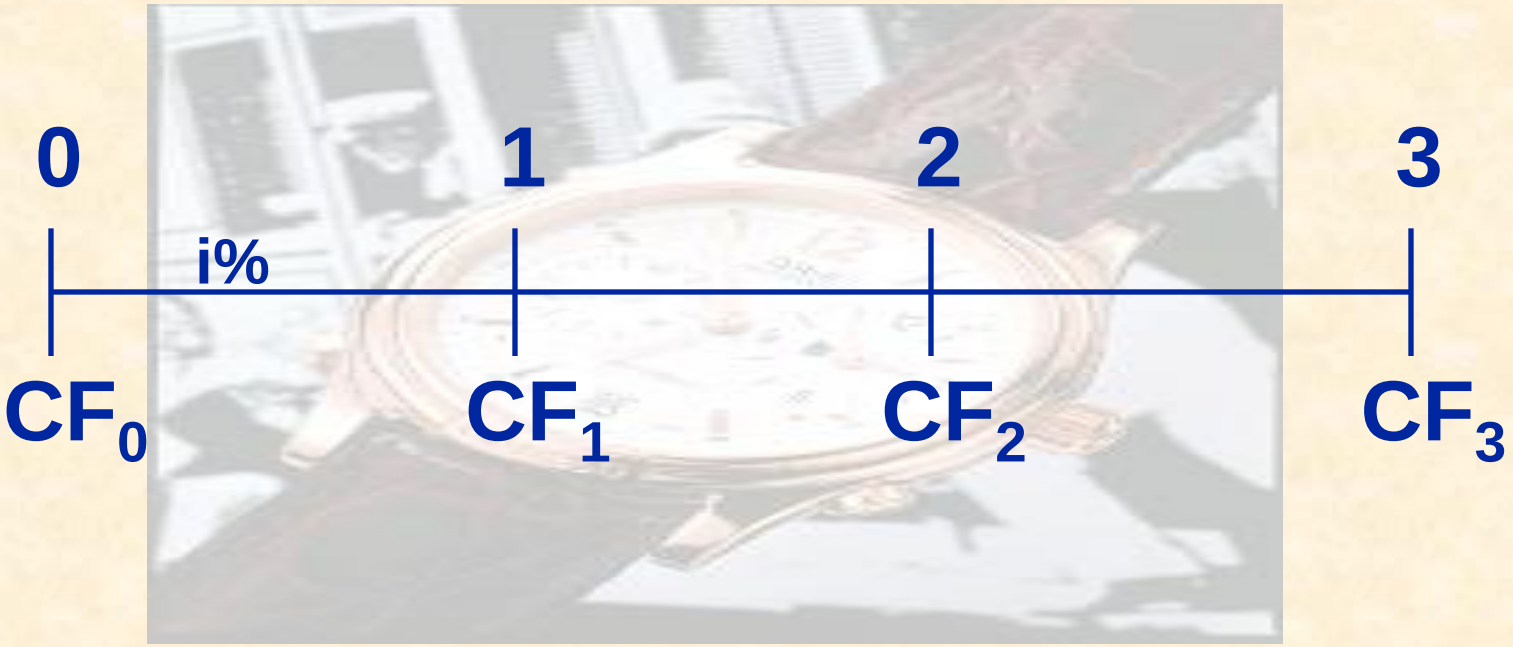
By

Dr. Rafiq Mansoor

CHAPTER 7
Time Value of Money

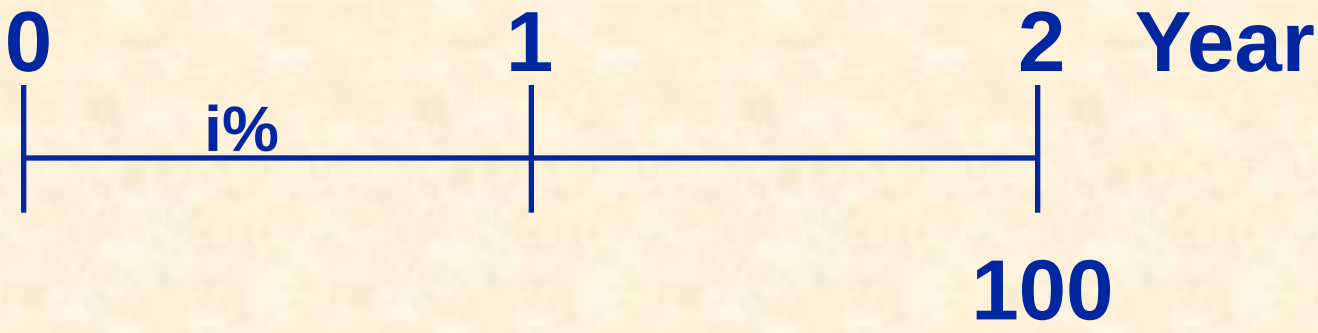
- Future value
- Present value
- Rates of return
- Amortization

Time lines show timing of cash flows.

