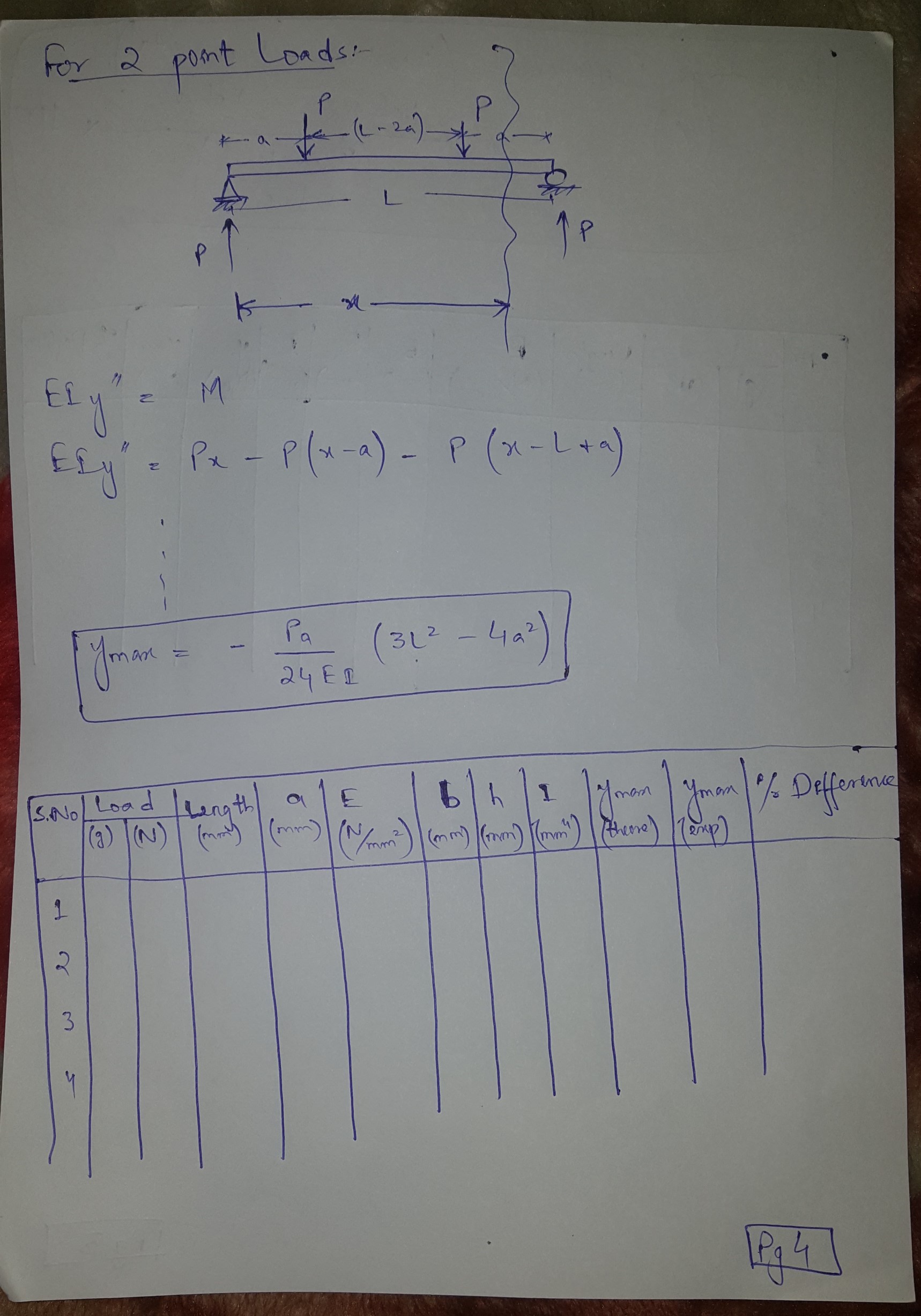
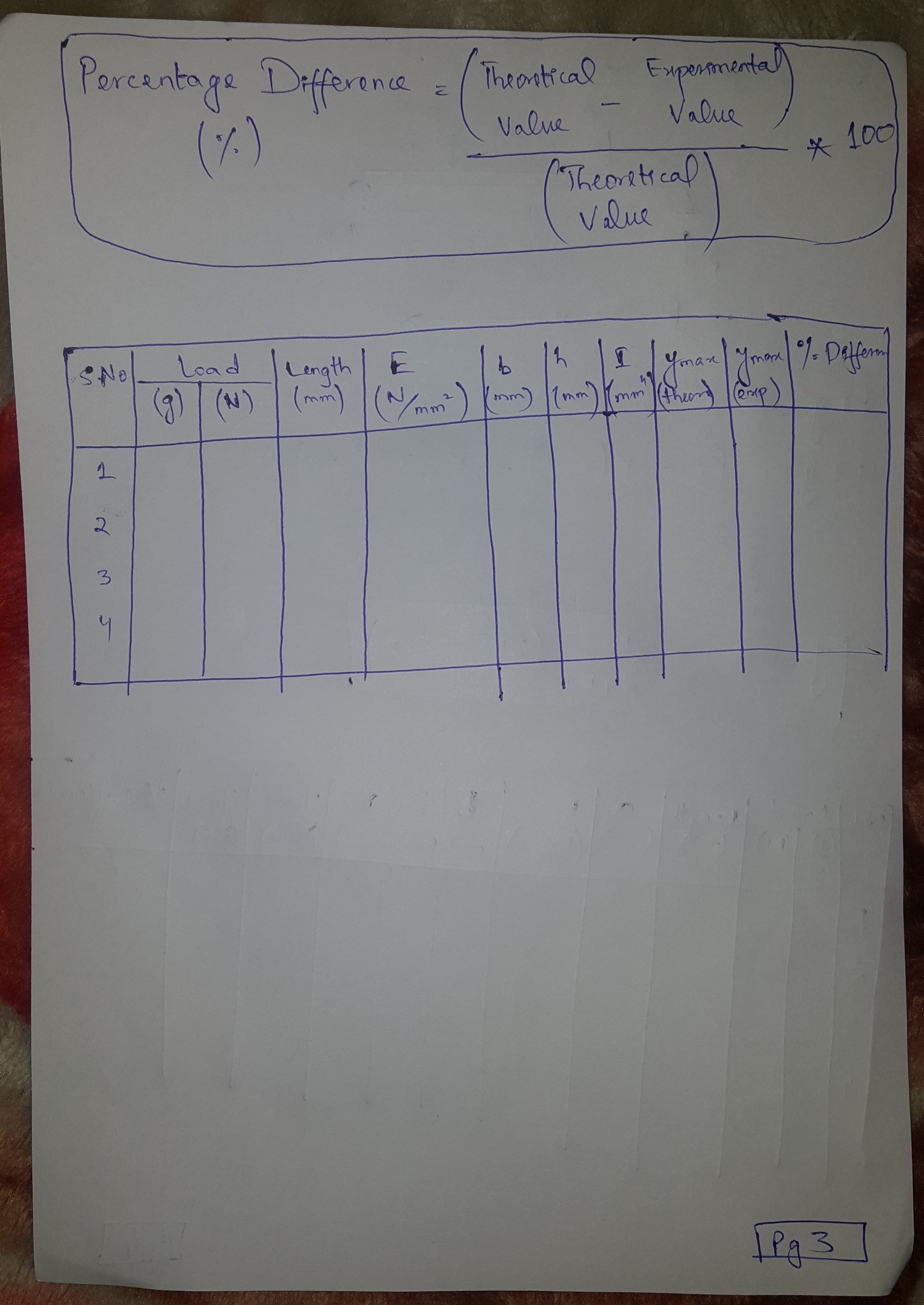
