**Lab # 9**

**Convolution of signals.**

**Objective:**

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**LTI systems:**

- Systems that are both linear and time invariant are called LTI systems.

- The behavior of LTI systems is completely characterized by their impulse response.
- The input and output of an LTI system is related by convolution sum/integral.

**Impulse Response:**

- Impulse response ' h[n] ', is the output of an LTI system, when the input is a unit impulse.

- Given the impulse response, we can find the output for any input using convolution.

**Difference equation:**

- A very common representation of LTI systems is in the form of difference equation.
- The general difference equation is$\Sigma$ aky[n-k] = $\Sigma$ bkx[n-k]

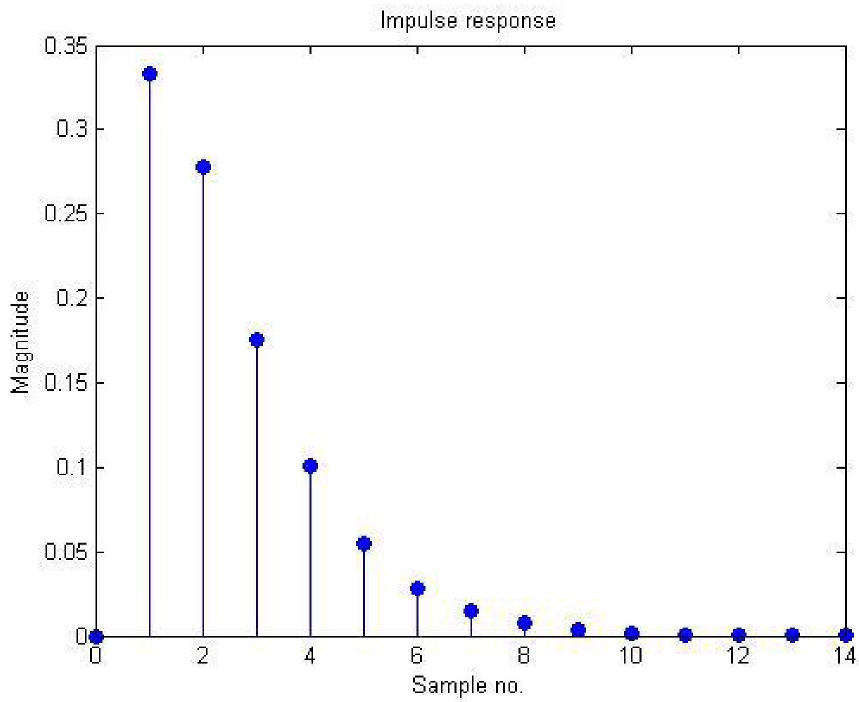
**Example:**

for , y[n] –5/6y[n-1] + 1/6y[n-2] = 1/3x[n-1]
a0=1, a1=-5/6, a2=1/6
and
bo=0, b1=1/3

**Plot the impulse response of the following difference equation**

- **y[n] –5/6y[n-1] + 1/6y[n-2] = 1/3x[n-1]**

```
a=[1 -5/6 1/6];
b=[0 1/3];
[H,T]=impz(b,a);
stem(T,H);
```




## Convolution

- Consider a discrete time system with input x[n] and output t[n].

- When Impulse response is given we can find out the system output by following relation

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

For continuous signal, output is computed through following relation.

$$y(t) = \int_{-\infty}^{\infty} x(T)h(t-T)dT$$

- Here y[n] is the output signal, x[n] is the input signal, and h[n] is the impulse response of the LTI system.

- In MATLAB use the instruction **' y=conv(x,h)'** to perform convolution.

- It assumes that the time increment is the same for both signals.