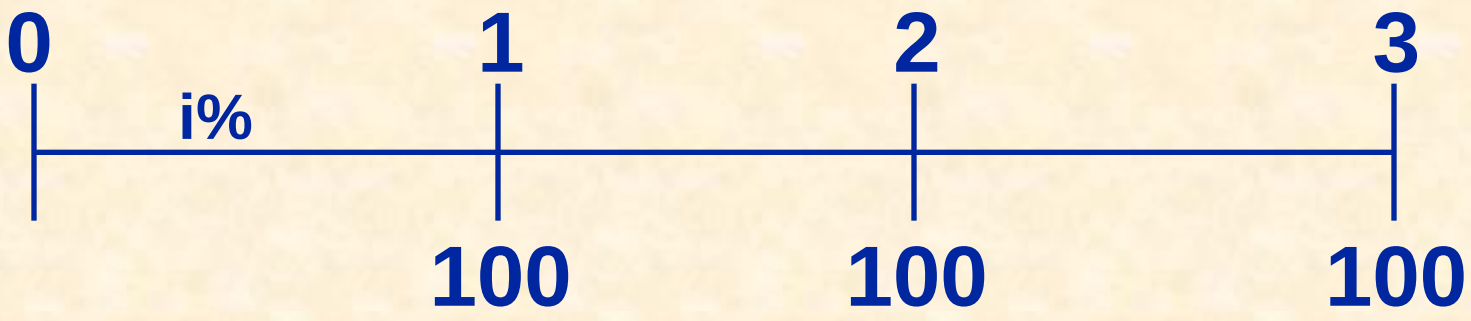


Tick marks at ends of periods, so Time 0 is today; Time 1 is the end of Period 1; or the beginning of Period 2.

Time line for a $100 lump sum due at the end of Year 2.

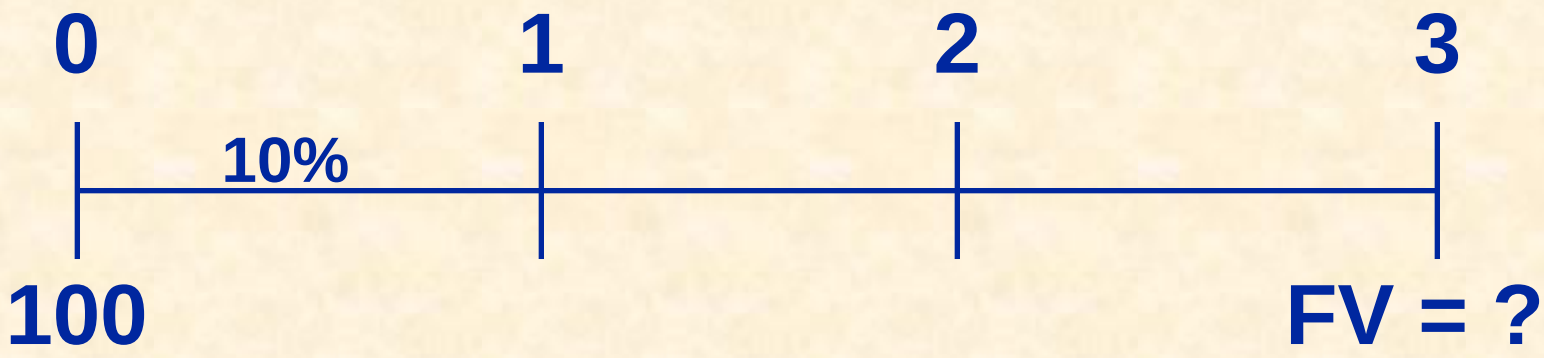


Time line for an ordinary annuity of $100 for 3 years.



Time line for uneven CFs -$50 at t = 0 and $100, $75, and $50 at the end of Years 1 through 3.

What's the FV of an initial $100 after 3 years if i = 10%?



Finding FVs is compounding.

After 1 year:

$$FV_1 = PV + INT_1 = PV + PV(i)$$
$$= PV(1 + i)$$
$$= \$100(1.10)$$
$$= \$110.00.$$

After 2 years:

$$FV_2 = PV(1 + i)^2$$
$$= \$100(1.10)^2$$
$$= \$121.00.$$

After 3 years:

$$FV_3 = PV(1 + i)^3$$
$$= \$100(1.10)^3$$
$$= \$133.10.$$

In general,

$$FV_n = PV(1 + i)^n.$$

Four Ways to Find FVs

- Solve the equation with a regular calculator.
- Use tables.
- Use a financial calculator.
- Use a spreadsheet.

Financial Calculator Solution

Financial calculators solve this equation:

$$FV_n = PV(1 + i)^n.$$

There are 4 variables. If 3 are known, the calculator will solve for the 4th.

