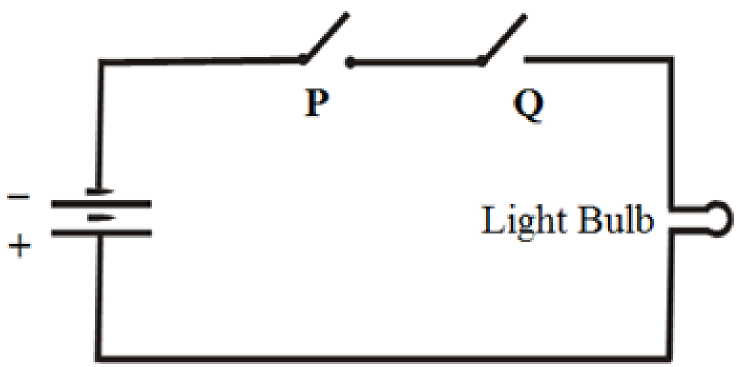
Lecture # 06

Discrete Structure

Switches in Series

- Circuit below consists of a battery, Light bulb, and two switches P and Q
- Both switches are open in both circuits
- An open switch means no current passes in the circuit
- A close switch means current passes in the circuit
- Switches are connected in series
- Light bulb is on only when both the switches are closed
- All the possibilities in the table below



Switches		Light Bulb
P	Q	State
Closed	Closed	On
Closed	Open	Off
Open	Closed	Off
Open	Open	Off

Switches in Series (Cont.)

- Replace **closed** and **on** by **T** and **open** and **off** by **F** then the table is same as truth table for conjunction ($P \wedge Q$)
- So both tables are equivalent

Switches		Light Bulb
P	**Q**	**State**
Closed	Closed	On
Closed	Open	Off
Open	Closed	Off
Open	Open	Off

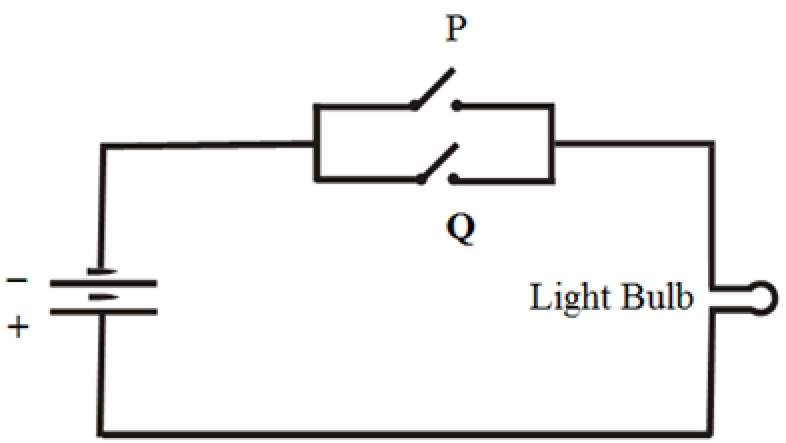
⇔

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Switches in Parallel

- Now switches are connected in parallel
- Light bulb is on when at least one switch is closed
- All the possibilities in the table below

Switches		Light Bulb
P	**Q**	**State**
Closed	Closed	On
Closed	Open	On
Open	Closed	On
Open	Open	Off



Switches in Parallel (Cont.)

- Replace **closed** and **on** by **T** and **open** and **off** by **F** then the table is same as truth table for disjunction ($P \vee Q$)
- So both tables are equivalent

Switches		Light Bulb
P	**Q**	**State**
Closed	Closed	On
Closed	Open	On
Open	Closed	On
Open	Open	Off

⟺

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Digital Logic Gates

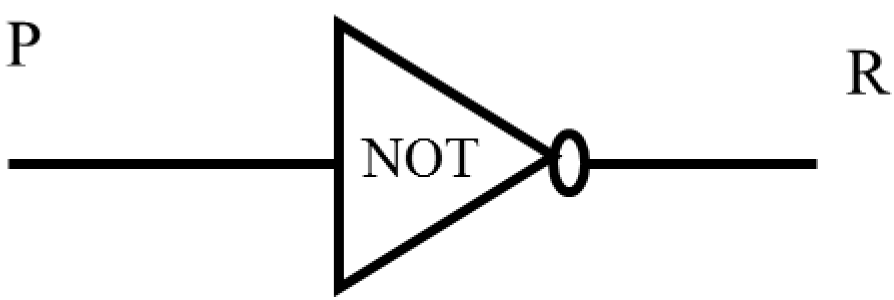
- Digital logic gates basic electronic components of a digital system
- Logic has important role in the digital circuits design
- Digital circuits process discrete, or separate, signals
- Generally bit 1 and bit 0 are used for T and F respectively

