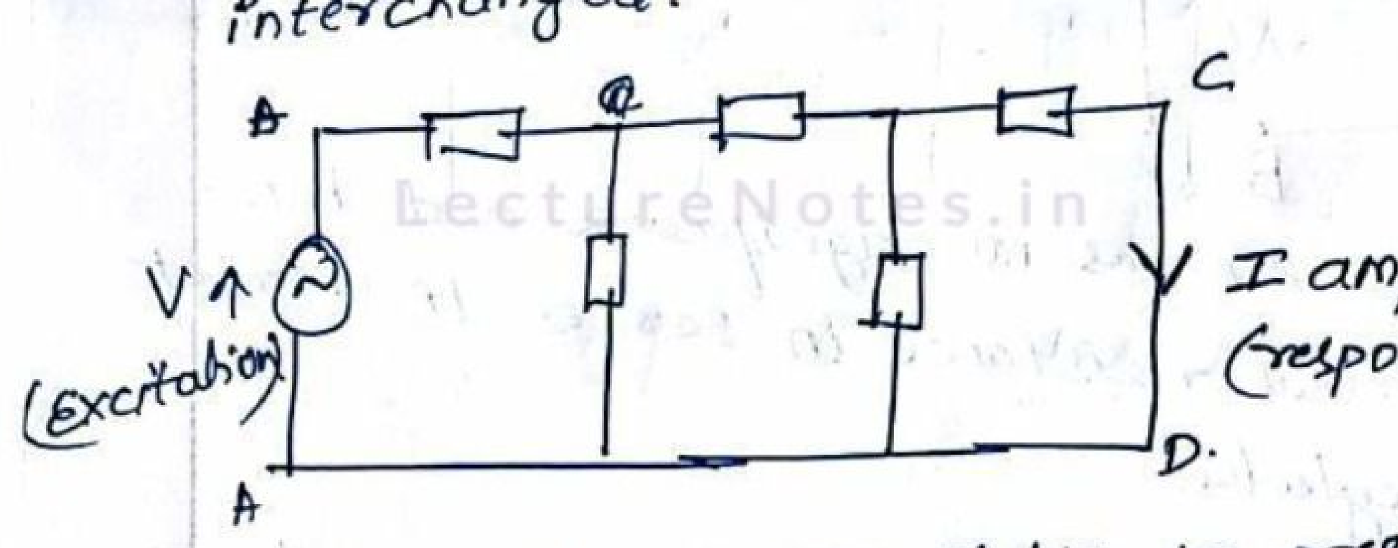
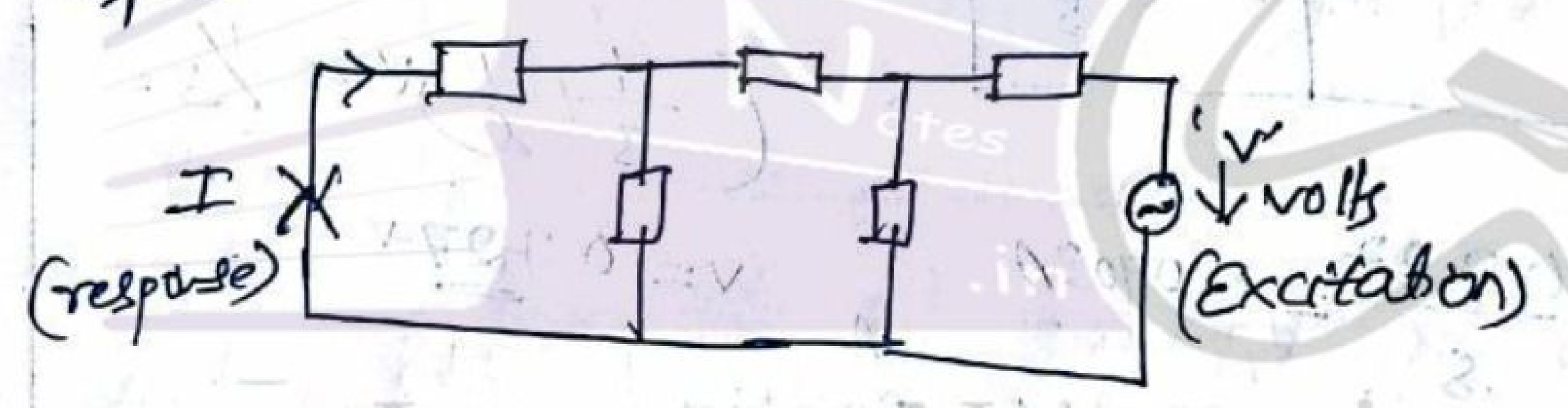
Reciprocity theorem

"If any linear, bilateral network containing only one independent source, the ratio of excitation to response remains constant, when their positions are interchanged."