Here's the setup to find FV:

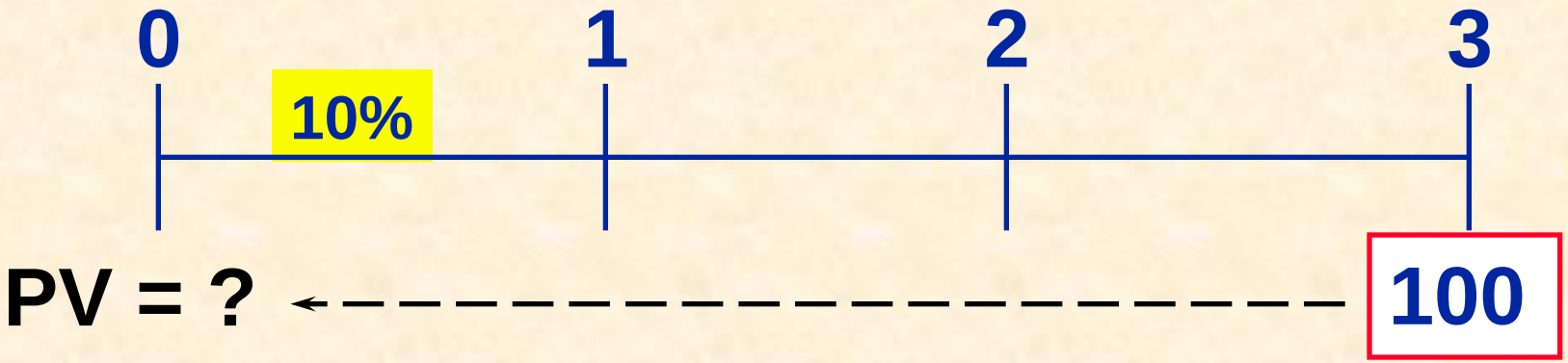
	N	I/YR	PV	PMT	FV
INPUTS	3	10	-100	0	
OUTPUT					133.10

Clearing automatically sets everything to 0, but for safety enter PMT = 0.

Set: P/YR = 1, END

What's the PV of $100 due in 3 years if i = 10%?

Finding PVs is discounting, and it's the reverse of compounding.



Solve $FV_n = PV(1 + i)^n$ for PV:

$$PV = \frac{FV_n}{(1 + i)^n} = FV_n\left(\frac{1}{1 + i}\right)^n.$$

$$PV = \$100\left(\frac{1}{1.10}\right)^3 = \$100(PVIF_{i,n})$$

$$= \$100(0.7513) = \$75.13.$$

Financial Calculator Solution

	N	I/YR	PV	PMT	FV
INPUTS	3	10		0	100
OUTPUT			-75.13		

Either PV or FV must be negative. Here PV = -75.13. Put in $75.13 today, take out $100 after 3 years.

If sales grow at 20% per year, how long before sales double?

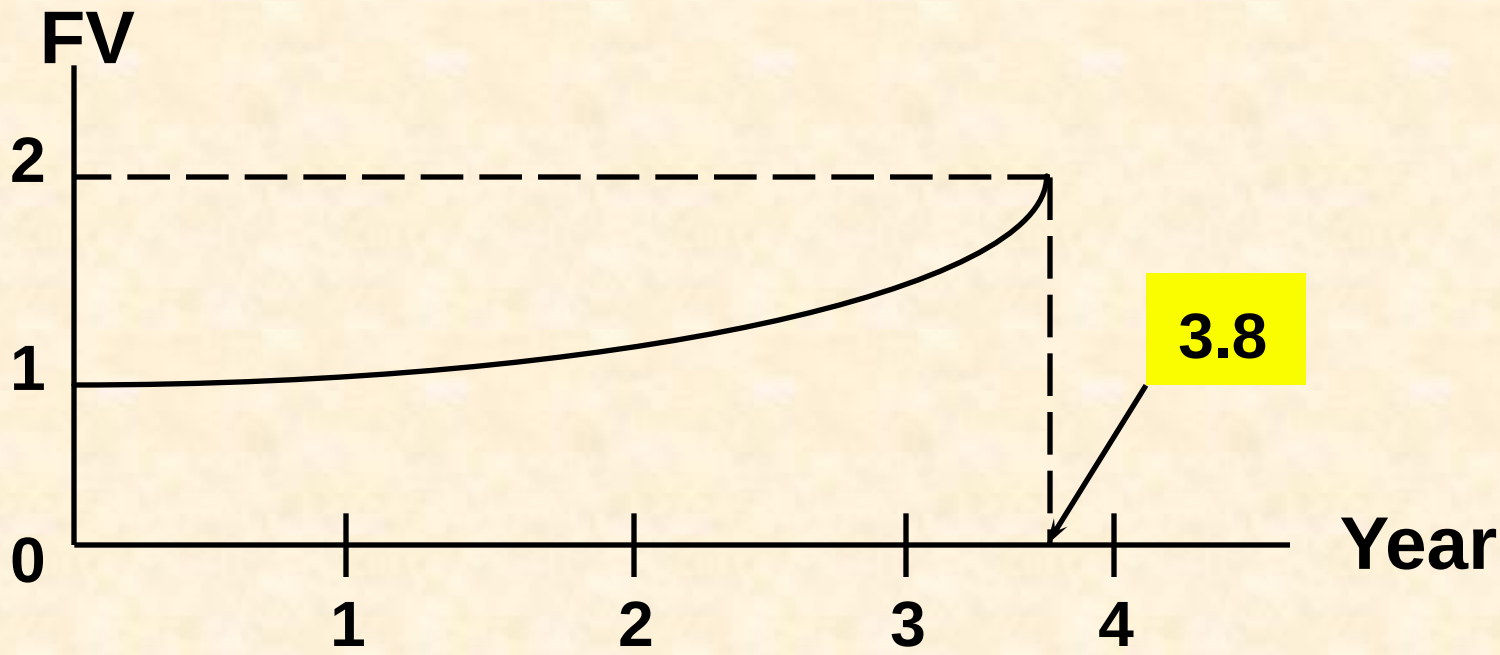
Solve for n:

$$FV_n = \$1(1 + i)^n;$$
$$\$2 = \$1(1.20)^n$$

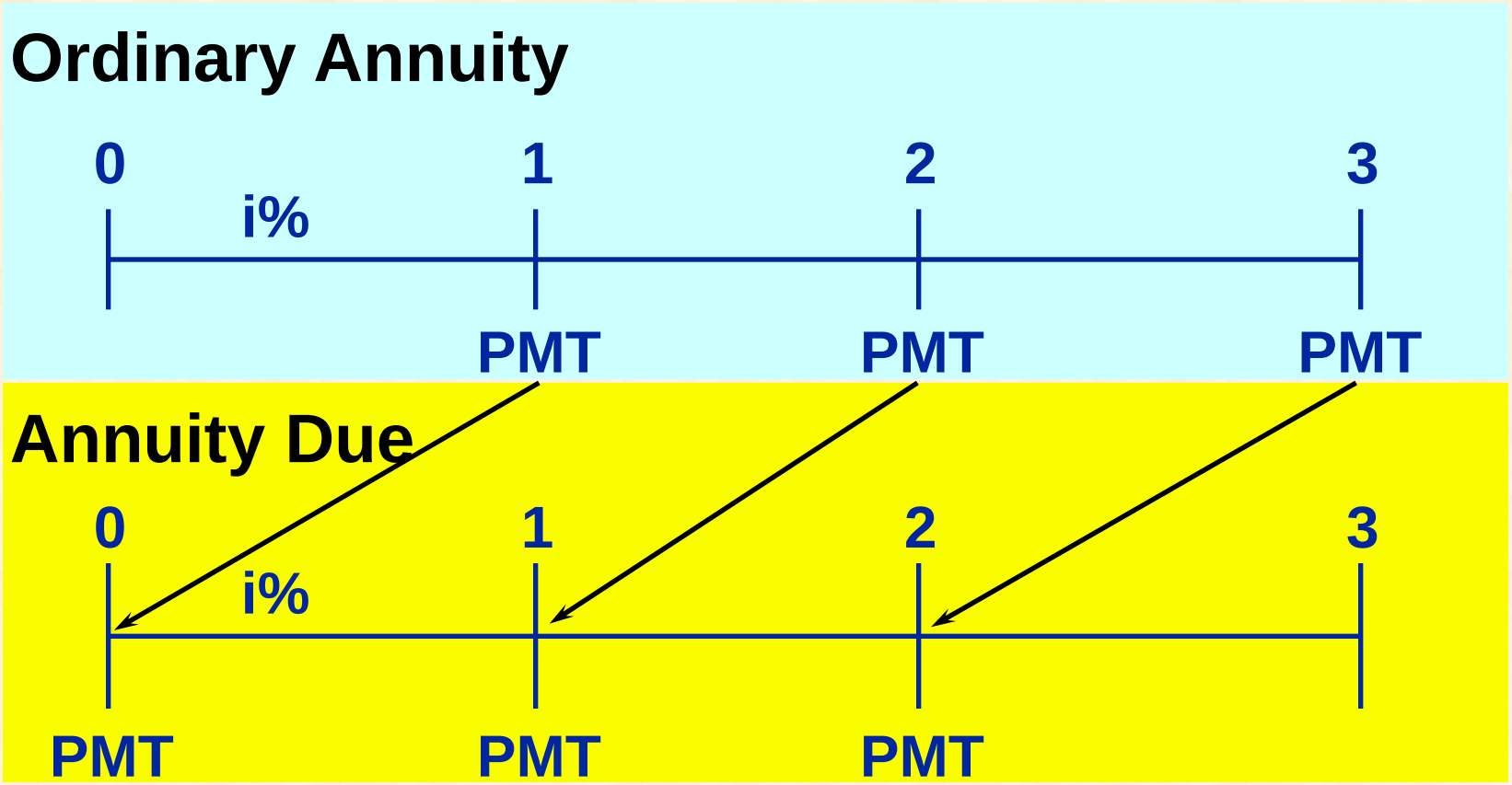
Use calculator to solve, see next slide.