Basic Logic Gates

NOT Gate

- NOT gate is a circuit with one input & one output
- Also known as inverter
- If the input signal is 1, the output signal is 0
- Conversely, if the input signal is 0, then the output signal is 1

Symbolic Representation



Input / Output Table

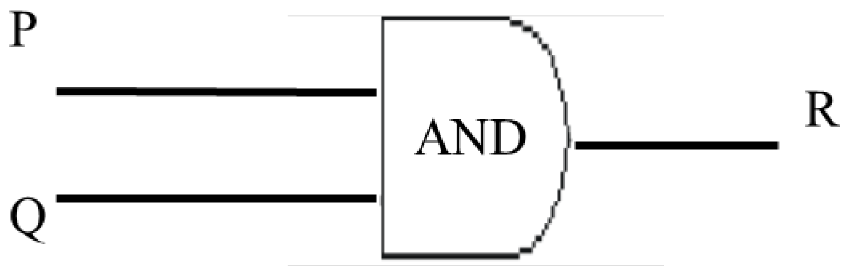
Input	Output
P	R
1	0
0	1

Basic Logic Gates (Cont.)

AND Gate

- AND gate is a circuit with two (or more) inputs and one output
- If both inputs are 1, the output is 1, else, the output is 0

Symbolic representation



Input / Output Table

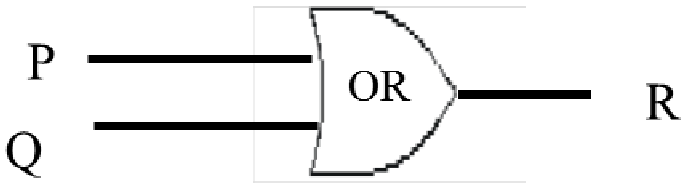
Input		Output
P	Q	R
1	1	1
1	0	0
0	1	0
0	0	0

Basic Logic Gates (Cont.)

OR Gate

- OR gate is a circuit with two (or more) inputs and one output
- If both inputs are 0, then the output is 0, else, the output is 1

Symbolic representation



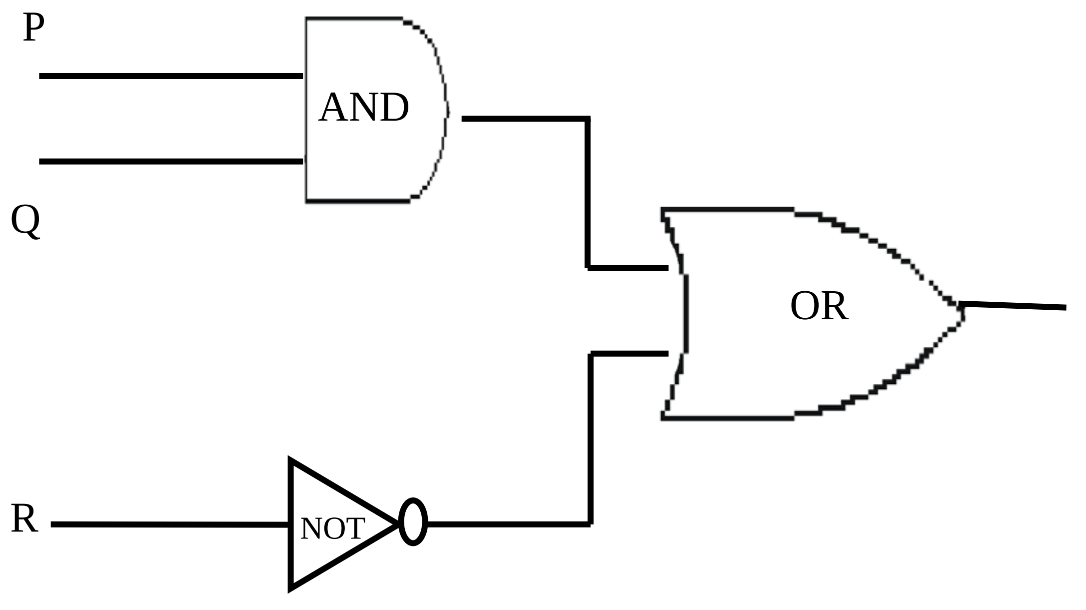
Input / Output Table

Input		Output
P	**Q**	**R**
1	1	1
1	0	1
0	1	1
0	0	0

Combinational Circuit

- A Combinational Circuit is a compound circuit consisting of the basic logic gates such as NOT, AND, OR
- Figure shows a combinational logic circuit for logic expression

