Lab #2

Matrices, solution of matrices and their operations using MATLAB

Objective:

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Matrix:

MATLAB treats every thing as a matrix

- 1-by-1 matrices are interpreted as scalars
- Matrices with only one row or one column are known as vectors
- A matrix is a rectangular array of numbers.

Accessing Matrices:

- The matrix element located in the i-th row and j-th column of "A" is referred to as, A(i,j)

Example

– How to enter a matrix-

For example, the matrix A which is mathematically defined by

$$A = \begin{bmatrix} 1 & 2 & 9 \\ 4 & 7 & 5 \\ 3 & 1 & 6 \end{bmatrix}$$ is described in MATLAB by

$$>> A = \left[\underbrace{1\,2\,9}_{first\ row}; \ \underbrace{4\,7\,5}_{second\ row}; \ \underbrace{3\,1\,6}_{third\ row} \right]$$ % create a 3-by-3 square matrix which is named A

This results in a 3x3 matrix, which looks like

```
A =
     1     2     9
     4     7     5
     3     1     6
```

MATLAB displays matrices without braces

Building Large Matrices:

Large matrices can be assembled from smaller matrix blocks:

Example

- Adding a row to an existing matrix-

```
>>A=[1 2 9; 4 7 5; 3 1 6];
>>B=[A; 11 12 13]                    % add one row to matrix A
 B =
        1     2     9
        4     7     5
        3     1     6
       11    12    13
```

```
>>C=[A; A; A]

 C =
     1     2     9
     4     7     5
     3     1     6
     1     2     9
     4     7     5
     3     1     6
     1     2     9
     4     7     5
     3     1     6
```

Size Command:

We can determine the size of a vector or matrix by using the size command.

```
>>size(A)          % return the size of A
>>size(A,1)        % return the number of rows in A
>>size(A,2)        % return the number of columns in A
```

Individual Elements:

Individual elements of a matrix can be referenced via indices enclosed within parentheses. The first index refers to the row number, and the second index refers to the column number.

Example

- Reference Individual Elements-

```
>>A(2,1)                  % reference the second element of the first row

results in

ans =
    4
```

Special Matrices:

Four Kinds of Special Matrices:

- A diagonal matrix is a matrix where only the diagonal entries are non-zero.
- An identity matrix, I, is the diagonal matrix with diagonal consisting of all 1's.
- An upper triangular matrix is a matrix whose entries lying below the diagonal are all zero.
- A lower triangular matrix is a matrix whose entries lying above the diagonal are all zero.

A. Diagonal Matrix:

The diagonal matrix A is one whose elements off the main diagonal are all equal to zero, while those along the main diagonal are non-zero. The command diag will generate a diagonal matrix with the specified elements on the main diagonal.

Practice

-Special Matrices: The "diag" Command for Diagonal Matrix-

```
>>A=diag([1 2 3])          % generate a diagonal matrix
```

B. Identity Matrix

If A is any matrix, the identity matrix for multiplication is a matrix I which satisfies the following relation. AI = A and IA = A This matrix, called the identity matrix, is the square matrix.

Commands for Special Matrices

MATLAB has several build-in matrices.

- The command **eye(n)** produces a n-by-n identity matrix.
- The **zero(n,m)** and **ones(n,m)** command will generates an n-by m matrices willed with zeros, and filled with ones, respectively.
- The **rand(n)** command will generate an n-by-n matrix whose elements are pseudo-random numbers uniformly distributed between 0 and 1, while **rand(n,m)** will create a n-by-m matrix with randomly generated entries distributed uniformly between 0 and 1.
- The **magic(n)** command generate a n-by-n square matrix whose entries constitute a magic square; i.e., the sum of elements along each row, column, or principal diagonal is the same value.

Practice

- Commands for Special Matrices-

(1)

```
>>D=eye(3)   <enter>            % create a 3-by-3 identity matrix

D =
       1     0     0
       0     1     0
       0     0     1

>>H=zeros(2,3)   <enter>        % create a 2-by-3 null matrix

H =
       0     0     0
       0     0     0

>>W=ones(2,3)   <enter>         % create a 2-by-3 matrix with entries equal to 1

W =
       1     1     1
       1     1     1
```

Matrix Operations:

Addition of Matrices Matrix addition can be accomplished only if the matrices to be added have the same dimensions for rows and columns. If A and B are two matrices of the same size, then the sum A+B is the matrix obtained by adding the corresponding entries in A and B.

Practice

- Matrix Operations: Addition of Matrices-

```
>>A=[3 4; 5 6];              % define matrix A
>>B=[1 2; 9 7];              % define matrix B
>>C=A+B   <enter>            % compute the sum

C =
      4       6
     14      13
```

Difference of Matrices:

If A and B are matrices with the same size, then the difference between A and B denoted A-B is the matrix defined by A-B=A+(-B).

Practice

- Matrix Operations: Difference of Matrices-

```
>>A=[3 4; 5 6];              % define matrix A
>>B=[1 2; 9 7];              % define matrix B
>>D=A-B   <enter>            % compute the difference

D =
      2       2
     -4      -1
```

Product of Matrices:

We can perform the product C=A x B only if the number of rows of B, the right matrix, is equal to the number of column of A, the left matrix.

Practice

-Matrix Operations: Product of Matrices

For the given matrices, obtain the product C=A*B

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix},\quad B=\begin{bmatrix}-2 & 3\\ 4 & 1\end{bmatrix}$$

```
>>A=[1 2; 3 4; 5 6];        % define matrix A
>>B=[-2 3; 4 1];            % define matrix B
>>C= A*B   <enter>          % compute the product matrix C

C =
      6      5
     10     13
     14     21
```

Multiplication by a Scalar:

If A is a matrix and k is a scalar, then the product k* A is defined to be the matrix obtained by multiplying each entry of A by the constant k.