	N	I/YR	PV	PMT	FV
INPUTS		20	-1	0	2
OUTPUT	3.8				

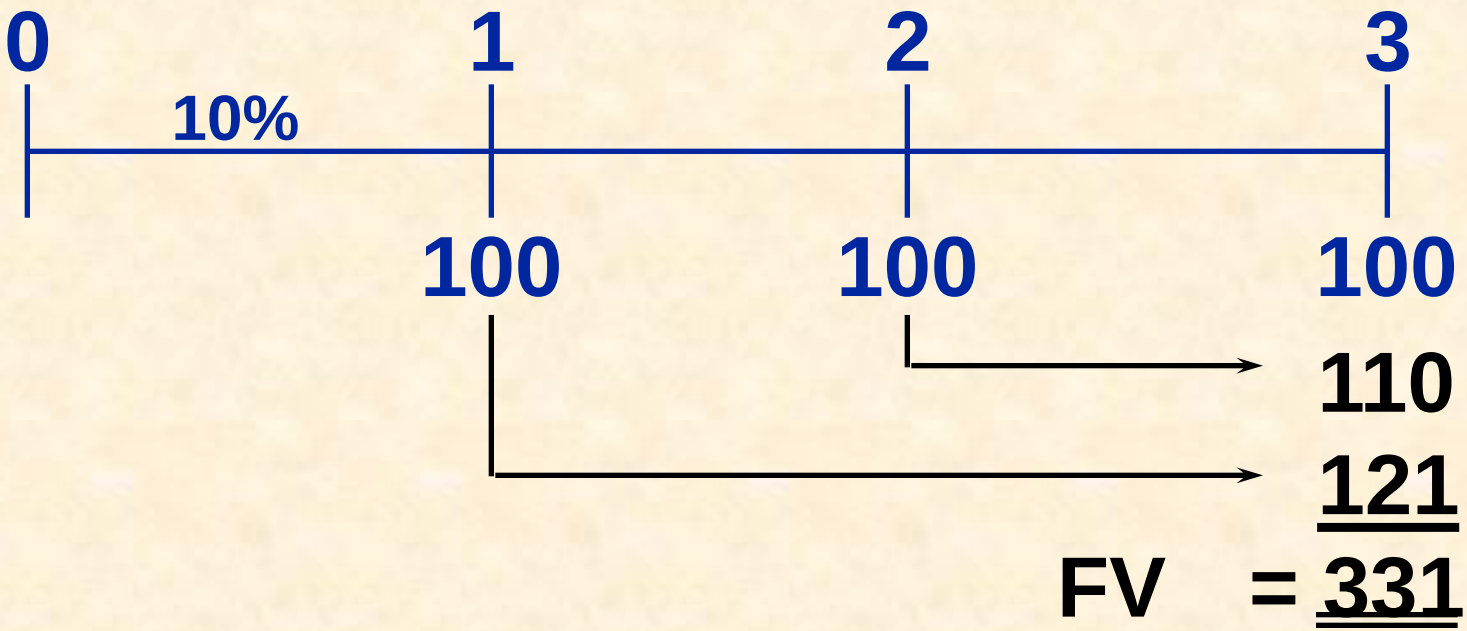
Graphical Illustration:



What's the difference between an ordinary annuity and an annuity due?



What's the FV of a 3-year ordinary annuity of $100 at 10%?

