Lecture # 08

Discrete Structure

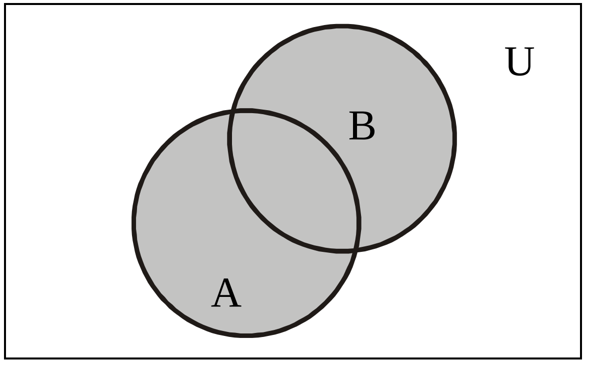
Union

- Let A and B are two sets
- A and B are subsets of a universal set U
- The union of A and B is the set of all elements in U that belong to A or to B or to both
- It is denoted $A \cup B$
- $A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$
- Union is commutative: $A \cup B = B \cup A$
- $A \subseteq A \cup B$ and $B \subseteq A \cup B$

Example: Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$, $B = \{d, e, f, g\}$

Then $A \cup B = \{a, c, d, e, f, g\}$



$A \cup B$ is shaded

Membership Table for Union

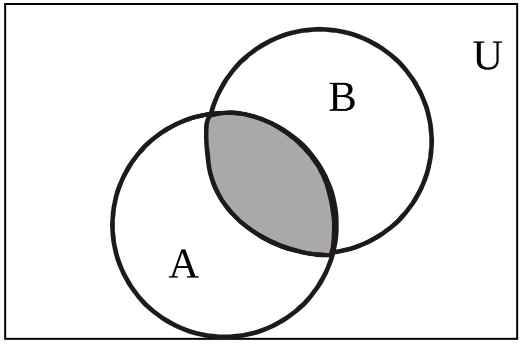
- The Membership table for the union of sets A and B is given below
- The truth table for disjunction of two statements P and Q is given below
- In the membership table of Union replace, 1 by T and 0 by F then the table is same as of disjunction
- So membership table for Union is similar to the truth table for disjunction ($\vee$)

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

P	Q	$P \vee Q$
T	F	T
T	F	T
F	T	T
F	F	F

Intersection

- Let A and B are two sets
- A and B are subsets of a universal set U
- The intersection of A and B is the set of all elements in U that belong to both A and B
- It is denoted $A \cap B$
- $A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$
- Intersection is commutative: $A \cap B = B \cap A$
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$
- If A and B are disjoint, then $A \cap B = \phi$



$A \cap B$ is shaded

Example: Let U = {a, b, c, d, e, f, g}

A = {a, c, e, g}, B = {d, e, f, g}

Then $A \cap B = \{e, g\}$

Membership Table For Intersection

- The Membership table for intersection of sets A and B is given below
- The truth table for conjunction of two statements P and Q is given below
- In the membership table of Intersection, replace 1 by T and 0 by F then the table is same as of conjunction
- So membership table for Intersection is similar to the truth table for conjunction ($\wedge$)

A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

P	Q	$P \wedge Q$
T	F	T
T	F	T
F	T	T
F	F	F

Difference

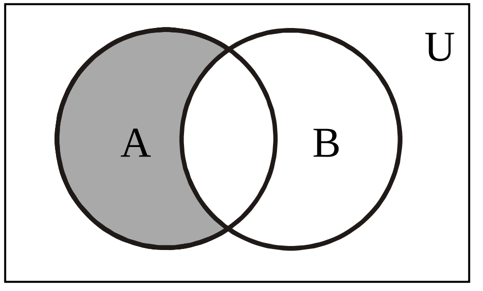
- Let A and B be subsets of a universal set U
- The difference of A and B is the set of all elements in U that belong to A but not to B
- It is denoted as $A - B$
- $A - B = \{x \in U \mid x \in A \text{ and } x \quad B\}$
- Set difference is not commutative: $A - B \neq B - A$
- $A - B \subseteq A$
- $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets

Example: Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$

$B = \{d, e, f, g\}$

Then $A - B = \{a, c\}$



A-B is shaded

Membership Table for Set Difference

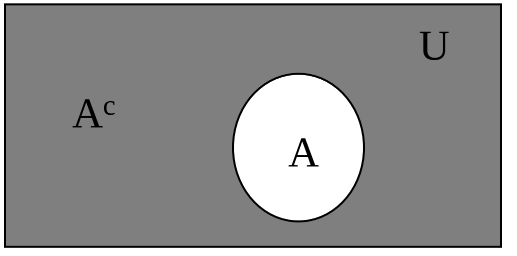
- The Membership table for the difference of sets A and B is given below
- The truth table for negation of implication of two statements P and Q is given below
- In the membership table of difference, replace 1 by T and 0 by F then the table is same as of negation of implication
- So membership table for difference is similar to the truth table for negation of implication

A	B	A – B
1	1	0
1	0	1
0	1	0
0	0	0

P	q	$p \to q$	$\sim (p \to q)$.
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Complement

- Let A be a subset of universal set U
- The complement of A is the set of all element in U that do not belong to A
- It is denoted by A^c
- $A^c = \{x \in U \mid x \notin A\}$
- $A^c = U - A$
- $A \cap A^c = \phi$
- $A \cup A^c = U$



A^c is shaded

Example: Let U = {a, b, c, d, e, f, g]