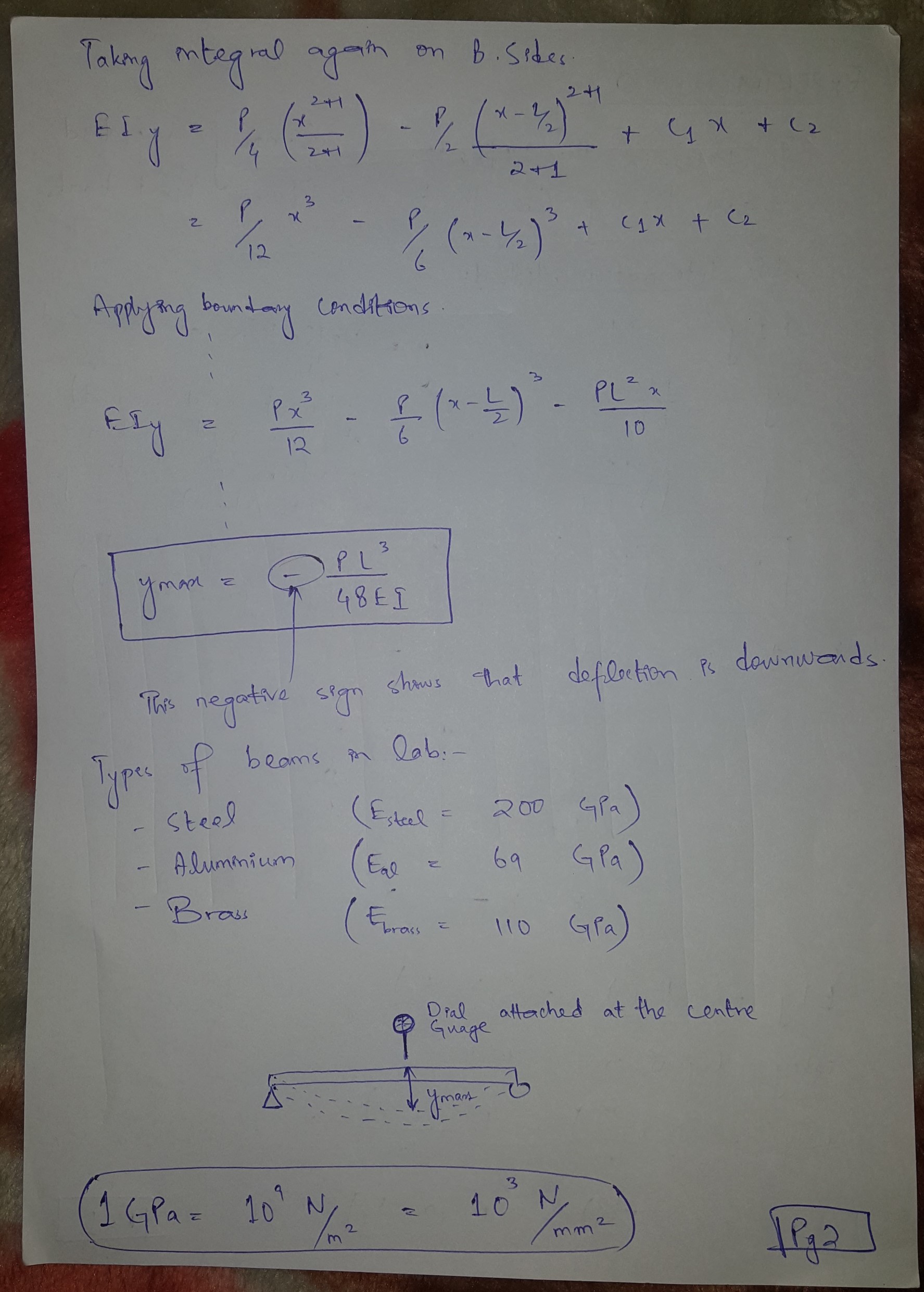
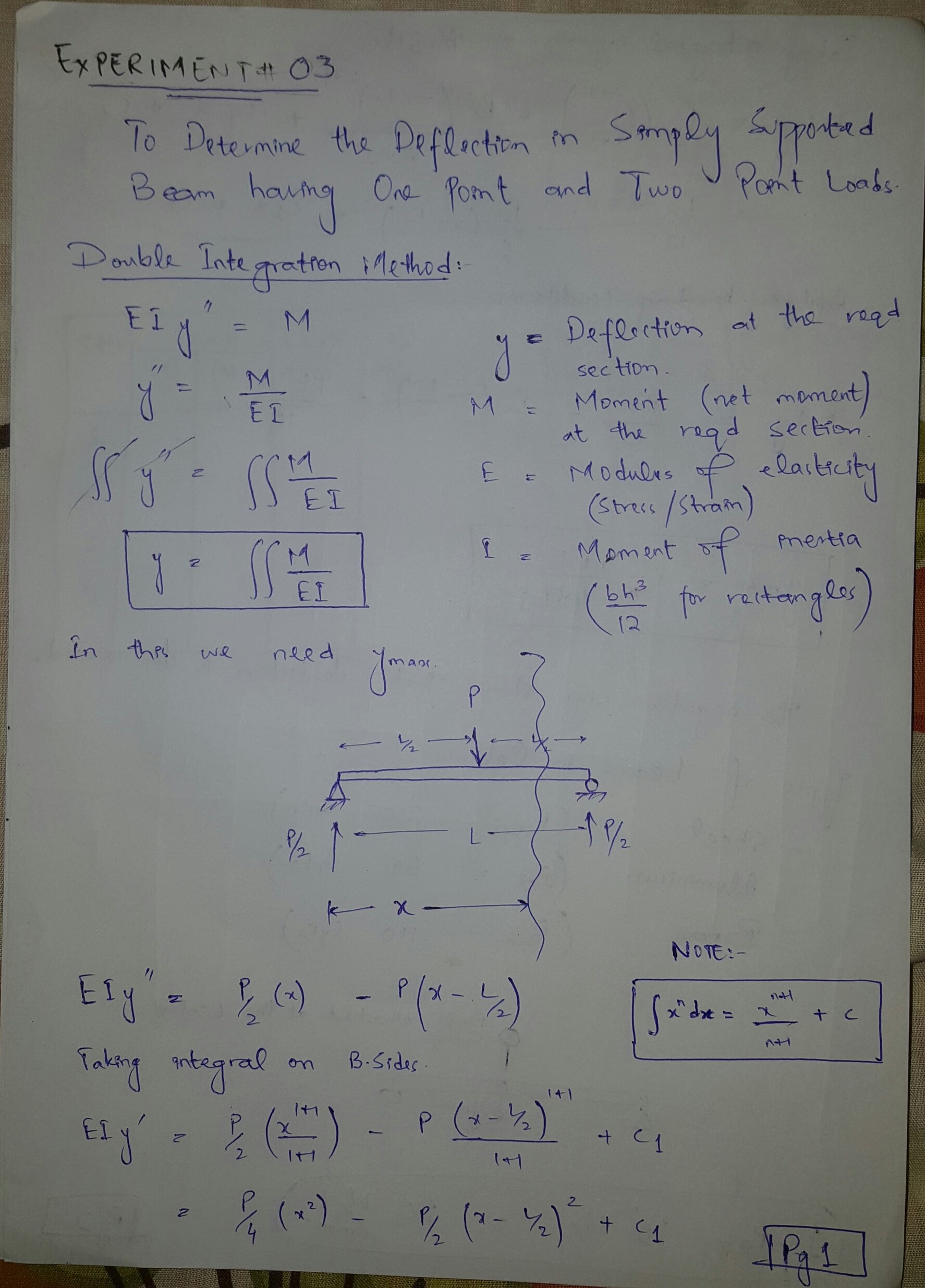
**Deflection of simply supported beam and cantilever**

