Lecture # 05
Discrete Structure

Argument

- **Argument** is a list of statements (**premises** or **assumptions** or **hypotheses**) followed by a statement (**conclusion)**

  P_1 Premise

  P_2 Premise

  . . . .

  P_n Premise

  _____

  $\therefore$ C Conclusion

Example:

An interesting teacher keeps me awake.

I stay awake in Discrete Mathematics class.

Therefore, my Discrete Mathematics teacher is interesting.

Valid & Invalid Argument

- Argument is **valid** if the conclusion is true when all the premises are true **or** if conjunction of its premises imply conclusion.

$$(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \rightarrow C \text{ is a tautology.}$$

- Argument is **invalid** if the conclusion is false when all the premises are true **or** if conjunction of its premises does not imply conclusion.

$$(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \rightarrow C \text{ is a Contradiction.}$$

- A **valid argument** may have:
  - true premises and a true conclusion
  - or false premises and a false conclusion
  - or false premises and a true conclusion
  - but it cannot have all true premises and yet a false conclusion
- Arguments may either valid or invalid; and statements may either true or false

Valid & Invalid Argument (Cont.)

Example: Show that the following argument form is valid.

$p \rightarrow q$

p

$\therefore \quad q$

Solution: In first row (critical row), both the premises are T and the conclusion is also T so the argument is valid.

p	q	p→q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Valid & Invalid Argument (Cont.)

Example: Show that the following argument form is invalid.

$p \rightarrow q$

p

$\therefore \quad q$

Solution: In first and third rows (critical rows) both premises are true, but the conclusion is false in third row so argument is invalid.

p	q	p→q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Valid & Invalid Argument (Cont.)

Example: If Tariq is not on team A, then Hameed is on team B.

If Hameed is not on team B, then Tariq is on team A.

$\therefore$ Tariq is not on team A or Hameed is not on team B.

Solution: Let

t = Tariq is on team A

h = Hameed is on team B

Then the argument is:

$\sim t \rightarrow h$

$\sim h \rightarrow t$

$\therefore \sim t \vee \sim h$

Argument is invalid

t	h	$\sim t \rightarrow h$	$\sim h \rightarrow t$	$\sim t \vee \sim h$
T	T	T	T	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Valid & Invalid Argument (Cont.)

Example: An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class. Therefore, my Discrete Mathematics teacher is interesting.

Solution: Let

t: my teacher is interesting

a: I stay awake

m: I am in Discrete Mathematics class

The argument is:

$t \rightarrow a$

$a \wedge m$

$\therefore \quad m \wedge t$

Argument is invalid

t	a	m	$t \rightarrow a$	$a \wedge m$	$m \wedge t$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
T	F	F	F	F	F
F	T	T	T	T	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F