A = {a, c, e, g}

Then A^c = {b, d, f}

Membership Table for Complement

- The Membership table for the complement of sets A is given below
- The truth table for negation of statement P is given below
- In the membership table of complement, replace 1 by T and 0 by F then the table is same as of negation
- So membership table for complement is similar to the truth table for negation

A	A^c
1	0
0	1

p	~p
T	F
F	T

Exercise 1

Let $U = \{1, 2, 3, ..., 10\}$, $X = \{1, 2, 3, 4, 5\}$

$Y = \{y \mid y = 2x, x \in X\}$, $Z = \{z \mid z^2 - 9z + 14 = 0\}$

Enumerate:

(i) $X \cap Y$ (ii) $Y \cup Z$ (iii) $X - Z$

(iv) Y^c (v) $X^c - Z^c$ (vi) $(X - Z)^c$

Solution:

Given $U = \{1, 2, 3, ..., 10\}$, $X = \{1, 2, 3, 4, 5\}$

$Y = \{y \mid y = 2x, x \in X\} = \{2, 4, 6, 8, 10\}$

$Z = \{z \mid z^2 - 9z + 14 = 0\} = \{2, 7\}$

Exercise 1 (Cont.)

(i) $X \cap Y = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} = \{2, 4\}$

(ii) $Y \cup Z = \{2, 4, 6, 8, 10\} \cup \{2, 7\} = \{2, 4, 6, 7, 8, 10\}$

(iii) $X - Z = \{1, 2, 3, 4, 5\} - \{2, 7\} = \{1, 3, 4, 5\}$

(iv) $Y^c = U - Y = \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

(v) $X^c - Z^c = \{6, 7, 8, 9, 10\} - \{1, 3, 4, 5, 6, 8, 9, 10\} = \{7\}$

(vi) $(X - Z)^c = U - (X - Z) = \{1, 2, 3, \ldots, 10\} - \{1, 3, 4, 5\}$

$= \{2, 6, 7, 8, 9, 10\}$

Exercise 2

Given the following universal set U and its two subsets P and Q, where

$U = \{x \mid x \in Z,\ 0 \leq x \leq 10\}$

$P = \{x \mid x \text{ is a prime number}\}$

$Q = \{x \mid x^2 < 70\}$

(i) Draw a Venn diagram for the above

(ii) List the elements in $P^c \cap Q$

Solution: First we write the sets in Tabular form.

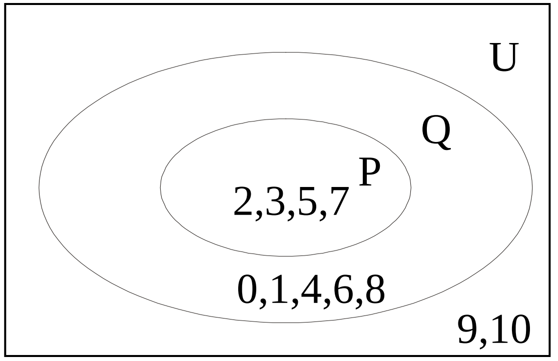
$$U = \{x \mid x \in Z,\ 0 \leq x \leq 10\} = \{0, 1, 2, 3, \ldots, 10\}$$

$$P = \{x \mid x \text{ is a prime number}\} = \{2, 3, 5, 7\}$$

$$Q = \{x \mid x^2 < 70\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

Exercise 2 (Cont.)

(i)Venn diagram



(ii) $P^c \cap Q = ?$

$P^c = U - P = \{0, 1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{0, 1, 4, 6, 8, 9, 10\}$

and $P^c \cap Q = \{0, 1, 4, 6, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{0, 1, 4, 6, 8\}$

Exercise 3

Let $U = \{1, 2, 3, 4, 5\}$, $C = \{1, 3\}$

and A and B are non empty sets. Find A in each of the following:

i. $A \cup B = U, A \cap B = \phi$ and $B = \{1\}$

ii. $A \subset B$ and $A \cup B = \{4, 5\}$

iii. $A \cap B = \{3\}$, $A \cup B = \{2, 3, 4\}$ and $B \cup C = \{1,2,3\}$

iv. A and B are disjoint, B and C are disjoint, and the union of A and B is the set $\{1, 2\}$.

i. $A \cup B = U$, $A \cap B = \phi$ and $B = \{1\}$

Solution:

Since $A \cup B = U = \{1, 2, 3, 4, 5\}$, and $A \cap B = \phi$

Therefore $A = B^c = \{1\}^c = \{2, 3, 4, 5\}$

Exercise 3 (Cont.)

ii. $A \subset B$ and $A \cup B = \{4, 5\}$ also $C = \{1, 3\}$

Solution:

When $A \subset B$, then $A \cup B = B = \{4, 5\}$

Also A being a proper subset of B implies

$A = \{4\}$ or $A = \{5\}$

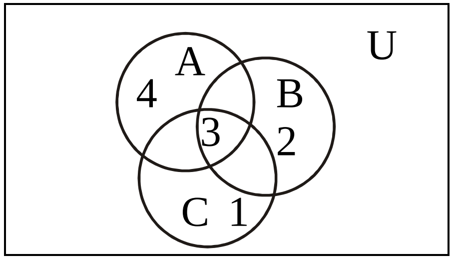
iii. $A \cap B = \{3\}$, $A \cup B = \{2, 3, 4\}$, $B \cup C = \{1,2,3\}$,

$C = \{1, 3\}$

Solution:

$A = \{3, 4\}$

$B = \{2, 3\}$



Exercise 3 (Cont.)

(iv) $A \cap B = \phi$, $B \cap C = \phi$, $A \cup B = \{1, 2\}$, $C = \{1, 3\}$

Solution:

$A = \{1\}$