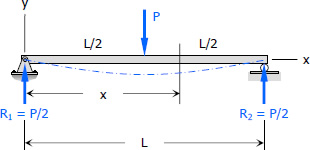


**DOUBLE INTEGRATION METHOD**

**Problem 605**

Determine the maximum deflection δ in a simply supported beam of length L carrying a concentrated load P at midspan.

**Solution 605**



$ EI \, y'' =  \frac{1}{2}Px - P\langle x - \frac{1}{2}L \rangle $

$ EI \, y' = \frac{1}{4}Px^2 -  \frac{1}{2}P\langle x - \frac{1}{2}L \rangle^2 + C_1 $

$ EI \, y = \frac{1}{12}Px^3 -  \frac{1}{6}P\langle x - \frac{1}{2}L \rangle^3 + C_1x + C_2 $

At x = 0, y = 0, therefore, C2 = 0

At x = L, y = 0  
$ 0 = \frac{1}{12}PL^3 -  \frac{1}{6}P\langle L - \frac{1}{2}L \rangle^3 + C_1L $

$ 0 = \frac{1}{12}PL^3 -  \frac{1}{48}PL^3 + C_1L $

$ C_1 = -\frac{1}{16}PL^2 $

Thus,  
$ EI \, y = \frac{1}{12}Px^3 -  \frac{1}{6}P\langle x - \frac{1}{2}L \rangle^3 - \frac{1}{16}PL^2x $

Maximum deflection will occur at x = ½ L (midspan)  
$ EI \, y_{max} = \frac{1}{12}P(\frac{1}{2}L)^3 - \frac{1}{6}P (\frac{1}{2}L - \frac{1}{2}L)^3 - \frac{1}{16}PL^2 (\frac{1}{2}L) $

$ EI \, y_{max} = \frac{1}{96}PL^3 - 0 - \frac{1}{32}PL^3 $

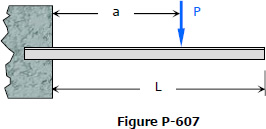
$ y_{max} = -\dfrac{PL^3}{48EI} $

The negative sign indicates that the deflection is below the undeformed neutral axis.

Therefore,  
$ \delta_{max} = \dfrac{PL^3}{48EI}\,\, $            ***answer***

**Problem 607**

Determine the maximum value of EIy for the cantilever beam loaded as shown in [Fig. P-607](http://www.mathalino.com/image/607-cantilever-beam-with-point-load). Take the origin at the wall.

