Market Equilibrium: CAPM and APT

Capital Market Theory

- After Markotwitz, William Sharpe presented capital market theory
- He analyzed the effect of risk-free asset on a portfolio
- He proved that when a risk-free asset is combined with a risky portfolio, the risk of the new portfolio considerably declines without significant decline in the return of the new portfolio

Risk-free asset

- The return on a risk-free asset is fixed hence the standard deviation of risk-free asset is zero
- The return on risk free asset does not change with change in return of other securities, so the covariance of risk-free asset with any other asset will be zero

$$\sigma_p = [w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B Cov_{AB}]^{1/2}$$

- Suppose risk free asset is A so SD of A =0
- The first term of the equation will be removed
- The COV term will also be 0, so the last term will also be removed
- Risk of the portfolio will be

$$\sigma_p = [0 + w_B^2 \sigma_B^2 + 0]^{1/2}$$

- The weight of W_B is equal to $W_{B\,=}$ $(1-W_A)$
- So we can write the above formula as
- SD of Portfolio =$(1-W_A)^2 Q^2{}_B]^{1/2}$ OR
- SD of Portfolio =$(1-W_A)Q_B$

Example

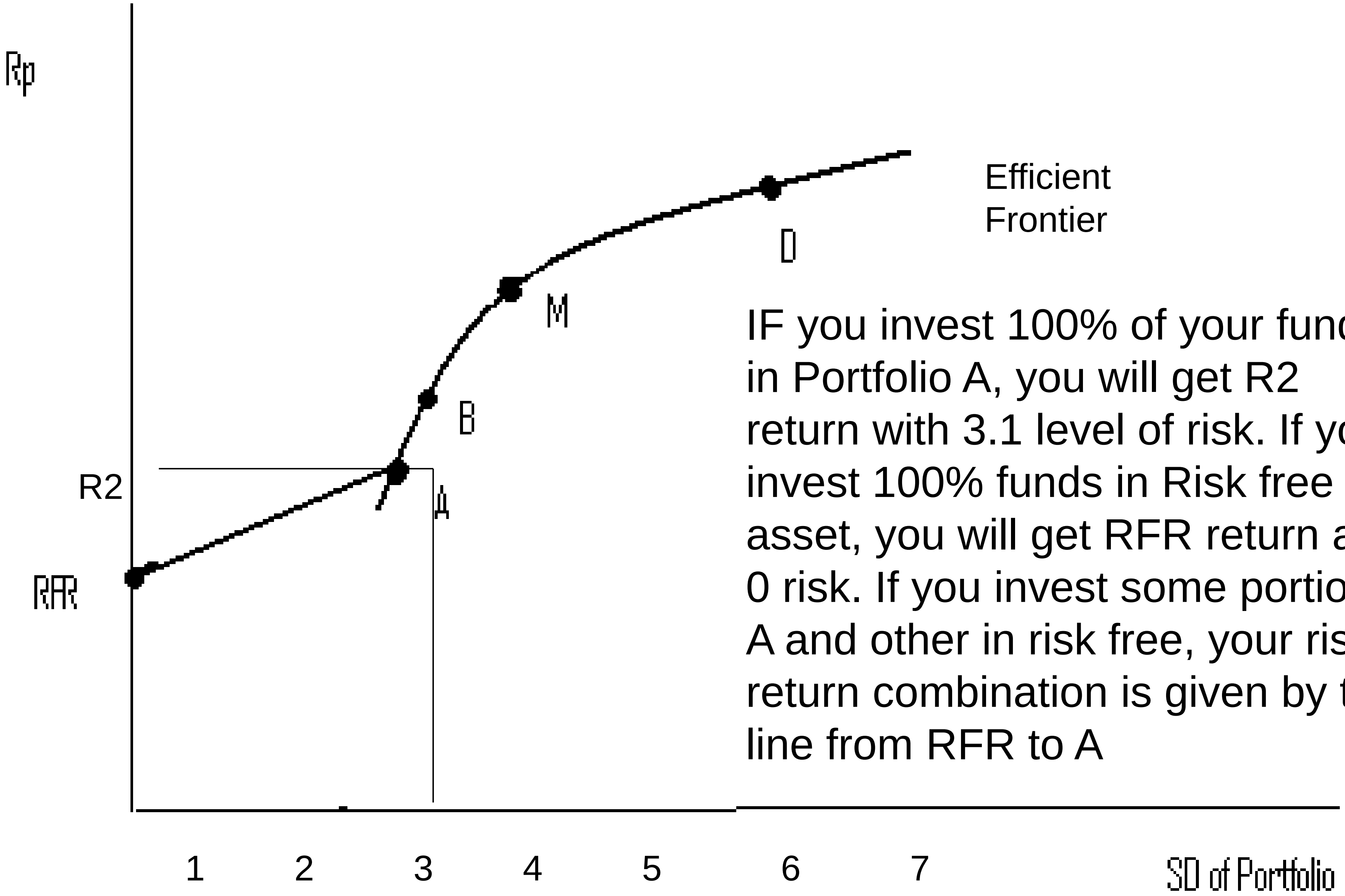
- Suppose we made a portfolio of POL, MCB, and Engro. After trying different weights, we found out that one efficient portfolio give SD of 8%; we can reduce this SD further by investing some portion of our funds in the above risky portfolio and remainder in risk-free asset.
- Suppose we invest 30% in risk free and 70% in the risky asset
- SD of portfolio = $(1-W_A)Q_B$ = (1-.3)x8 = 0.7x8 = 5.6
- IF we invest 50% in risk free and 50% in risky
- SD of portfolio = $(1-W_A)Q_B$ = (1-.5)x8 = 0.5x8 = 4
- If we invest 80% in risk free,
- SD of portfolio = $(1-W_A)Q_B$ = (1-.8)x8 = 0.2x8 = 1.6