**Convolution using Matlab:**

- To perform discrete time convolution, x[n]*h[n], define the vectors x and h with elements in the sequences x[n] and h[n]
- Then use the command y = conv(x,h) This command assumes that the first element in x and the first element in h correspond to n = 0, so that the first element in the resulting output vector corresponds to n = 0 .
- If this is not the case, then the output vector will be computed correctly, but the index will have to be adjusted.
- The command Conv () can also be used to multiply polynomials.
- Suppose the coefficients of the polynomial a are given in vector A and that of b are given in B. then coefficients of the output polynomial can be found out as:

For Example

```
a(s)=S+1
b(s)=S+2

then
A=[1 1 ];
B=[1 2 ];
ab=conv(A,B)

output comes out to be
ab=[1 3 2]
```

**Example:**

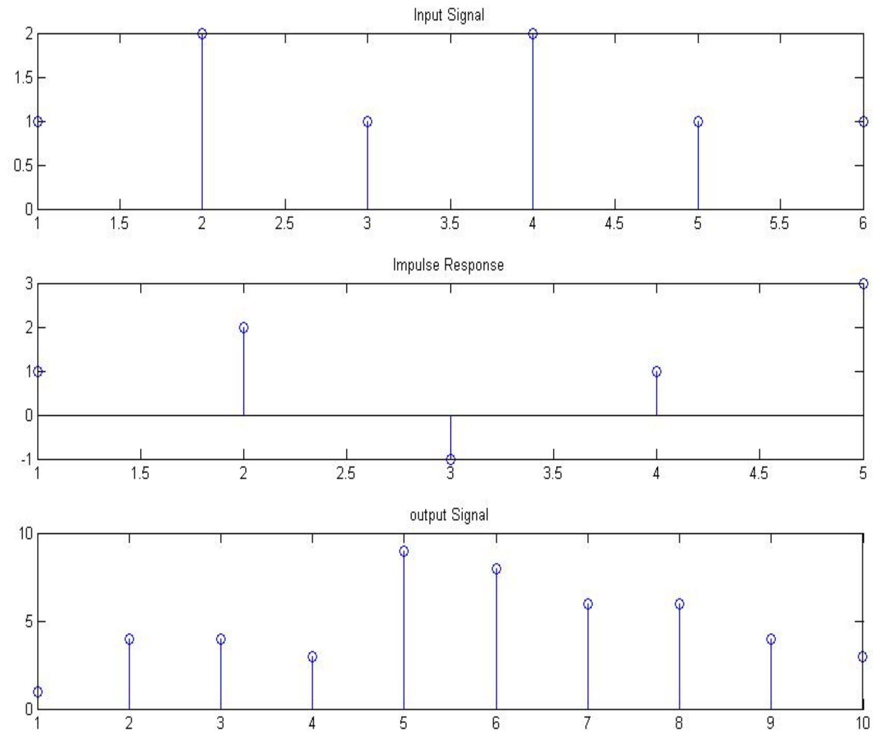
Given the following input signal for discrete LTI system and impulse response.

$x[n]=[1,2,1,2,1,1]$

$h[n]=[1,2,-1,1,3]$

Code to find the convolved signal is given below.

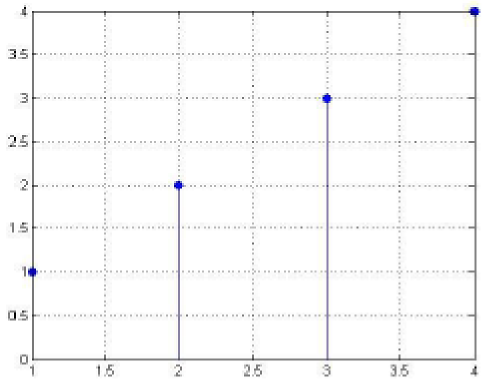
```
x=[1,2,1,2,1,1];
h=[1,2,-1,1,3];
y=conv(x,h);
subplot(311);stem(x);title('Input Signal');
subplot(312);stem(h);title('Impulse Response');
subplot(313);stem(y);title('output Signal');
```