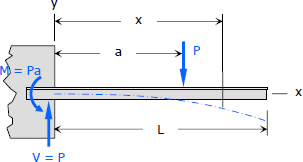


**Solution 607**

$ EI \, y'' = -Pa + Px - P\langle x - a \rangle $

$ EI \, y' = -Pax + \frac{1}{2} Px^2 - \frac{1}{2} P\langle x - a \rangle^2 + C_1 $

$ EI \, y = -\frac{1}{2}Pax^2 + \frac{1}{6} Px^3 - \frac{1}{6} P\langle x - a \rangle^3 + C_1x + C_2 $



At x = 0, y' = 0, therefore C1 = 0  
At x = 0, y = 0, therefore C2 = 0

Therefore,  
$ EI \, y = -\frac{1}{2}Pax^2 + \frac{1}{6} Px^3 - \frac{1}{6} P\langle x - a \rangle^3 $

The maximum value of EI y is at x = L (free end)

$ EI \, y_{max} = -\frac{1}{2}PaL^2 + \frac{1}{6} PL^3 - \frac{1}{6} P(L - a)^3 $

$ EI \, y_{max} = -\frac{1}{2}PaL^2 + \frac{1}{6} PL^3 - \frac{1}{6} P(L^3 - 3L^2a + 3La^2 - a^3) $

$ EI \, y_{max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}PL^3 + \frac{1}{2}PL^2a - \frac{1}{2}PLa^2 + \frac{1}{6}Pa^3 $

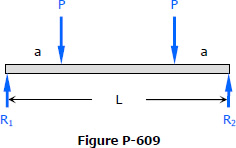
$ EI \, y_{max} = -\frac{1}{2}PLa^2 + \frac{1}{6}Pa^3 $

$ EI \, y_{max} = -\frac{1}{2}PLa^2 + \frac{1}{6}Pa^3 $

$ EI \, y_{max} = -\frac{1}{6}Pa^2(3L - a) \,\, $            ***answer***

**Problem 609**

As shown in [Fig. P-609](http://www.mathalino.com/image/609-syymetrically-placed-concentrated-loads), a simply supported beam carries two symmetrically placed concentrated loads. Compute the maximum deflection δ. Check your answer by letting a = ½ L and comparing it with the answer to [Problem 605](http://www.mathalino.com/reviewer/mechanics-and-strength-of-materials/solution-to-problem-605-double-integration-method).



**Solution 609**

By symmetry  
$ R_1 = R_2 = P $

Experiment (A)

**Aim**: Deflection of simply supported beam with concentrated point load on the mid of beam

**Apparatus**: knife edge, load hanger, movable digital dial, test indicator, movable knife edge, clamp, hanger with mass, steel structure mild steel bar.

**Theory**:

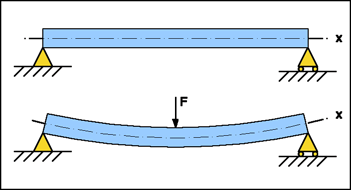
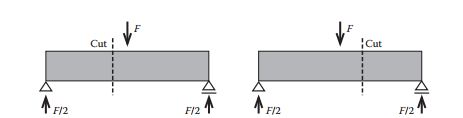


Fig: simply supported beam



a)Cut to the left of the load b) Cut to the right of the load

M+F\*()-

Where clockwise moments are defined as being positive.

We shall evaluate the deflection of a simply supported beam. Dividing the bending-moment distribution by EI, we obtain the distribution of curvature.

for 0<x<

Integrating this function,

+ θ

Where θ is a constant of integration representing the slope at the left end. Because the slope should be zero at mid span, we have

Integrating this equation with the boundary condition v = 0 at x = 0 leads to

for 0*<x<*

The deflection at mid span (x = L/2) is

Procedure:

(Load kept at center of simply supported beam is varying, but distance between applied load and fixed end is same)

A mid steel bar is clamped on steel structure with the help of knife edge bolts and movable clamped.

Using the load hanger mass is added at the center of beam.

The deflection of beam is noted using the digital dial.

Zero correction is noted using the digital dial before adding any weight.

Now load is increased as per the table and the deflection of beam is noted.

The process is repeated again and again for different weight.