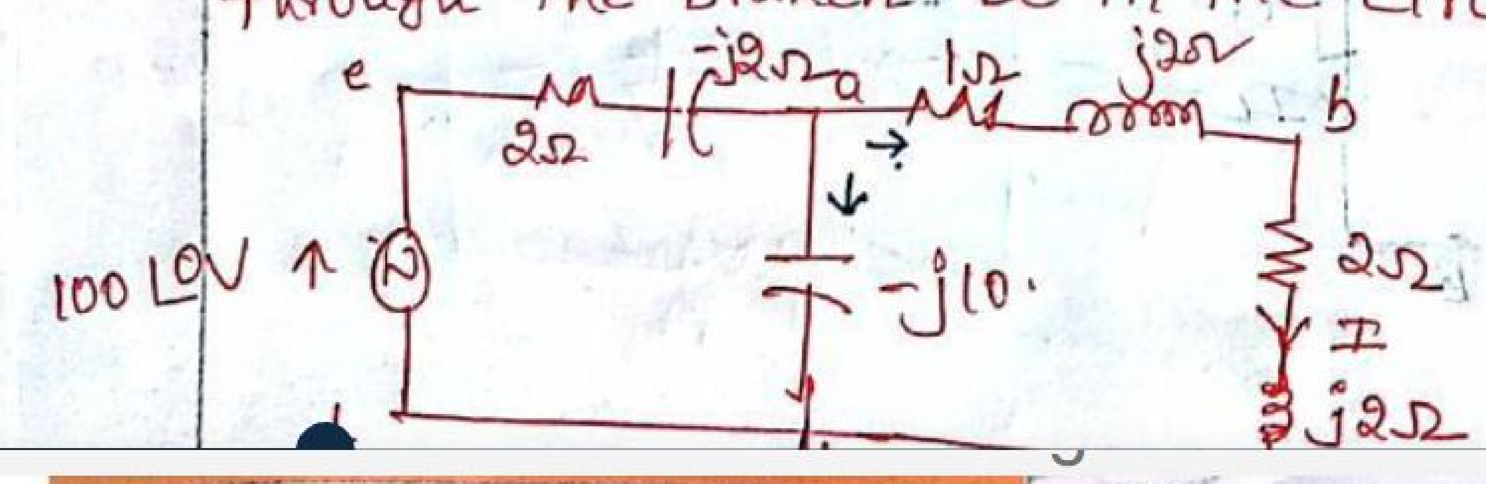
The ratio of excitation to response is $\frac{V}{I}$

Now, interchange positions of excitation and response as shown below,



Then according to theorem, ratio of excitation to response is $\frac{V}{I}$ remains same.

① Verify reciprocity theorem by finding the current through the branch bc in the circuit shown in fig



