

Lab # 1

Transfer Function Representation, Transfer Function From Poles and Zeros and Zeros And Poles From Transfer Function

Objective:

Transfer Function:

Transfer function of a system is defined as the ratio of output to input in frequency domain. The transfer functions are defined in MATLAB by storing the coefficients of numerator and denominator in vector form.

The transfer function is commonly used in the analysis of single-input single-output electronic system, for instance. It is mainly used in signal processing, communication theory, and control theory. The term is often used exclusively to refer to linear time-invariant systems (LTI). In its simplest form for continuous time input signal $x(t)$ and output $y(t)$, the transfer function is the linear mapping of the Laplace transform of the input, $X(s)$, to the output $Y(s)$.

Zeros and Poles:

Zeros are the value(s) for z where the numerator of the transfer function equals zero. The complex frequencies that make the overall gain of the filter transfer function zero. Poles are the value(s) for z where the denominator of the transfer function equals zero. The complex frequencies that make the overall gain of the filter transfer function infinite.

Transfer Function Representation:

Generally a transfer function is represented as

$$H(s) = B(s) / A(s)$$

Where B and A are defined as

$$B = b_m S^m + b_{m-1} S^{m-1} + b_{m-2} S^{m-2} + \dots$$
$$A = a_m S^m + a_{m-1} S^{m-1} + a_{m-2} S^{m-2} + \dots$$

Coefficients of A and B are stored in vectors and then used forward.

Example:

Given the following transfer function find the zeros, poles and gain:

$$H(S) = \frac{2S^2 + 3}{S^3 + 4S^2 + 5}$$

We implement the transfer function in Matlab as

```
num=[2 0 3];           %Coefficients of the numerator  
den=[1 4 0 5];        %Coefficients of the denominator
```

To convert the vectors in transfer function we use:

```
h=tf(num,den);         %Transfer function
```

Output:

```
num =  
      2      0      3  
  
den =  
      1      4      0      5  
  
h =  
  
      2 s^2 + 3  
-----  
      s^3 + 4 s^2 + 5
```

Zero,poles and gain can be found using the following

```
[z,p,k]=tf2zp(num,den);
```

```
z =
```

```
0 + 1.2247i
```

```
0 - 1.2247i
```

```
p =  
-4.2737  
0.1369 + 1.0729i  
0.1369 - 1.0729i
```

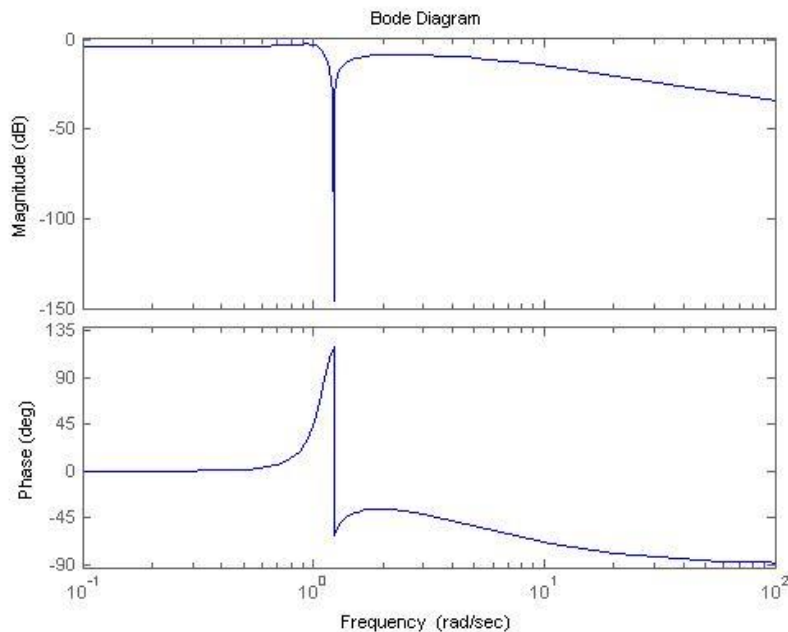
```
k =  
2
```

Transfer function can be obtained back from zero poles and gain.

```
[num,den]=zp2tf(z,p,k);  
num =  
0 2.0000 0 3.0000  
  
den =  
1.0000 4.0000 0 5.0000  
  
tf(num,den)  
  
Transfer function:  
2 s^2 + 3  
-----  
s^3 + 4 s^2 + 5
```

To draw the poles, zeros, gain and phase of the transfer function, we draw bode plot using following function

```
Bode(num,den)
```



Laplace Transform

The general procedure to find the transfer function of a linear differential equation from input to output is to take the Laplace Transforms of both sides assuming zero conditions, and to solve for the ratio of the output Laplace over the input Laplace.

Laplace transform is used to transform a function/signal from time domain to s domain (complex domain). System can be analyzed and performed different operations in S domain and then can be taken back into t domain. Laplace transform converts the function expression into simple algebraic polynomial. Following expression is used to calculate the Laplace transform of a time domain function f(t).

$$F(S) = \int_{-\infty}^{\infty} f(t)e^{-st} dt$$

In Matlab we solve the Laplace transform using symbolic math toolbox. Function used for finding the Laplace is Laplace().

Example:

Let f(t)=t and we have to find the Laplace transform of the function. Matlab code for this is given below.

```
Syms t;
```

F=laplace(t)

Output returned is

$$F=1/S^2$$

Example:

Laplace transform of e^{-at} is calculated as:

Syms a t s

f=exp(-a*t);

F=laplace(f);

Output comes out to be

$$F=1/s+a$$

Inverse Laplace Transform

Inverse Laplace transform is used to transform s domain(complex domain) to time domain. Following is the relation for the inverse laplace transform.

$$f(t) = \frac{1}{2\pi i} \lim_{T \rightarrow \infty} \int_{\gamma-iT}^{\gamma+iT} F(S)e^{st} ds$$

In Matlab we find out the inverse laplace transform using symbolic math toolbox. ilaplace is function for finding the inverse laplace.

Example:

Given below the transfer function

$$H(S) = \frac{(S+3)}{(S+1)(S+2)}$$

Find the inverse laplace.

Matlab Code is given below

Syms s

H=(s+3)/((s+1)*(s+2));

F=ilaplace(H);

Output comes out as:

$$F=2*\exp(-t)-\exp(-2*t)$$

In finding the inverse Laplace often we encounter improper fractions having numerator power greater than denominator. For such fractions we first divide the polynomials (numerator and denominator) and then make partial fractions before taking inverse Laplace.

Post Lab Questions

a) What is Transfer function?

b) Which command is used to find Zero,poles and gain?

c) What is zp2tf used for?

d) What is command 'bode' used for?

Lab Tasks

Task 1

- a) Find the poles and zeros of the following transfer function. Find the transfer function back from the obtained zeros and poles.

1.

$$G(s) = \frac{s^2 - 3s - 4}{s^2 + 5s + 6}$$

2.

$$\frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$$

Task 2

- a) Using the following parameters find the transfer function, also plot the given transfer function:
numerator = 1
denominator = 2 ; 5 ; 3;

Task 3

- a) Find the Laplace Transform of the following:

i) $f(t) = 3e^{-8t}$

ii) $f(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-2t}$