**Specification of beam**:

Beam height = 3.3mm

Beam width = 18.5mm

= 207GPa

I = 5.54

**Calculation**:

**Observation table**:

1. Simply supported beam , length = 400mm

|  |  |  |  |
| --- | --- | --- | --- |
| serial no. | mass (gm) | actual deflection (mm) | theoretical deflection (mm) |
| 1 | 0 | 0 | 0 |
| 2 | 100 | 0.06 | 0.114 |
| 3 | 200 | 0.22 | 0.2281 |
| 4 | 300 | 0.37 | 0.3421 |
| 5 | 400 | 0.48 | 0.456 |
| 6 | 500 | 0.65 | 0.572 |

**Graph**:

Now repeat the some procedure by reducing the length of beam

**Observation table**

1. Simply supported beam , length = 200mm

|  |  |  |  |
| --- | --- | --- | --- |
| serial no. | mass (gm) | actual deflection (mm) | theoretical deflection (mm) |
| 1 | 0 | 0 | 0 |
| 2 | 100 | 0.03 | 0.0142 |
| 3 | 200 | 0.06 | 0.0285 |
| 4 | 300 | 0.09 | 0.0427 |
| 5 | 400 | 0.13 | 0.057 |
| 6 | 500 | 0.17 | 0.0712 |

**Graph**:

Experiment (B)

**Aim**: To determine the deflection in a cantilever beam when a load is applied at the center.

**Apparatus**: knife edge, load hanger, movable digital dial, indicator, movable knife edge, clamp, hanger with mass, steel structure

**Theory**:

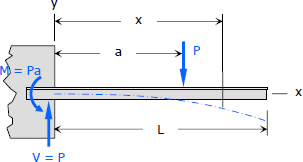


Fig: cantilever beam

We shall evaluate the deflection of a cantilever beam. Dividing the bending-moment distribution by EI, we obtain the distribution of curvature.

Boundary condition

Therefore, we obtain the solution as

Integrating this equation with the boundary condition v = 0 at *x* = 0 leads to

*v=*

Now calculate deflection for *x* =

*v=*

**Procedure**:

1. Load kept at center of cantilever is varying, but distance between applied load and fixed end is same.

Mild steel is clamped at one end and the other end left free.

At a distance of 200mm from the fixed end a load is applied .

The deflection at the point of application of load is noted.

The load is varied as per mention in table.

1. Load at center of cantilever is kept constant i.e. 500gm, distance between dial indicator and clamped end is changing

Mild steel is clamped at one end and the other end left free.

Keep the dial indicator exactly at the fixed end of mild steel beam.

Make the reading of dial indicator ‘zero’.

Now put the load of 500gm initial at 200mm from fixed end and then shift the dial indicator by 50mm.

Continuously shift the dial indicator by keeping the constant difference of 50mm and note down the deflection.

**Specification of beam**:

Beam height = 3.3mm

Beam width = 18.5mm

= 207GPa

I = 5.54

**Calculation**: L = 200mm

**Observation table(A):**

1. Load kept at center of cantilever is varying , but distance between applied load and fixed end is same

|  |  |  |  |
| --- | --- | --- | --- |
| serial no. | mass(gm) | actual deflection(mm) | theoretical deflection(mm) |
| 1 | 0 | 0 | 0 |
| 2 | 100 | 0.38 | 0.228 |
| 3 | 200 | 0.7 | 0.456 |
| 4 | 300 | 1.06 | 0.684 |
| 5 | 400 | 1.34 | 0.912 |
| 6 | 500 | 1.68 | 1.14 |

**Graph(A):**

**Observation table (B)** : L = 400mm

1. load at center of cantilever is kept constant i.e. 500gm , distance between dial indicator and clamped end is changing

|  |  |  |
| --- | --- | --- |
| serial no | position from fixed end (mm) | actual deflection (mm) |
| 1 | 0 | 0 |
| 2 | 50 | 0.597 |
| 3 | 100 | 1.66 |
| 4 | 150 | 3.04 |
| 5 | 200 | 4.51 |
| 6 | 250 | 5.79 |
| 7 | 300 | 6.56 |

**Graph(B):**

**Conclusion:**

1. Linearity observed in both experiment between the mass and deflection, as increases the deflection of beam decreases.
2. Linearity observed in second experiment between the position of dial indicator from the fixed end to the deflection , as the dial indicator shifted toward left side its deflection increases.

For video tutorial of hands on experiment, do check the following videos:

1. Simply supported beam (<https://www.youtube.com/watch?v=jpzHgRASjNU> )
2. Cantilever beam ( <https://www.youtube.com/watch?v=P3rcO1UMxYw> )

Thank you