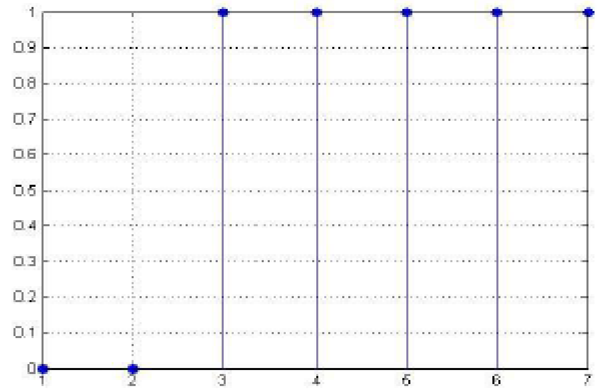


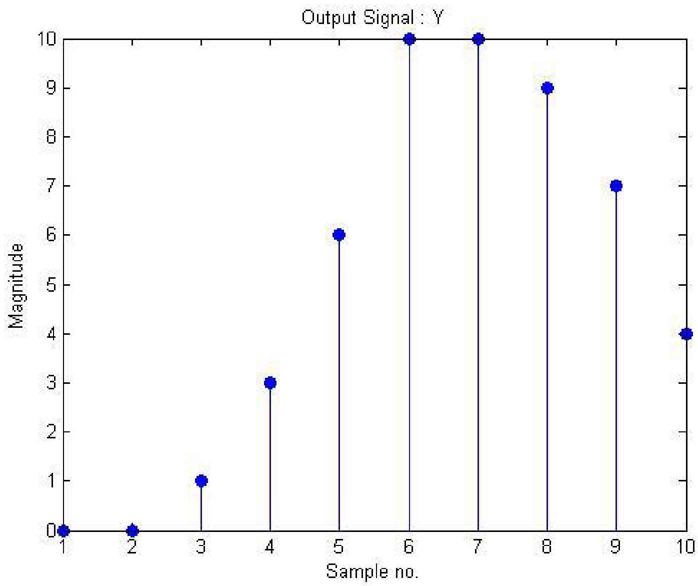

If x[n] and h[n] are of different lengths or different starting points then the output will be computed correctly but the indices would have to be adjusted. For example: if x[n] starts from n=-1 and h[n] starts from n=-3 then the output signal will start from n=-4.

<u>Convolve the following two sequences in MATLAB.</u>

1. h=[1 2 3 4];
2. subplot(3,1,1)
3. stem(h)
4. x=[0 0 1 1 1 1 1];
5. subplot(3,1,2)
6. stem(x)
7. y=conv(x,h);
8. subplot(3,1,3)
9. stem (y)








**NOTE**

- MATLAB assumes that both the convolving signals are starting from zero index, hence the time/sample no. of the output signal is not correct always

- The input signals are finite-length, so the result of the convolution should have a length equal to the sum of the lengths of the inputs– which turns out to be:

  Length of y = length of (x) + length of (h) - 1 and the starting index for y will be the sum of starting indices of x and h

# Post Lab Questions

a) **What is an LTI System?**

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b) **Define Convolution.**

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c) **Show the following with the help of a block diagram:**

i. y(t)=x(t)*h(t)

ii. yn)=x(n)*h(n)

# **Lab Tasks**

## **Task 1**

a) Find the output of the LTI system when x(n)= {0,1 2,3,4} and h(n)={0,1,2,3}

b) Convolution is associative. Given the three signals x1[n], x2[n], and x3[n] as:
x1= [ 3,1,1]
x2= [ 4,2,1]
x3= [ 3,2,1,2,3]
Show that **(x1*x2)*x3= x1*(x2*x3)**

c) Convolution is commutative. Given x and as:
x=[1,3,2,1]
h=[1,1,2]
**Show that x* h = h*x**

## **Task 2**

a) Find the output signal y[n] for any range of 'n' using convolution, for the following case: (assume -10 ≤n≤10)

i. $h[n] = 5(-1/2)^n u[n]$ , $x[n] = (1/3)^n u[n]$