  $(P \wedge Q) \vee \sim R$



Rules of Forming Combinational Circuit

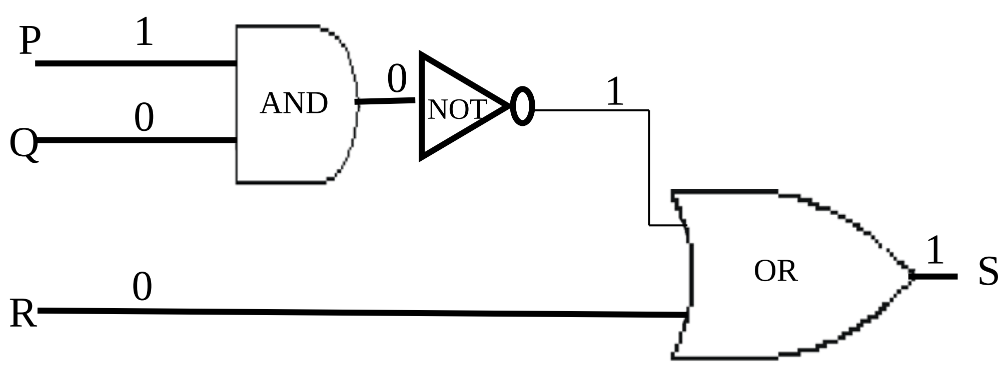
1. Inputs cannot be combined directly.
2. A single input can be split for two separate gates.
3. An output of a gate can be used as input for another gate.
4. No output of a gate can eventually feed back into that gate.

Determining Output for a Given Input

Example: Indicate the output of the circuit below when the input signals are P = 1, Q = 0 and R = 0

Solution:

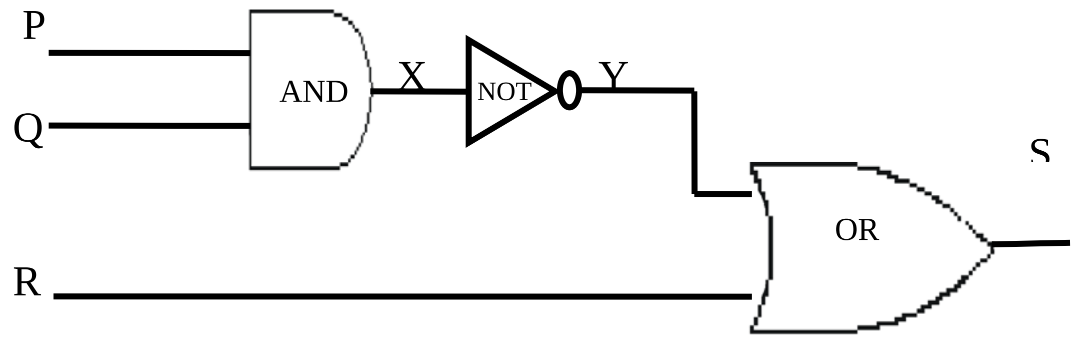
- Inputs are P = 1, Q = 0 and R = 0
- P=1 and Q=0 are Inputs to AND gate so the output will be 0
- This 0 output is the input to the NOT gate ,so output of the is 1
- This 1 output of NOT gate and R = 0 are input to OR gate thus producing a final output of 1



Output S = 1

Constructing the I/O Table for a Circuit

- First label variables to inputs, intermediate outputs, and output as shown in figure
- There are 3 inputs P, Q and R, so I/O table have total $2^3 = 8$ rows
- First, fill the X column, which is the conjunction of P & Q
- Second, fill the Y column which is the negation of X
- Finally, fill the S column, which is the disjunction of Y and R



