Lecture # 07

Discrete Structure

Sets

- Set is a collection of well defined distinct objects
- Objects are called the elements or members of the set
- Sets are denoted by capital letters A, B, C ..., X, Y, Z
- Elements of a set are represented by small letters a, b, c, ... , x, y, z
- If an object x is a member of a set A then it denoted by $x \in A$, which reads "x belongs to A"
- If an object x is not a member of a set A then it denoted by $x \notin A$, which reads "x does not belong to A"

Tabular Form

- Listing all the elements of a set, separated by commas and enclosed within curly brackets { }

Examples:

- Set of first five Natural Numbers A = {1, 2, 3, 4, 5}
- Set of Even numbers up to 50 B = {2, 4, 6, 8, …, 50}
- Set of positive odd numbers C = {1, 3, 5, 7, 9, …}

The symbol "…" is called an ellipsis, short for "and so forth"

Descriptive Form

- Stating the elements of a set in words

Examples:

- A = set of first five Natural Numbers
- B = set of positive even integers less or equal to fifty
- C = set of positive odd integers

Set Builder Form

- Writing the common characteristics shared by all the elements of the set in symbolic form

Examples:

- $A = \{x \in \mathrm{N} \mid x<=5\}$
- $B = \{x \in \mathrm{E} \mid 0 < x <=50\}$
- $C = \{x \in \mathrm{O} \mid 0 < x \}$

Sets of Numbers

1. **Set of Natural Numbers**

   N = {1, 2, 3, ... }

2. **Set of Whole Numbers**

   W = {0, 1, 2, 3, ... }

3. **Set of Integers**

   Z = {..., -3, -2, -1, 0, +1, +2, +3, ...}

   = {0, $\pm$1, $\pm$2, $\pm$3, ...}

4. **Set of Even Integers**

   E = {0, $\pm$ 2, $\pm$ 4, $\pm$ 6, ...}

5. **Set of Odd Integers**

   O = {$\pm$ 1, $\pm$ 3, $\pm$ 5, ...}

Sets of Numbers (Cont.)

6. **Set of Prime Numbers**

   $P = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$

7. **Set of Rational Numbers (or Quotient of Integers)**

   $Q = \{x \mid x = \quad ; p, q \in Z, q \neq 0\}$

8. **Set of Irrational Numbers**

   $Q' = \{ x \mid x \text{ is not rational}\}$

   For example, $\sqrt{2}$, $\sqrt{3}$, π, e, etc.

9. **Set of Real Numbers**

   $R = Q \cup Q'$

10. **Set of Complex Numbers**

    $C = \{z \mid z = x + iy;\ x, y \in R\}$

Subset

- Let A and B are two sets
- Then A is called a subset of B, if, and only if, every element of A is also an element of B
- It is denoted as $A \subseteq B$
- Symbolically: $A \subseteq B \Leftrightarrow$ if $x \in A$ then $x \in B$
- When $A \subseteq B$, then B is called a superset of A
- When $A \nsubseteq B$, then there exist at least one $x \in A$ such that $x \notin B$
- Every set is a subset of itself

Subset (Cont.)

Examples:

Let

$A = \{1, 3, 5\}$ $B = \{1, 2, 3, 4, 5\}$

$C = \{1, 2, 3, 4\}$ $D = \{3, 1, 5\}$

Then

$A \subseteq B$ (As every element of A is in B)

$C \subseteq B$ (As every element of C is also an element of B)

$A \subseteq D$ (As every element of A is also an element of D and also note that every element of D is in A so $D \subseteq A$)

and $A \nsubseteq C$ (Because there is an element 5 of A which is not in C)

Proper Subset

- A is a proper subset of B, if, and only if, every element of A is in B but there is at least one element of B that is not in A
- It is denoted as $A \subset B$

Example:

Let A = {1, 3, 5}, B = {1, 2, 3, 5}

Then $A \subset B$ (Because there is an element 2 of B which is not in A)

Example:

It is very easy to note that

$N \subset Z \subset Q \subset R \subset C$

Equal Sets

- Set A and B are equal if, and only if, every element of A is in B and every element of B is in A
- It is denoted as A = B
- Symbolically: A = B iff A $\subseteq$ B and B $\subseteq$ A

Example:

Let A = {1, 2, 3, 6}   B = the set of positive divisors of 6

C = {3, 1, 6, 2}   D = {1, 2, 2, 3, 6, 6, 6}

Then A, B, C, and D are all equal sets.

Null Set

- A set which contains no element is called a null set, or an empty set or a void set
- It is denoted by the Greek letter $\varnothing$ (phi) or { }
- $\varnothing$ is regarded as a subset of every set

Example:

A = {x | x is a person taller than 10 feet} = $\varnothing$

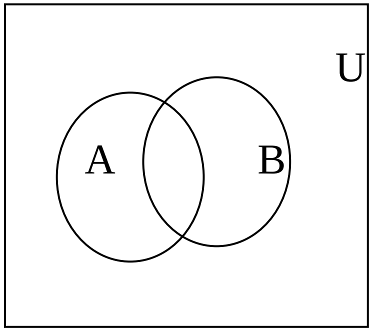
B = {x | $x^2 = 4$, x is odd} = $\varnothing$

Universal Set

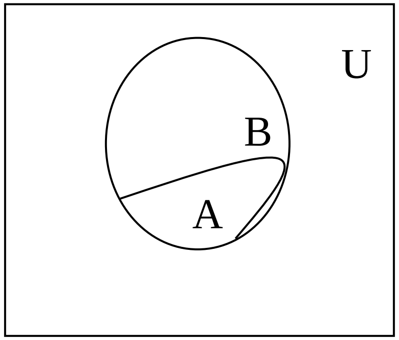
- It is the set of all elements under consideration
- The Universal Set is usually denoted by U

Venn Diagram

- It is a graphical representation of sets by regions in the plane
- The Universal Set is represented by the interior of a rectangle
- Other sets are represented by circles lying within the rectangle



- In Venn diagram above , sets A and B intersect each other
- In Venn diagram below, set A is totally contained in set B, so $A \subseteq B$



Finite and Infinite Sets

- A finite set S contains exactly m distinct elements
- In such case we write $|S| = m$ or $n(S) = m$
- A set is said to be infinite if it is not finite

Examples

1. The set S of letters of English alphabets is finite and $|S| = 26$
2. The null set $\varnothing$ has no elements, is finite and $|\varnothing| = 0$
3. The set of positive integers $\{1, 2, 3,\ldots\}$ is infinite.

Membership Table

- A table displaying the membership of elements in sets
- To indicate that an element is in a set, a 1 is used
- To indicate that an element is not in a set, a 0 is used
- Membership tables can be used to prove set identities

A	A^c
1	0
0	1

- In table if an element belongs to A then it can`t belongs to A^c
- Thus if there is 1 for A then 0 for A^c in that row of table