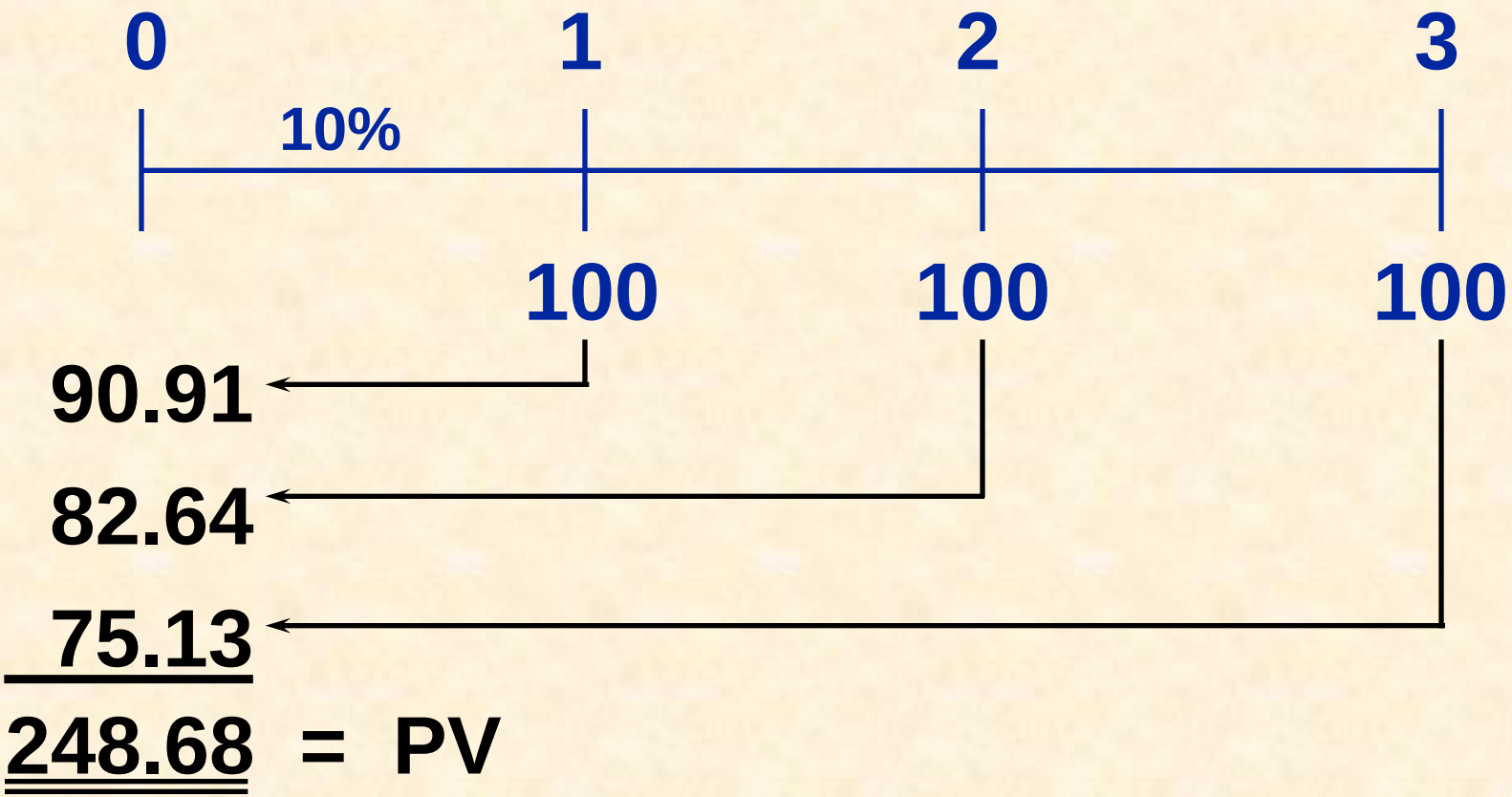


Financial Calculator Solution

	N	I/YR	PV	PMT	FV
INPUTS	3	10	0	-100	
OUTPUT					331.00

Have payments but no lump sum PV, so enter 0 for present value.

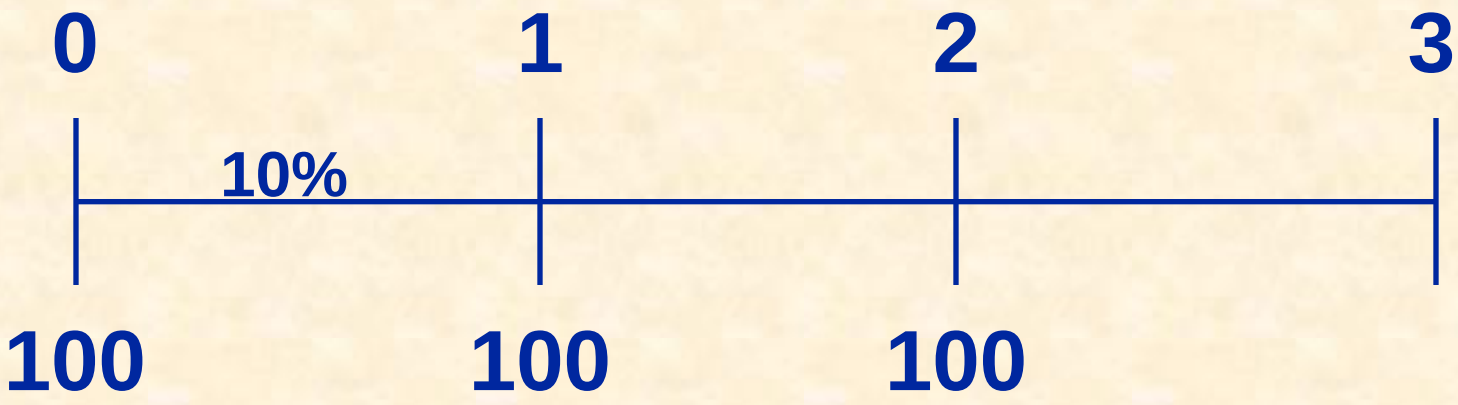
What's the PV of this ordinary annuity?



	N	I/YR	PV	PMT	FV
INPUTS	3	10		100	0
OUTPUT			-248.69		

Have payments but no lump sum FV, so enter 0 for future value.