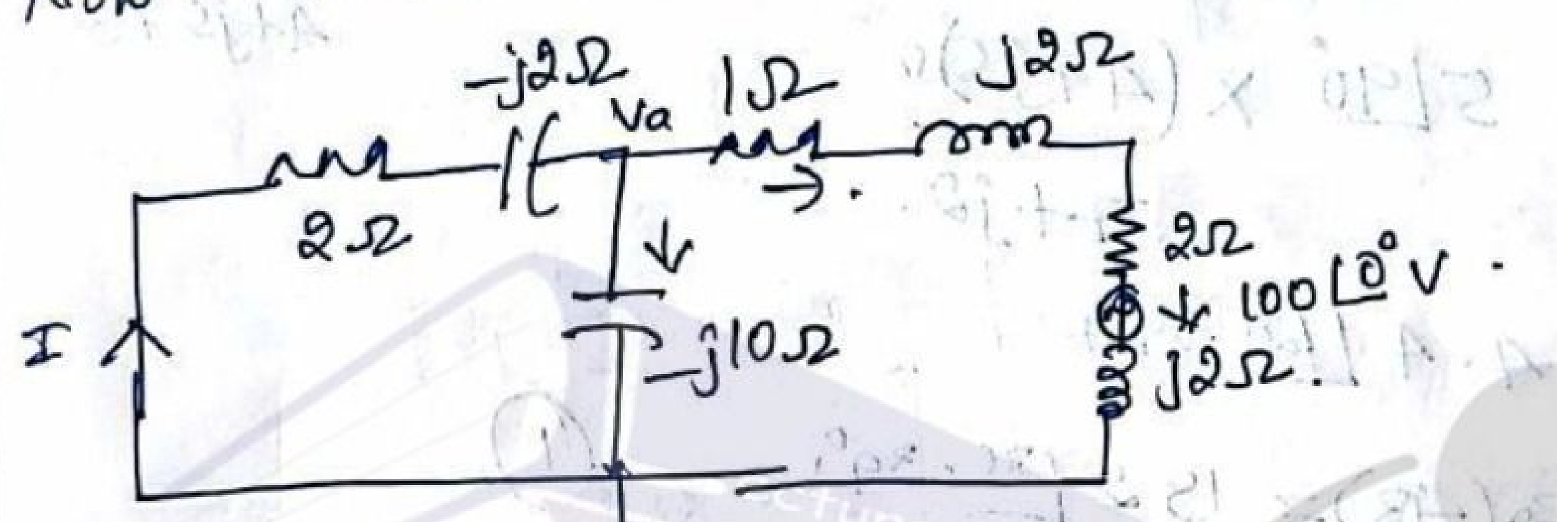
The node equation at 'a'

$$\left(\frac{1}{2-2j}+\frac{1}{-j10}+\frac{1}{3+j4}\right)V_a=\frac{100}{2-2j}$$

$V_a = 84.29\angle 17.82^\circ V$.

$I = \frac{V_a}{3+j4} = \frac{84.29}{3+j4} = 16.89\angle -35.31^\circ A$. —— ①

Now $100\angle 0^\circ V$ is shifted to position I as shown below



The node equations at 'a' is

$$\left(\frac{1}{2-j2}\right)V_a + \frac{V_a}{-j10} + \frac{V_a - 100\angle 0^\circ}{3+j4\,\Omega}$$

$$\left(\frac{1}{2-j2}+\frac{1}{-j10}+\frac{1}{3+j4}\right)V_a = \frac{-100}{3+j4}$$

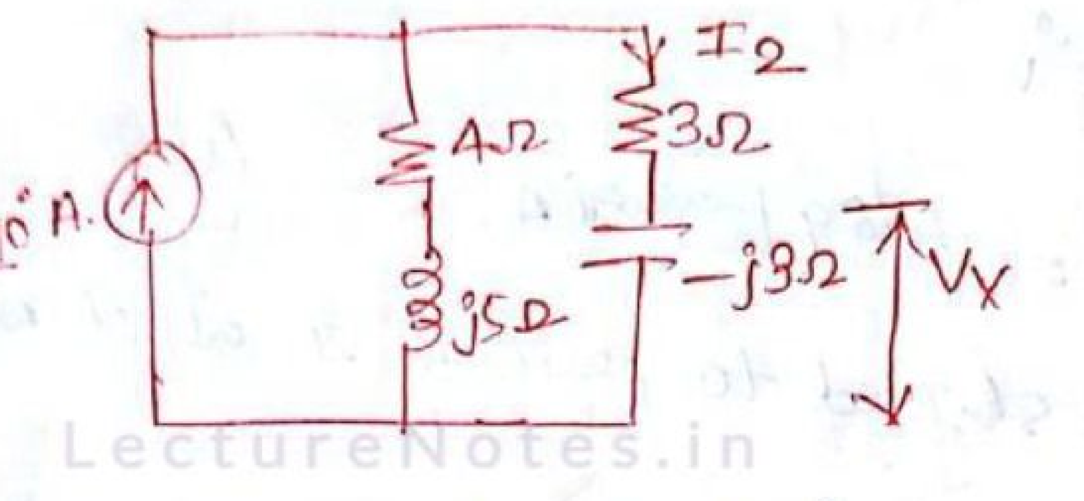
$\therefore V_a = -47.62\angle -80.31^\circ V$

$I = \frac{0-V_a}{2-j2\Omega} = \frac{-47.62\angle -80.31^\circ}{2-j2}$

$I = 16.86\angle -35.31^\circ A$ —— ②

eq ① is same as eq ②. Hence theorem is approved.