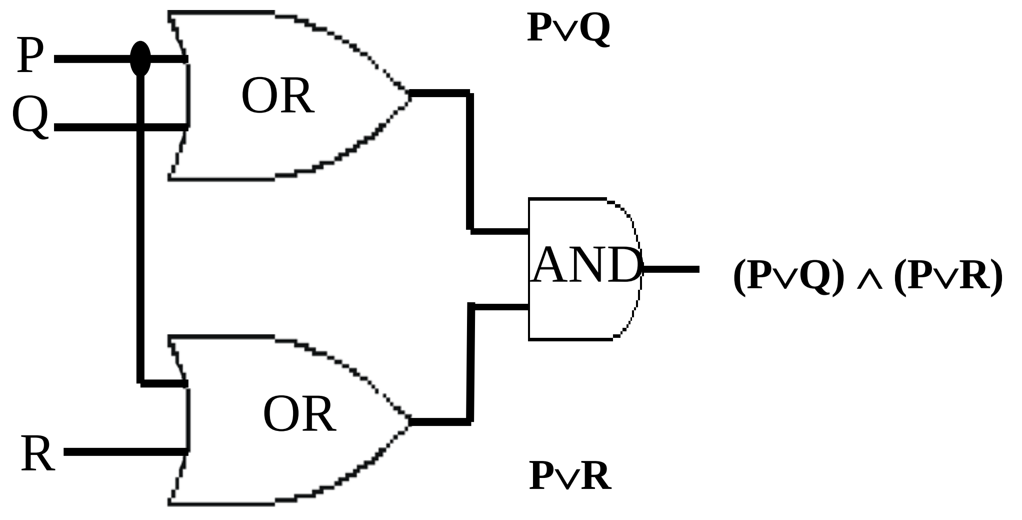
P	Q	R	X	Y	S
1	1	1	1	0	1
1	1	0	1	0	0
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
0	0	0	0	1	1

Boolean Expression

- Boolean expression is composed of Boolean variables and connectives ~, $\wedge$, and $\vee$ is called a Boolean expression
- Boolean variables that can take only two values i.e. 0 and 1
- In logic, variables such as *p*, *q*, and *r* represent statements, and a statement can have only two truth values
- input in a digital circuits, also take only one of the two values of 0 or 1
- So a statement variable or an input is treated as a Boolean variable

Finding a Boolean Expression for a Circuit

- Trace through the circuit from left to right, writing down the output of each logic gate
- Hence **(P∨Q) ∧(P∨R)** is the Boolean expression for this circuit

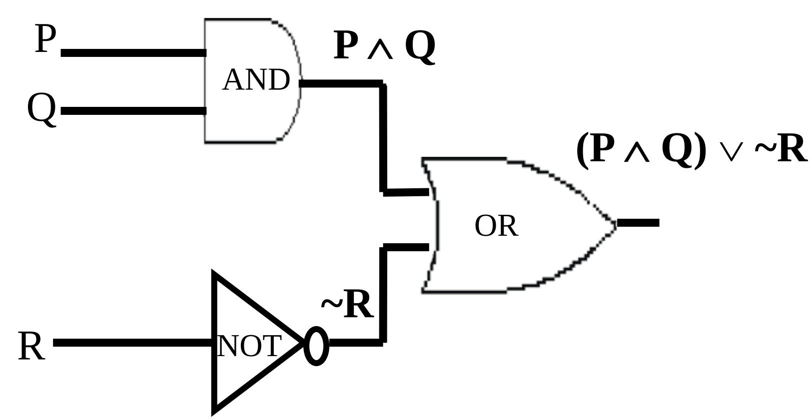


Circuit Corresponding to a Boolean Expression

Example: Construct circuit for the Boolean expression $(P \wedge Q) \vee \sim R$

Solution:

- There are three inputs P ,Q and R in the Boolean expression
- The conjunction b/w P & Q in the parenthesis tells that their must be an AND gate with inputs P and Q
- Then the negation of R gives information that we must have a "NOT gate" to convert the input R into ~R
- Finally, the disjunction b/w $P \wedge Q$ and ~R can be implemented as OR gate with inputs $(P \wedge Q)$ and ~R
- Circuit diagram below, is the implementation of the given Boolean expression