Find the FV and PV if the annuity were an annuity due.

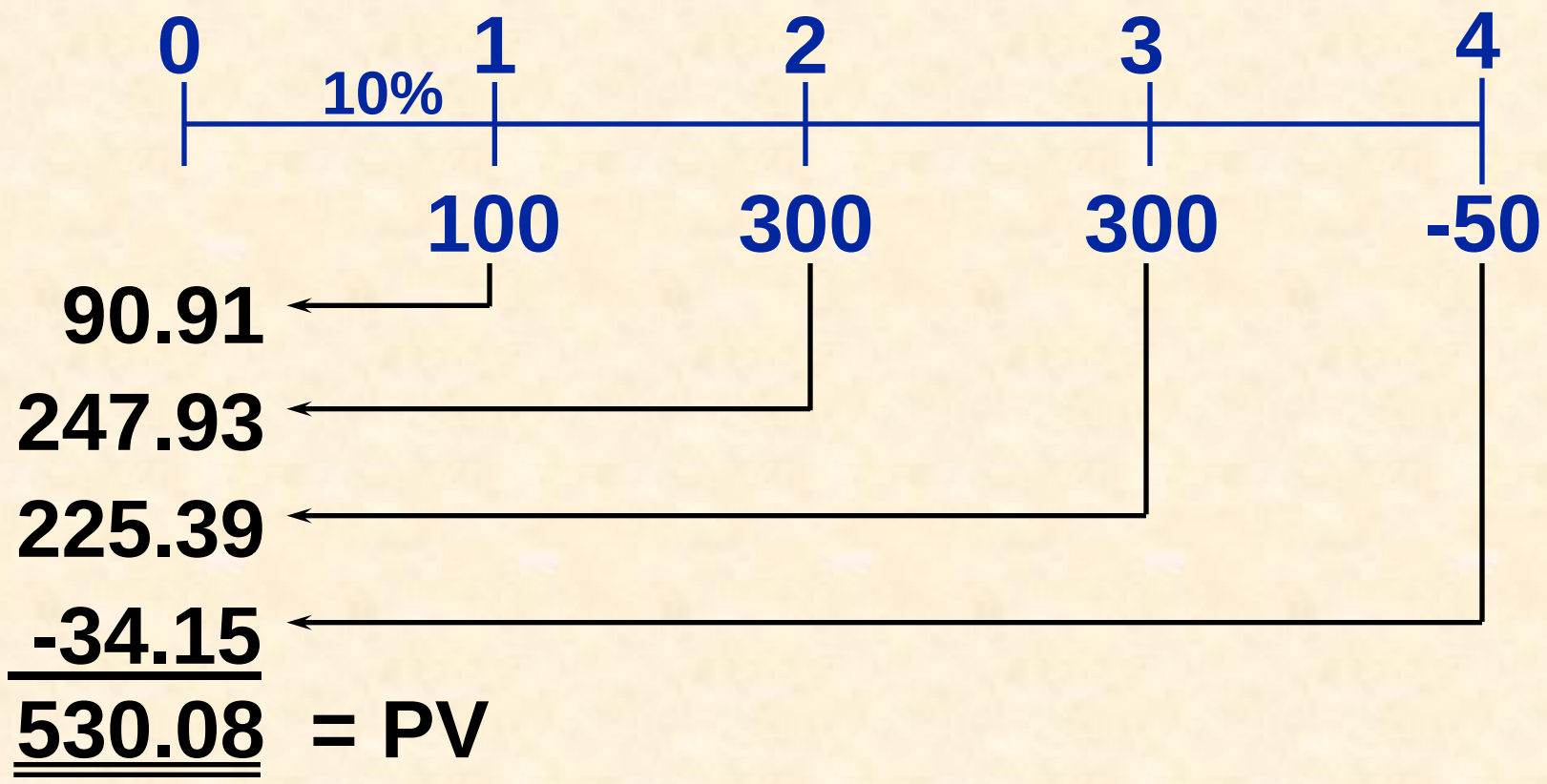


Switch from "End" to "Begin." Then enter variables to find PVA_3 = $273.55.

	N	I/YR	PV	PMT	FV
INPUTS	3	10		100	0
OUTPUT			-273.55		

Then enter PV = 0 and press FV to find FV = $364.10.

What is the PV of this uneven cash flow stream?



- **Input in "CFLO" register:**

  $CF_0 = 0$

  $CF_1 = 100$

  $CF_2 = 300$

  $CF_3 = 300$

  $CF_4 = -50$

- **Enter I = 10, then press NPV button to get NPV = $530.09. (Here NPV = PV.)**

What interest rate would cause $100 to grow to $125.97 in 3 years?

$$\$100\,(1 + i)^3 = \$125.97.$$

	N	I/YR	PV	PMT	FV
INPUTS	3		-100	0	125.97
OUTPUT		8%			

The Power of Compound Interest

A 20-year old student wants to start saving for retirement. She plans to save $3 a day. Every day, she puts $3 in her drawer. At the end of the year, she invests the accumulated savings ($1,095) in an online stock account. The stock account has an expected annual return of 12%.