Practice

- Matrix Operations: Multiplication by a Scalar-

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix},\quad B=\begin{bmatrix}7 & 0\\ 5 & 9\end{bmatrix}$$

```
>>A=[1 2; 3 4];             % define matrix A
>>B=[7 0; 5 9];             % define matrix B
>>C=2*A                     % scale A by 2
>>D=3*A+2*B                 % scale A by 3, B by 2, and add the results
```

answers

```
C =
      2      4
      6      8
D =
     17      6
     19     30
```

Determinant of A Matrix:

Associated with any square matrix A is a scalar quantity called the determent of the matrix A. A matrix whose determinant is non-zero is called a non-singular or invertible, otherwise it is called singular.

The command **det** evaluates the determinant of a square matrix.

Practice

-Determinant of a Matrix-

Find the determinant of the square matrix A depicted below.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 1 & 2 \end{bmatrix}$$

```
>>A=[ 1 2 3; 4 5 6; 7 1 2];          % specify matrix A
>>det(A)                             % compute the determinant

ans =
      -21
```

Rank of A Matrix:

The rank of a matrix, A, equals the number of linearly independent rows or columns. The rank can be determined by finding the highest-order square sub-matrix that is non-singular. The command rank provides the rank of a given matrix.

Practice

- Rank of a Matrix-

```
>>A=[1 2 3; 4 5 6; 7 8 9];           % define matrix A
>>rank(A)                            % determine the rank of matrix A
```

Inverse of A Matrix:

If A and B are square matrices such that AB=BA=I, then matrix B is called the inverse of A and we usually write it as 1 B A− = . Only a square matrix whose determinant is not zero has an inverse. Such a matrix is called nonsingul

Practice

-Inverse of Matrix: The "inv" Command-

Find the inverse of the matrix A given by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 2 \end{bmatrix}$$

```
>>A=[1 2 3; 4 5 6; 7 8 2];          % define matrix A
>>B=inv(A)   <enter>                % compute the inverse of A store result in B
B =
   -1.8095     0.9524    -0.1429
    1.6190    -0.9048     0.2857
   -0.1429     0.2857    -0.1429
```

Transpose of A Matrix:

The transpose of a matrix is the result of interchanging rows and columns. The transpose is found by using the prime operator (apostrophe), [']. In particular, the transpose of a row vector is equal to a column vector and vice versa.

Practice

- Transpose of a Matrix-

Find the transpose of the matrix A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 2 \end{bmatrix}$$

```
>>A=[1 2 3; 4 5 6; 7 8 2];       % specify matrix A
>>B=A'   <enter>                 % compute the transpose of A and store result in B

B =
      1      4      7
      2      5      8
      3      6      2
```

Trace of a Matrix:

The trace of a square matrix is equal to the sum of its diagonal elements. The trace command provides the trace of a given matrix.

Practice

- Trance of a Matrix: The "trace" Command-

```
>>% compute the trace of matrix
 >>A=[1 2 3; 4 5 6; 7 8 2];          % define matrix A
 >>trace(A)   <enter>                % calculate the trace of matrix A

ans =
      8
```

Solving Systems Of Linear Equations:

To solve the linear system Ax=b, where A is known n-by-n matrix, b is known column vector of length n, and x is an unknown column vector of length n.

Ax=b

Multiply each side by the inverse matrix A^{-1}

$\therefore A^{-1}Ax = A^{-1}b$

but $A^{-1}A = I \Rightarrow Ix = A^{-1}b$

$\therefore x = A^{-1}b$

Example:

A system of 3 linear equations with 3 unknowns (x1, x2, x3):

$3x_1 + 2x_2 + x_3 = 10$

$-x_1 + 3x_2 + 2x_3 = 5$

$x_1 - x_2 - x_3 = -1$

Let:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 3 & 2 \\ 1 & -1 & -1 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad b = \begin{bmatrix} 10 \\ 5 \\ -1 \end{bmatrix}$$

$$Ax = b$$

Solution by left division in MATLAB:

```
>> A = [ 3 2 -1; -1 3 2; 1 -1 -1];

>> B = [ 10; 5; -1];

>> x = A\B

x=

-2.0000

5.0000

-6.0000
```

Solution by Matrix Inverse in MATLAB:

```
>> A = [3 2 -1; -1 3 2; 1 -1 -1];

>> B = [10; 5; -1];

>> x = inv (A)*B

x=

-2.0000

5.0000

-6.0000
```

Post Lab Questions

1. **Which is the operator by which element to element multiplication can be done using MATLAB?**

   ____

2. **A(i,j), What does i and j represent?**

   ____

   ____

3. **What do the following basic Matrix functions represent**

det	
rank	
ones	
rand	

Lab Tasks

Task 1

a) Generate a vector of 50 elements having random values between 0 and 50.

b) What do the following commands generate:

- m=magic(4)
- sum(m)
- diag(m)
- trace(m)
- ones(3),ones(3,2)
- zeros(3),zeros(3,2)

Task 2

$$A = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix} \qquad B = \begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{matrix} \qquad C = \begin{matrix} 1 & 2 & 1 & 2 \end{matrix}$$

a) Practice the following Matrix operations on the given Matrices A, B and C:

- A+B
- A’
- A*B
- 2*A
- A/2
- C.^2

b) Find the size of Matrix C and also generate an identity Matrix.

Task 3

Solve the following system of linear equation.

$$\begin{cases} x+5y+7z=1 \\ 3x+2y+4z=2 \\ 7x+9y+z=3 \end{cases}$$