2) In the circuit shown in fig below, find the voltage V_x and verify reciprocity theorem.



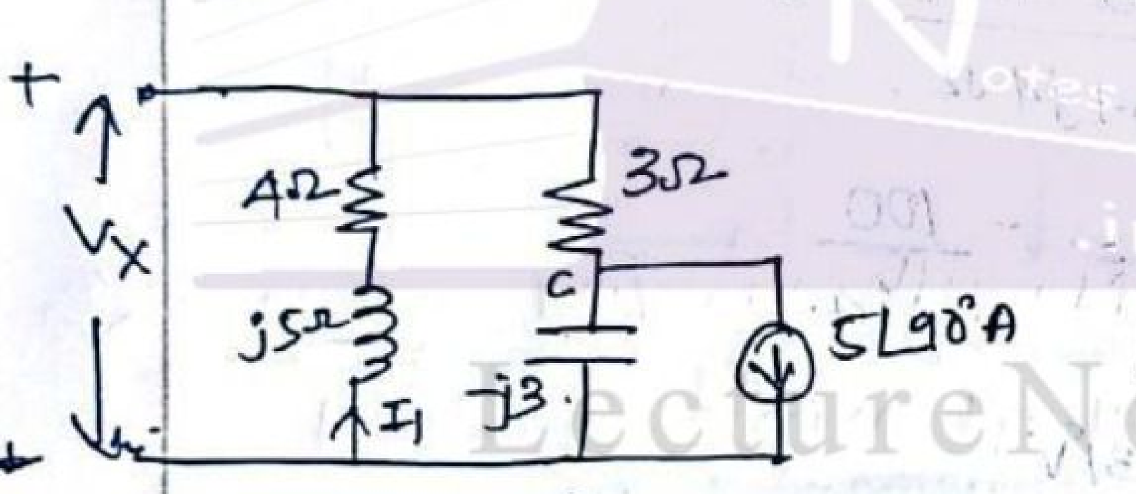
$I_2 = 5\angle 90 \times \frac{(4+j5)}{4+j5+3-j3}$

Sol:- $I_2 = 5\angle 90^\circ \times \frac{(4+j5)\Omega}{7+j2}$.

$I_2 = 4.4\angle 125.39^\circ A$.

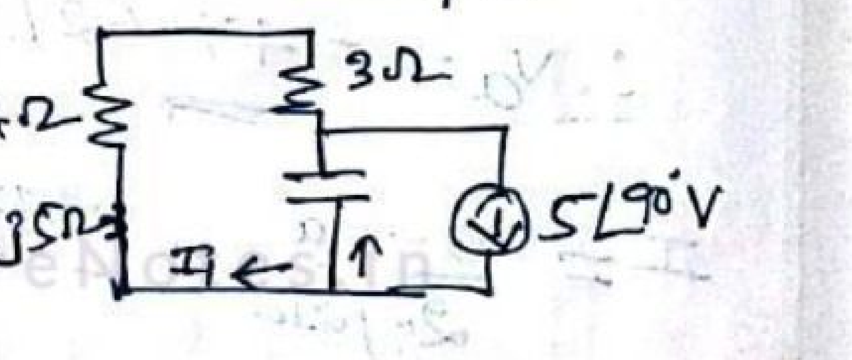
$V_x = I_2(-j3) = 13.2\angle 35.39^\circ V$. —— ①

Now interchange positions of $5\angle 90^\circ V$ and V_x



$I_1 = 5\angle 90^\circ \times \frac{-j3}{7+j2}$.

$I_1 = 2.06\angle 35.49^\circ$.



$V_x = (4+j5)\,I_1$

$V_x = 13.2\angle 35.39^\circ V$ —— ②

Hence theorem is verified.