Interpretation

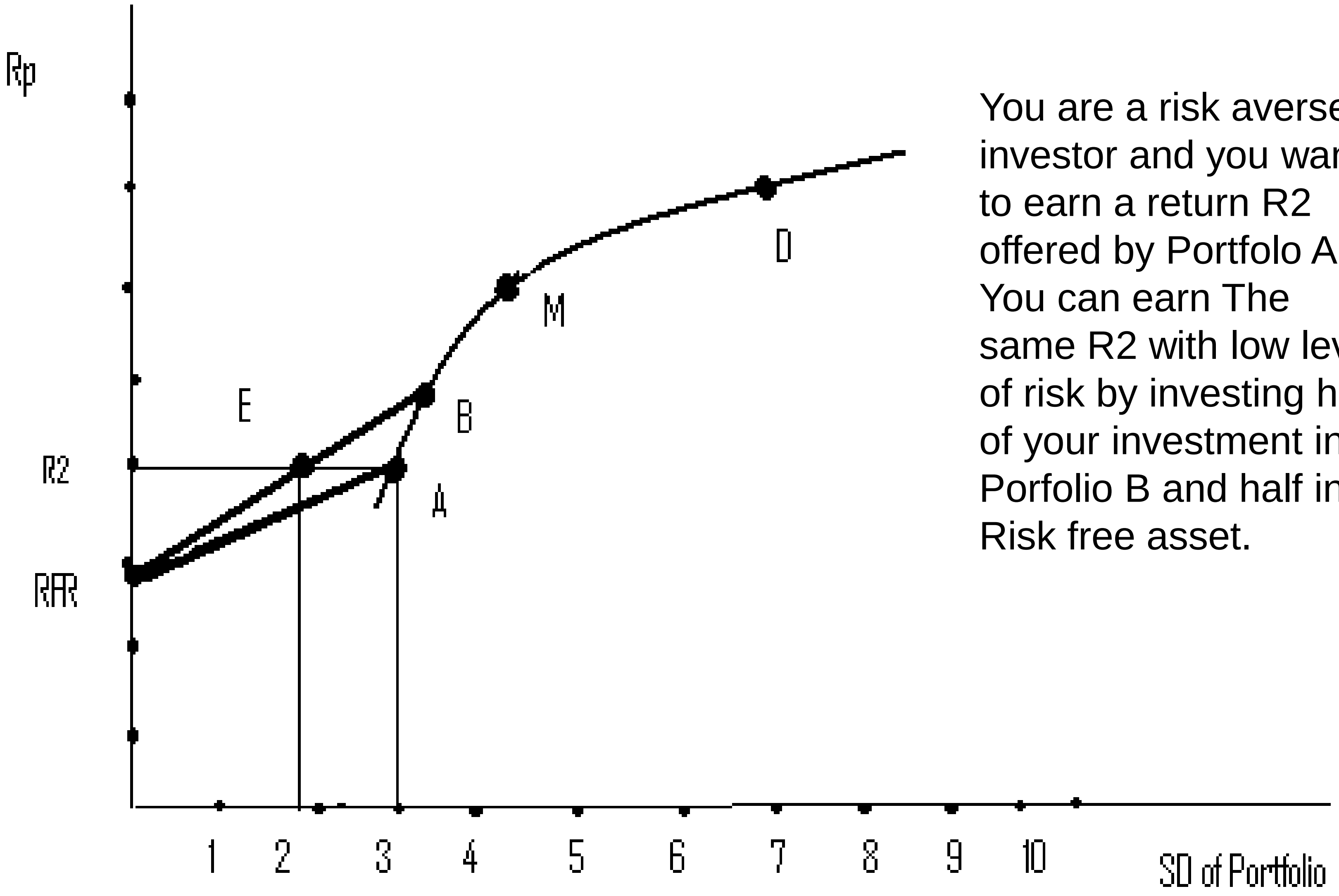
- The formula shows that the risk of the risky portfolio falls proportionately with weight of the risk-free security
- In other words, if we add 50% risk free asset to our portfolio, the overall risk of the risky portfolio will decrease by 50%
- The return of the portfolio will also decrease but not by 50%:

	SD	Return
Risk Free	0	4
Risky Portfolio	6	10

Risk free weight	Risky Wieght	New Portfolio	
Wa	**Wb**	**SD**	**Return**
0	1	6	10
0.1	0.9	5.4	9.4
0.2	0.8	4.8	8.8
0.3	0.7	4.2	8.2
0.4	0.6	3.6	7.6
0.5	0.5	3	7
0.6	0.4	2.4	6.4
0.7	0.3	1.8	5.8
0.8	0.2	1.2	5.2
0.9	0.1	0.6	4.6
1	0	0	4

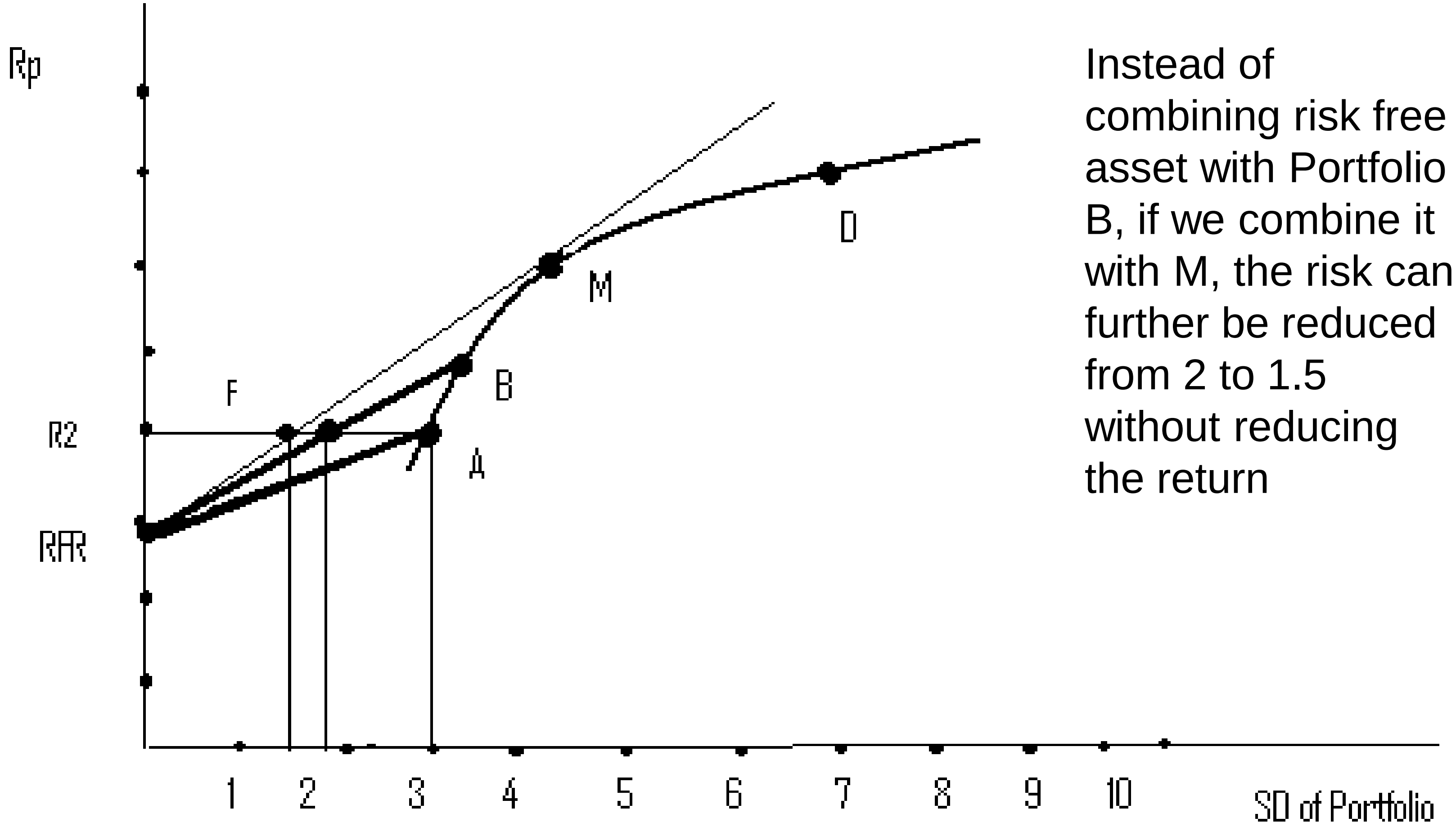


IF you invest 100% of your funds in Portfolio A, you will get R2 return with 3.1 level of risk. If you invest 100% funds in Risk free asset, you will get RFR return and 0 risk. If you invest some portion in A and other in risk free, your risk return combination is given by the line from RFR to A



You are a risk averse investor and you want to earn a return R2 offered by Portfolo A. You can earn The same R2 with low level of risk by investing half of your investment in Porfolio B and half in Risk free asset.

Can risk further be reduced?



Instead of combining risk free asset with Portfolio B, if we combine it with M, the risk can further be reduced from 2 to 1.5 without reducing the return

Capital Asset Pricing Model (CAPM)

- The capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965) marks the birth of asset pricing theory (resulting in a Nobel Prize for Sharpe in 1990)
- Four decades later, the CAPM is still widely used in applications, such as estimating the cost of capital for firms and evaluating the performance of managed portfolios.

The Logic of the CAPM

- The CAPM builds on the model of portfolio choice developed by Harry Markowitz (1959).
- In Markowitz's model, an investor selects a portfolio at time *t*-1 that produces a stochastic return at *t*.
- The model assumes investors are risk averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return. As a result, investors choose "mean-variance-efficient" portfolios, in the sense that the portfolios:
  1. minimize the variance of portfolio return, given expected return, and
  2. maximize expected return, given variance.