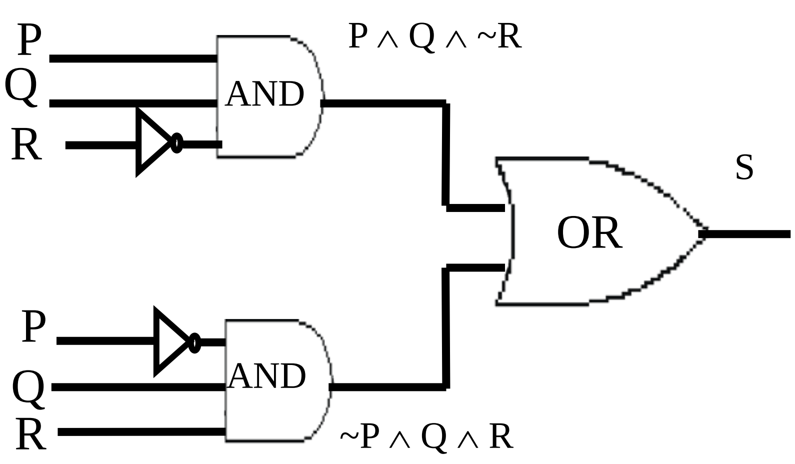


Circuit Corresponding to a Given I/O Table

Example: Design a circuit for the following input/output table.

Solution:

- First, find out the rows in the output columns where output is 1
- Second, write down the Boolean expression for these rows
- Finally, join these expressions using OR operator
- Hence the Boolean expression is $(P \wedge Q \wedge {\sim}R) \vee ({\sim}P \wedge Q \wedge R) = S$
- This Boolean expression can be implemented as logic circuit



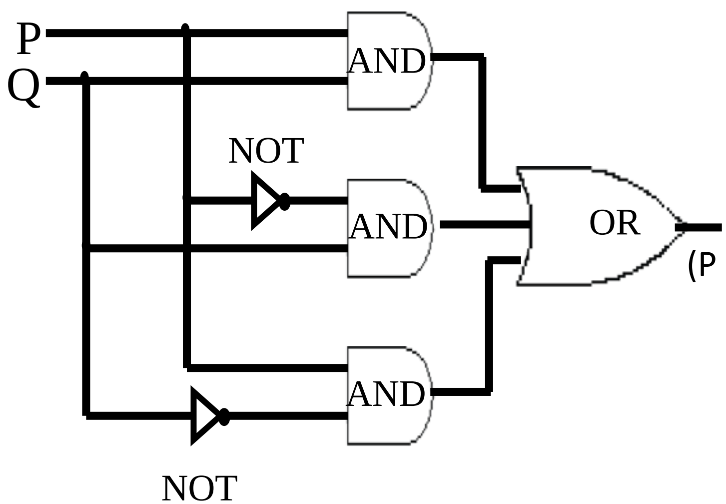
INPUTS			OUTPUT	
P	Q	R	S	
1	1	1	0	
1	1	0	1	$P \wedge Q \wedge {\sim}R$
1	0	1	0	
1	0	0	0	
0	1	1	1	${\sim}P \wedge Q \wedge R$
0	1	0	0	
0	0	1	0	
0	0	0	0	

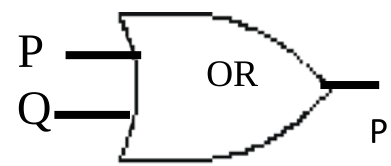
Equivalent Circuits

Example: Show that the following pair of circuits are equivalent.

Solution:

- First, find the Boolean expressions for the circuits
- Then, show that they are logically equivalent using truth table or laws of logic





PTO

Equivalent Circuits (Cont.)

STATEMENT	REASON
$(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$	
$\equiv (P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$	
$\equiv (P \vee \sim P) \wedge Q \vee (P \wedge \sim Q)$	Distributive law
$\equiv t \wedge Q \vee (P \wedge \sim Q)$	Negation law
$\equiv Q \vee (P \wedge \sim Q)$	Identity law
$\equiv (Q \vee P) \wedge (Q \vee \sim Q)$	Distributive law
$\equiv (Q \vee P) \wedge t$	Negation law
$\equiv (Q \vee P) \wedge t$	
$\equiv Q \vee P$	Identity law
$\equiv P \vee Q$	Commutative law

Thus $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \equiv P \vee Q$

Accordingly, the two circuits are equivalent.