How much money by the age of 65?

	N	I/YR	PV	PMT	FV
INPUTS	45	12	0	-1095	
OUTPUT					1,487,261.89

If she begins saving today, and sticks to her plan, she will have $1,487,261.89 by the age of 65.

How much would a 40-year old investor accumulate by this method?

	N	I/YR	PV	PMT	FV
INPUTS	25	12	0	-1095	
OUTPUT					146,000.59

Waiting until 40, the investor will only have $146,000.59, which is over $1.3 million less than if saving began at 20. So it pays to get started early.

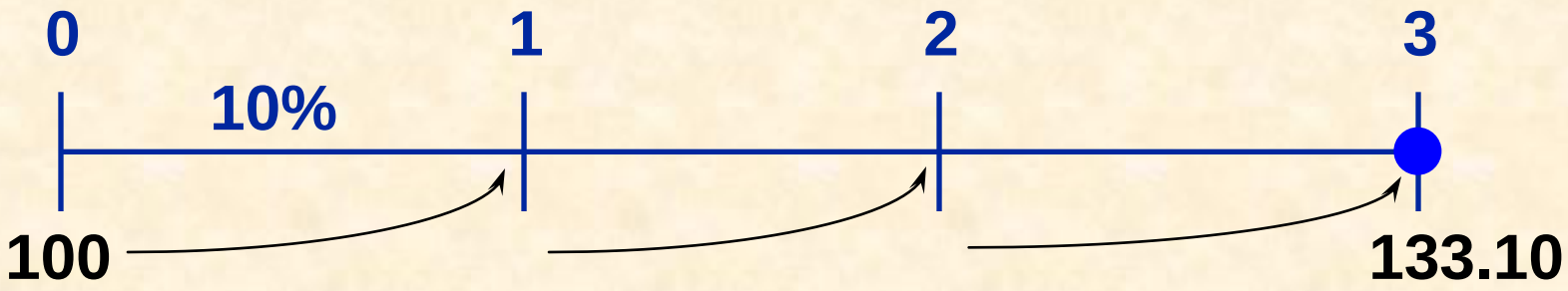
How much would the 40-year old investor need to save to accumulate as much as the 20-year old?

	N	I/YR	PV	PMT	FV
INPUTS	25	12	0		1487261.89
OUTPUT				-11,154.42	

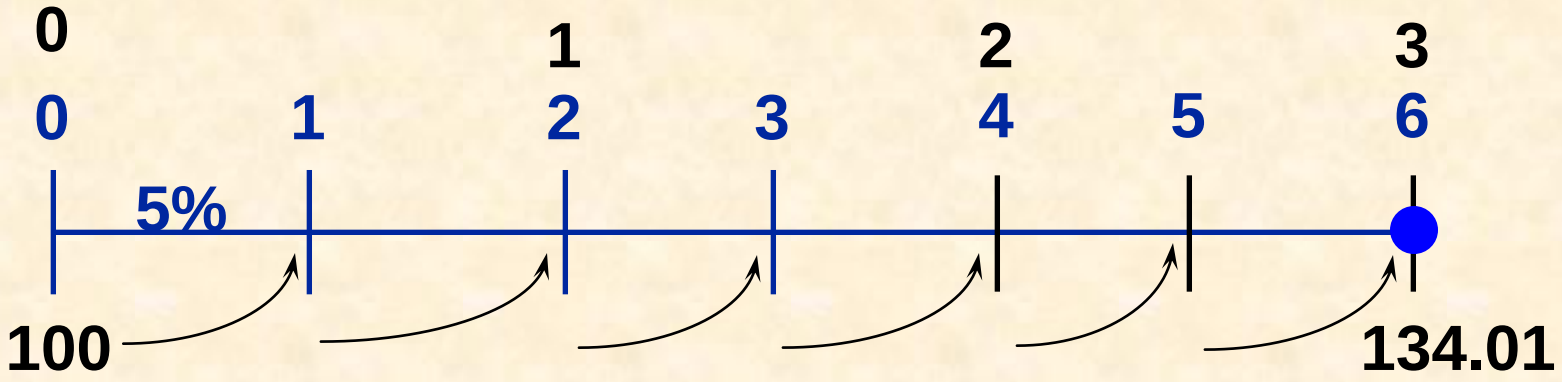
The 40-year old investor would have to save $11,154.42 every year, or $30.56 per day to have as much as the investor beginning at the age of 20.

Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated I% constant? Why?

LARGER! **If compounding is more frequent than once a year--for example, semiannually, quarterly, or daily--interest is earned on interest more often.**



Annually: $FV_3 = \$100(1.10)^3 = \$133.10.$



Semiannually: $FV_6 = \$100(1.05)^6 = \$134.01.$