- The portfolio model provides an algebraic condition on asset weights in mean-variance-efficient portfolios.
- The CAPM turns this algebraic statement into a testable prediction about the relation between risk and expected return by identifying a portfolio that must be efficient if asset prices are to clear the market of all assets.
- Sharpe (1964) and Lintner (1965) add two key assumptions to the Markowitz model to identify a portfolio that must be mean-variance-efficient
    1. *complete agreement: investors agree on the joint distribution of asset returns from t -1 to t*
    2. *borrowing and lending at a risk-free rate*

CAPM equation

$$ER_i = R_f + \beta_i(ER_m - R_f)$$

where:

ER_i = expected return of investment

R_f = risk-free rate

β_i = beta of the investment

$(ER_m - R_f)$ = market risk premium

- Investors expect to be compensated for risk and the time value of money
- The risk-free rate accounts for the time value of money
- The other components account for the investor taking on additional risk.
- Beta gives a measure how much returns of a security varies when the return on market portfolio varies

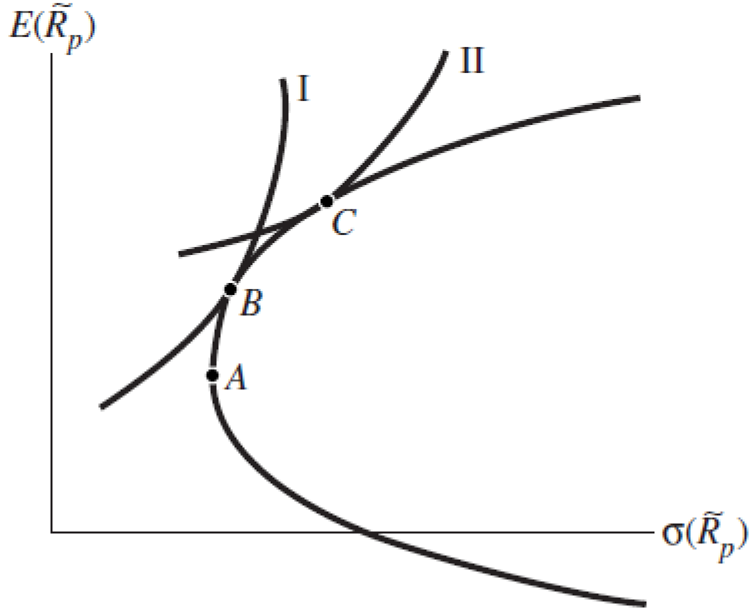
Capital Asset Pricing Model (CAPM)

The CAPM is developed in a hypothetical world where the following assumptions are made about investors and the opportunity set:

1. Investors are risk-averse individuals who maximize the expected utility of their wealth.
2. Investors are price takers and have homogeneous expectations about asset returns that have a joint normal distribution.
3. There exists a risk-free asset such that investors may borrow or lend unlimited amounts at a risk-free rate.
4. The quantities of assets are fixed. Also, all assets are marketable and perfectly divisible.
5. Asset markets are frictionless, and information is costless and simultaneously available to all investors.
6. There are no market imperfections such as taxes, regulations, or restrictions on short selling.

The Efficiency of the Market Portfolio

- Proof of the CAPM requires that in equilibrium the market portfolio must be an efficient portfolio
- The portfolio must lie on the upper half of the minimum variance opportunity set (efficient frontier)
- Because investors have homogeneous expectations, they will all perceive the same minimum variance opportunity set



Properties of CAPM

1. **In equilibrium, every asset must be priced so that its risk-adjusted required rate of return falls exactly to the *security market line*.**
   - Because not all the variance of an asset's return is of concern to risk-averse investors.

   As **total risk = systematic risk + unsystematic risk**

   - Investors can always diversify away all risk except the covariance of an asset with the market portfolio (systematic risk).

