Lecture # 05 Discrete Structure

Argument

- Argument is a list of statements (premises or assumptions or hypotheses) followed by a statement (conclusion)
 - P₁ Premise
 - P₂ Premise

P_n Premise

.

.:. C Conclusion

Example:

An interesting teacher keeps me awake.

I stay awake in Discrete Mathematics class.

Therefore, my Discrete Mathematics teacher is interesting.

Valid & Invalid Argument

• Argument is **valid** if the conclusion is true when all the premises are true **or** if conjunction of its premises imply conclusion.

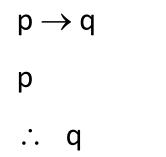
 $(P_1 \land P_2 \land P_3 \land \ldots \land P_n) \rightarrow C \text{ is a tautology.}$

 Argument is invalid if the conclusion is false when all the premises are true or if conjunction of its premises does not imply conclusion.

 $(P_1 \land P_2 \land P_3 \land \ldots \land P_n) \rightarrow C$ is a Contradiction.

- A valid argument may have:
 - true premises and a true conclusion
 - or false premises and a false conclusion
 - or false premises and a true conclusion
 - but it cannot have all true premises and yet a false conclusion
- Arguments may either valid or invalid; and statements may either true or false

Example: Show that the following argument form is valid.



Solution: In first row (critical row), both the premises are T and the conclusion is also T so the argument is valid.

р	q	p→q	р	q
Т	Т	Т	Т	Т
Т	F	F	Т	F
F	Т	Т	F	Т
F	F	Т	F	F

Example: Show that the following argument form is invalid.

 $p \rightarrow q$ p $\therefore q$

Solution: In first and third rows (critical rows) both premises are true, but the conclusion is false in third row so argument is invalid.

р	q	p→q	q	p
Т	Т	Т	Т	Т
Т	F	F	F	Т
F	Т	Т	Т	F
F	F	Т	F	F

Example: If Tariq is not on team A, then Hameed is on team B.

If Hameed is not on team B, then Tariq is on team A.

.:. Tariq is not on team A or Hameed is not on team B.

Solution: Let

t = Tariq is on team A

h = Hameed is on team B

Then the argument is:

 $^{\sim} t \rightarrow h$ $^{\sim} h \rightarrow t$ ∴ $^{\sim} t \lor ^{\sim} h$

Argument is invalid

	t	h	$\sim t \rightarrow h$	$\sim h \rightarrow t$	~t v~h
	Т	Т	Т	Т	F
	Т	F	Т	Т	Т
1	F	Т	Т	Т	Т
1	F	F	F	F	Т

Example: An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class. Therefore, my Discrete Mathematics teacher is interesting.

Solution: Let

t: my teacher is interesting

a: I stay awake

m: I am in Discrete Mathematics class

The argument is:

 $t \rightarrow a$

 $\mathsf{a} \wedge \mathsf{m}$

 $\therefore \quad m \wedge t$

Argument is invalid

t	a	m	$t \rightarrow a$	$a \wedge m$	$m \wedge t$
Т	Т	Т	Т	Т	Т
Т	Т	F	Т	F	F
Т	F	Т	F	F	Т
Т	F	F	F	F	F
F	Т	Т	Т	Т	F
F	Т	F	Т	F	F
F	F	Т	Т	F	F
F	F	F	Т	F	F