Properties of CAPM

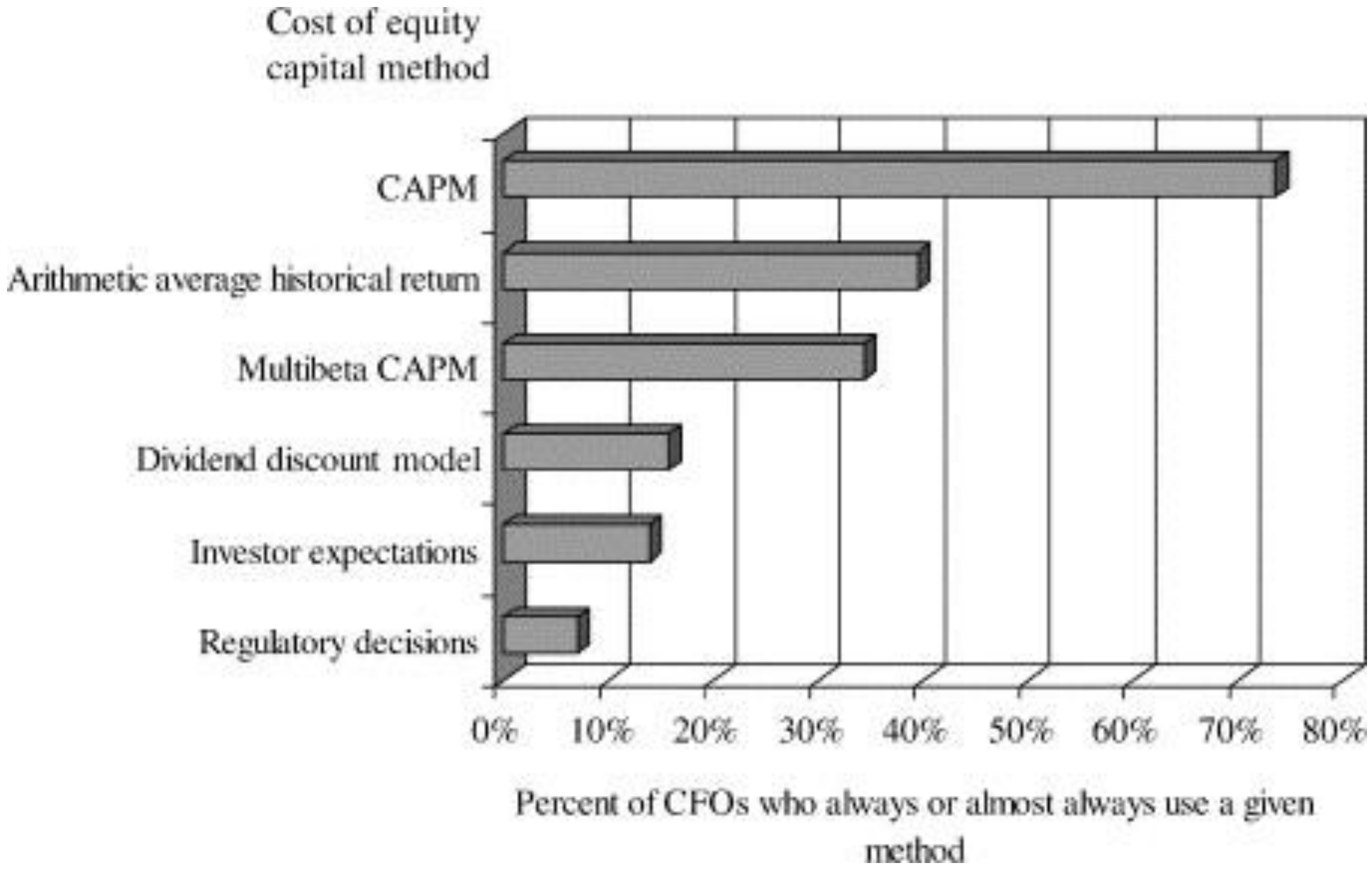
2. The measure of risk for individual assets is linearly additive when the assets are combined into portfolios

- For example, if we put *a*% of our wealth into asset *X*, with systematic risk of β_x, and *b*% of our wealth into asset *Y*, with systematic risk of β_y, then the beta of the resulting portfolio, β_p, is simply the weighted average of the betas of the individual securities:

$$\beta_p = a\beta_x + b\beta_y$$

- This implies that portfolio betas are linearly weighted combinations of individual asset betas

Practical use of CAPM



Source: Graham & Harvey (2001)

Early Empirical Tests

- Tests of the CAPM are based on three implications of the relation between expected return and market beta implied by the model.

1. Expected returns on all assets are linearly related to their betas, and no other variable has marginal explanatory power.
2. The beta premium is positive, meaning that the expected return on the market portfolio exceeds the expected return on assets whose returns are uncorrelated with the market return.
3. In the Sharpe-Lintner version of the model, assets uncorrelated with the market have expected returns equal to the risk-free interest rate, and the beta premium is the expected market return minus the risk-free rate.